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PREDICTIVE CONTROL OF SUPERCAVITATING VEHICLES BASED ON TIME-DELAY LPV MODEL

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Abstract. *Supercavitation technology can greatly improve the speed of underwater vehicles by reducing resistance. However, the time-delay characteristics of supercavitating vehicle make the control extremely difficult. In this paper, the linear parameter variation (LPV) time-delay model of supercavitating vehicle is established by analyzing the time-delay characteristics of planing force. In addition, considering that the external irregular disturbance will cause the perturbation of time-varying parameters in the model, a supercavitating LPV delay model depicted as a polytope is established. Based on this model, a Quasi-Min-Max robust model predictive controller based on time-delay characteristics is designed. The algorithm decomposes the "worst case" in infinite time domain into current performance index and terminal state penalty term, calculates the upper bound of the new maximum problem, and deduces the sufficient condition for the closed-loop asymptotic stability of the system. The current control input can be obtained by solving the convex optimization of linear matrix inequality (LMI) online. Finally, the simulation results show that the algorithm has good stability and robustness.*

Keywords: Supercavitating vehicle, Planing force, LPV time-delay model, Quasi-Min-Max predictive controller, LMI, Robustness

1. **Introduction.** The supercavitation technology can greatly reduce the water resistance by forming a cavitation layer on the water and the surface of the vehicle, so as to realize the speed leap of the underwater vehicle. However, the inclusion of cavitation makes the dynamic characteristics of the vehicle become more complex while reducing the resistance. The tail beat phenomenon caused by unpredictable disturbances in motion will lead to nonlinear planing force [1]. On the basis of fully understanding the mechanism of cavitation formation, scholars all over the world have carried out research on supercavitating vehicle motion model [2,3] and control [4,5], among which planing force has become one of the main difficulties in supercavitating vehicle modeling and stability control due to its strong nonlinear and time-delay characteristics.

[6] proposed a nonlinear robust integrated control method to provide strong performance against large planing force and parameter uncertainty. The performance index can suppress external interference through disturbance. [7] studied the nonlinear dynamic behavior of a supercavitating vehicle with time-delay and varying system parameters, and analyzed its local and global stability from the perspective of different parameter values and initial conditions. A new fractional order model of supercavitating vehicle with memory property is proposed in [8], which considers the effect of advection delay while maintaining the nonlinearity of mathematical equations. In [9], a robust predictive controller is designed based on supercavitating LPV time-delay model described by polytope. Different Lyapunov functions and free control variables are introduced to solve the LMI

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to find the optimal solution at a given time. In [10], an adaptive sliding mode control strategy based on radial basis function (RBF) neural network was proposed for the model uncertainty of supercavitating vehicle and considering external interference.

In this paper, considering the memory effect of the supercavitating vehicle, combined with the advantages of strong robustness, good stability, excellent dynamic performance of predictive control [12], the time-delay LPV model of the supercavitating vehicle is established. Based on this model, a Quasi-Min-Max robust predictive controller based on time-delay characteristics is designed. Compared with other controllers, the controller designed in this paper obtains the optimal control law online, which can improve the dynamic performance of the system. At the same time, the controller also reduces the burden of online calculation by obtaining the upper bound of the new maximum problem, and effectively solves the problems of time delay, cavitation disturbance and controller constraints of supercavitating vehicles.

2. **Supercavitating Vehicle Models.** In this study, the analysis of the delay characteristics of the system is added on the basis of the model in [1]. Assuming that the speed of the vehicle remains constant and the vehicle is completely enveloped by cavitation, according to the dynamic analysis of the pitch plane of supercavitating vehicle, the following model can be established:

$$M_0 \begin{bmatrix} \dfrac{dw}{dt} \\ \dfrac{dq}{dt} \end{bmatrix} = A_0 \begin{bmatrix} w \\ q \end{bmatrix} + B_0 \begin{bmatrix} \delta_f \\ \delta_c \end{bmatrix} + F_g + \hat{F}_p \begin{bmatrix} 1 \\ L \end{bmatrix}; \ \frac{dz}{dt} = w - V\theta; \ \frac{d\theta}{dt} = q \tag{1}$$

where $[z,\ w,\ \theta,\ q]$ are respectively the longitudinal depth, vertical velocity, pitch angle and pitch angular velocity of the vehicle relative to the launch point; t is the time, V is the forward velocity, δ_c and δ_f are respectively the deflection angle of the axis of the cavitator relative to the centerline of the vehicle and the deflection angle of the tail rudder relative to the centerline of the vehicle.

$$A_0 = CV \begin{bmatrix} \dfrac{-n}{m} & \dfrac{-n}{m} + \dfrac{7}{9C} \\ \dfrac{-n}{m} & \dfrac{-nL}{m} + \dfrac{17}{36C}L \end{bmatrix}; \ B_0 = CV^2 \begin{bmatrix} \dfrac{-n}{mL} & \dfrac{-1}{mL} \\ \dfrac{-n}{m} & 0 \end{bmatrix}; \ F_g = \begin{bmatrix} \dfrac{7}{9} \\ \dfrac{17L}{36} \end{bmatrix} g$$

$$M_0 = \begin{bmatrix} \dfrac{7}{9} & \dfrac{17L}{36} \\ \dfrac{17L}{36} & \dfrac{11}{16}R^2 + \dfrac{133}{405}L^2 \end{bmatrix}; \ C = \frac{1}{2}C_x \left(\frac{R_n}{R}\right)^2 = \frac{1}{2}C_{x0}(1+\sigma)\left(\frac{R_n}{R}\right)^2 \tag{2}$$

where m is the ratio of vehicle to fluid density; L is the length of the vehicle; R_n is the cavitator radius, R is the vehicle radius, n represents the effectiveness coefficient of the fins rudder, σ is the cavitation number and C_{x0} is the lift coefficient.

In [11], it is verified that when the vehicle's time-delay model and non time-delay model are both stable, there is a big difference between the response speed and the tail beat frequency. Therefore, the nonlinear planing force should be decomposed to build a time-delay model for analysis. The submergence depth of the vehicle tail h'_p and planing angle α_p are defined by the following formula

$$h'_p = \begin{cases} [z(t) + \theta(t)L - z(t-\tau) + R - R_c]/R, & \text{bottom contact} \\ 0, & \text{inside cavity} \\ [R - R_c - z(t) - \theta(t)L + z(t-\tau)]/R, & \text{top contact} \end{cases} \tag{3}$$

$$\alpha_p = \begin{cases} \theta(t) - \theta(t-\tau) + \left[w(t-\tau) - \dot{R}_c\right] \Big/ V, & \text{bottom contact} \\ 0, & \text{inside cavity} \\ \theta(t) - \theta(t-\tau) + \left[w(t-\tau) + \dot{R}_c\right] \Big/ V, & \text{top contact} \end{cases} \tag{4}$$

where R_c denotes the cavitation radius; $\dot{R}_c$ is the contraction rate of the cavitation at the planing location; $\tau = L/V$ is approximately the time delay.

The conditions for the existence of planing force are as follows:

$$\begin{cases} R - R_c < z(t) + \theta(t)L - z(t-z), & \text{bottom contact} \\ R_c - R > z(t) + \theta(t)L - z(t-z), & \text{top contact} \end{cases} \tag{5}$$

The planing force acting on the vehicle in the vertical direction can be expressed as

$$\begin{aligned} &F_p = \pi\rho R_c^2 V^2 \sin\alpha_p \cos\alpha_p \cdot \left[\left(1+h_p'\right) / \left(1+2h_p'\right)\right] \Big/ \left[1 - \left(R' / \left(h_p' + R'\right)\right)^2\right] \\ &\hat{F}_p = F_p / \pi\rho R^2 mL \end{aligned} \tag{6}$$

Define the following transformation:

$$\begin{aligned} &\gamma_1 = \left(\frac{1+h_p'}{1+2h_p'}\right)\left[1 - \left(\frac{R'}{h_p' + R'}\right)^2\right] \\ &\gamma_2 = \begin{cases} -\dot{R}_c/V, & \text{bottom contact} \\ 0, & \text{inside cavity} \\ \dot{R}_c/V, & \text{top contact} \end{cases} \\ &\gamma_3 = z(t) + \theta(t)L - z(t-\tau) \\ &\pi_1 = V^2\gamma_1/(mL);\ \pi_2 = \gamma_2/\gamma_3;\ \pi_3 = \pi_1\pi_2 \end{aligned} \tag{7}$$

When the supercavitating vehicle has planing force, the planing force can be decomposed as follows:

$$\begin{aligned} \hat{F}_p &= -\frac{V^2}{mL}\frac{1+h_p'}{1+2h_p'}\left[1 - \left(\frac{R'}{h_p' + R'}\right)^2\right]\alpha_p \\ &= -\pi_1[\theta(t) - \theta(t-\tau) + w(t-\tau)/V + \gamma_2] \\ &= -\pi_1[\theta(t) - \theta(t-\tau) + w(t-\tau)/V] - \pi_3[z(t) + \theta(t)L - z(t-\tau)] \end{aligned} \tag{8}$$

The LPV time-delay model of supercavitating vehicle is obtained

$$\begin{aligned} &\dot{x}(t) = A(\pi_1, \pi_3)x(t) + A_d(\pi_1, \pi_3) \cdot x(t-\tau) + Bu(t) \\ &y(t) = Cx(t) \end{aligned} \tag{9}$$

where $\dot{x}(t) = Ax(t) = [z(t), w(t), \theta(t), q(t)]^{\mathrm{T}}$ is the system state matrix, $x(t-\tau)$ is the delay part of the system, and represents the state variable with the delay as τ; $A(\pi_1, \pi_3)$ and $A_d(\pi_1, \pi_3)$ are time-varying matrices, which change with the two time-varying parameters of π_1 and π_3, B and C are constant matrix.

If π_1 and π_3 are assumed to be independent, the supercavitating vehicle model in (9) is a time-delay LPV system with affine dependence on scheduling parameters π_1 and π_3.

3. **Design of Predictive Controller.** When the vehicle satisfies the conditions of contacting the cavitation wall, the depth and angle of penetration of the vehicle will change, the time-varying parameters π_1 and π_3 will no longer be zero, and the system will have planing force, which makes the control more difficult. According to [9], the time-delay system of the supercavitating vehicle with (9) can be described by the following discrete linear time-varying model:

$$x(k+1) = A(k)x(k) + A_d(k)x(k-d) + Bu(k)$$

$$y = Cx(k)$$
$$[A(k)\ A_d(k)] \in \Omega;\ \Omega = \boldsymbol{Co}\{[A_1\ A_{d1}], [A_2\ A_{d2}], \ldots, [A_i\ A_{di}]\} \tag{10}$$

In the equation, $\boldsymbol{Co}$ is the convex hull symbol. No matter how the parameters change, they will eventually fall in the region of convex polyhedron with $[A_1\ A_{d1}], \ldots, [A_i\ A_{di}]$ as the vertex. The time-varying parameters $A(k)$ and $A_d(k)$ can be expressed as

$$[A(k)\ A_d(k)\ B] = \sum_{i=1}^{4} \eta_i [A_i\ A_{di}\ B];\ \sum_{i=1}^{4} \eta_i = 1,\ \eta_i \geq 0 \tag{11}$$

According to [6], the value of π_1 is $\pi_1 \in [0, 865.625]$, considering that the perturbation of cavitation radius is $\pm 20\%$, $\pi_3 \in [0, 1416.35]$ a convex set with $[0, 0]$, $[0, 1416.35]$, $[865.625, 0]$, $[865.625, 1416.35]$ as the vertex can be constructed, and in Equation (11) can be expressed as

$$\beta_1 = \pi_1/865.625;\ \beta_2 = \pi_3/1416.35;\ \eta_1 = (1-\beta_1)(1-\beta_2)$$
$$\eta_2 = (1-\beta_1)\beta_2;\ \eta_3 = \beta_1(1-\beta_2);\ \eta_4 = \beta_1\beta_2 \tag{12}$$

The performance index of infinite time domain is decomposed into current performance index and terminal penalty term, which can be expressed as the following Min-Max optimal solution problem:

$$\begin{aligned} J_\infty^0(k) &= \sum_{i=0}^{\infty} x(k+i|k)^{\mathrm{T}} Q_1 x(k+i|k) + u(k+i|k)^{\mathrm{T}} R u(k+i|k) \\ &= \sum_{i=1}^{\infty} (x(k+i|k)^{\mathrm{T}} Q_1 x(k+i|k) + u(k+i|k)^{\mathrm{T}} R u(k+i|k)) \\ &\quad + x(k|k)^{\mathrm{T}} Q_1 (k|k) + u(k|k)^{\mathrm{T}} R u(k|k) \\ &= J_1^\infty(k) + x(k|k)^{\mathrm{T}} Q_1 (k|k) + u(k|k)^{\mathrm{T}} R u(k|k) \end{aligned} \tag{13}$$

Based on Lyapunov-Krasovskii function, the upper bound of infinite time domain robust performance for time-delay systems is proposed.

$$\begin{aligned} J_1^\infty(k) \leq x(k+1|k)^{\mathrm{T}} P x(k+1|k) + \sum_{i=1}^{d} x(k+1-i|k)^{\mathrm{T}} S \\ \times x(k+1-i|k);\ P > 0,\ S > 0 \end{aligned} \tag{14}$$

After transformation, the performance index from time $k+1$ to infinite time can be replaced by the upper bound of a new maximum problem, so the objective function of the Quasi-Min-Max robust predictive control algorithm can be optimized as follows:

$$\begin{aligned} &\min_{U_0^\infty(k)} \max_{[A(k+i), A_d(k+i), B(k+i)] \in \Omega} J_0^\infty(k) \\ &= \min_{u(k|k), U_1^\infty(k), P} \Big\{ x(k|k)^{\mathrm{T}} Q_1 x(k|k) + u(k|k)^{\mathrm{T}} R u(k|k) + x(k+1|k)^{\mathrm{T}} P x(k+1|k) \\ &\quad + \sum_{i=1}^{d} x(k-i+1|k)^{\mathrm{T}} S x(k-i+1|k) \Big\} \end{aligned} \tag{15}$$

The control sequence is $U_0^\infty(k) = [u(k|k), U_1^\infty(k)]$, where $u(k|k)$ is the current component applied to the controlled object, and other control components are based on the linear feedback control rate: $U_1^\infty(k) : u(k+i|k) = F(k)x(k+i|k),\ i \geq 1$.

Lemma 3.1. *Assume that $x(k|k)$ is the state measurement value of Formula (11) at time k, and the robust asymptotic stability achieved by Formula (13) can be converted into the* *optimization problem of whether variable* $\gamma > 0$, $Q > 0$, $W > 0$, $F = YQ^{-1}$, Y, $u(k|k)$ *exists and satisfies the following LMI constraints:*

$$\min_{\gamma,u(k|k),Q,W,Y(k)} \gamma \tag{16}$$

$$s.t. \begin{bmatrix} 1 & x(k|k)^{\mathrm{T}}Q_1^{1/2} & u(k|k)^{\mathrm{T}}R^{1/2} & m^{\mathrm{T}} & x^{\mathrm{T}}(k|k) & \cdots & x^{\mathrm{T}}(k-d+1|k-d+1) \\ Q_1^{1/2}x(k|k) & \gamma I & 0 & 0 & 0 & \cdots & \\ R^{1/2}u(k|k) & 0 & \gamma I & 0 & 0 & \cdots & \\ m & 0 & 0 & Q & 0 & \cdots & \\ x(k|k) & 0 & 0 & 0 & W & & \\ \cdots & \cdots & \cdots & & & W & \\ x(k-d+1|k-d+1) & 0 & \cdots & & & & W \end{bmatrix} \geq 0 \tag{17}$$

$$m = A(k)x(k|k) + A_d(k)x(k-d|k) + B(k)u(k|k)$$

$$\begin{bmatrix} Q & Y^{\mathrm{T}}R^{1/2} & QQ_1^{1/2} & Q & \left(QA_i^{\mathrm{T}} + Y^{\mathrm{T}}B_i^{\mathrm{T}}\right) & 0 \\ R^{1/2}Y & \gamma I & 0 & 0 & 0 & 0 \\ Q_1^{1/2}Q & 0 & \gamma I & 0 & 0 & 0 \\ Q & 0 & 0 & W & 0 & 0 \\ (A_iQ + B_iY) & 0 & 0 & 0 & Q & A_{d,i} \\ 0 & 0 & 0 & 0 & A_{d,i}^{\mathrm{T}} & W^{-1} \end{bmatrix} \geq 0 \tag{18}$$

$$\begin{bmatrix} u_{\max}^2 I & Y \\ (Y)^{\mathrm{T}} & Q \end{bmatrix} > 0 \tag{19}$$

$$\|u(k|k)\|_2 \leq u_{\max} \tag{20}$$

Proof: 1) Define a quadratic Lyapunov Krasovskii function.

$$V(x(k)) = x(k)^{\mathrm{T}}Px(k) + \sum_{i=1}^{d} x(k-i)^{\mathrm{T}}Sx(k-i); \quad P \geq 0,\, S \geq 0 \tag{21}$$

If the $J_1^{\infty}(k)$ robust performance index has an upper bound, the following inequality shall be satisfied.

$$\begin{aligned} &V(x(k+i+1|k)) - V(x(k+i|k)) \\ &\leq -\left[x(k+i|k)^{\mathrm{T}}Q_1x(k+i|k) + u(k+i|k)^{\mathrm{T}}Ru(k+i|k)\right], \quad i \geq 1 \end{aligned} \tag{22}$$

When the system is asymptotically stable, then there are $x(\infty) = 0$, $V(x(\infty)) = 0$. Overlay from $i = 1$ to $i = \infty$.

$$\begin{aligned} J_1^{\infty}(k) &\leq V(x(k+1|k)) \\ &= x(k+1|k)^{\mathrm{T}}Px(k+1|k) + \sum_{i=1}^{d} x(k+1-i|k)^{\mathrm{T}}Sx(k+1-i|k) \\ x(k|k) &= x(k);\ x(k-j|k) = x(k-j|k-j),\ j = 1, \ldots, d \end{aligned} \tag{23}$$

Variable γ is introduced as the upper bound of robust performance, and the following form is obtained.

$$\begin{aligned} J_0^{\infty}(k) &\leq x(k|k)^{\mathrm{T}}Q_1x(k|k) + u(k|k)^{\mathrm{T}}Ru(k|k) + x(k+1|k)^{\mathrm{T}}Px(k+1|k) \\ &\quad + \sum_{i=1}^{d} x(k+1-i|k)^{\mathrm{T}}Sx(k+1-i|k) \leq \gamma \end{aligned} \tag{24}$$

Assume that $A(k)$, $A_d(k)$ and $B(k)$ are online measurable, let $Q = \gamma P^{-1} > 0$, $W = \gamma S^{-1} > 0$, and use Schur complement lemma to transform Equation (15) into LMI (17).

2) Stability constraints. Take Equation (10) into Inequality (22) to get

$$\begin{bmatrix} (A(k)+B(k)F)^{\mathrm{T}}P(A(k)+B(k)F)+S-P & (A(k)+B(k)F)^{\mathrm{T}}PA_d(k) \\ A_d^{\mathrm{T}}(k)P(A(k)+B(k)F) & A_d^{\mathrm{T}}(k)PA_d(k)-S \end{bmatrix} < 0 \quad (25)$$

LMI (18) can be obtained by using Schur complement and multiplying $\begin{bmatrix} Q & 0 \\ 0 & I \end{bmatrix}$ before and after the inequality.

3) Input constraint: it is divided into current component and terminal invariant set constraint.

a) $\|u(k|k)\|_2 \le u_{\max}$.

b) The following inequality is obtained from Inequality (24).

$$x(k+1|k)^{\mathrm{T}}Px(k+1|k)+\sum_{i=1}^{d}x(k-i+1|k)^{\mathrm{T}}Sx(k-i+1|k) \le \gamma \quad (26)$$

According to Inequality (22), $x(k+1|k)^{\mathrm{T}}Px(k+1|k) \le \gamma$ is an invariant elliptic set, equivalent to Inequality (22).

Therefore, the Quasi-Min-Max robust predictive control (15) can be reduced to the optimization problem of solving the LMI constraints of Inequalities (17)-(20). The selection of P and S is converted into the solution of Q and W. The LMI above is convex optimization, and the online solution efficiency is high.

4. **Simulation Analysis.** The Quasi-Min-Max predictive controller is simulated when the bubble radius is perturbed by ±20% to verify the control performance and robustness. The time-delay for the simulation is $\tau = L/V = 0.024$ s. The sampling time of the system is set to $T = 0.008$ s. The initial state of the vehicle is set to $[z, w, \theta, q] = [0, 0, 0, 0]$, $u = [0, 0]$, the command signal of the system is $[1, 0, 0, 0]$, the control quantity constraint of the system is set as $[-20°, 20°]$, and the matrix Q_1 and matrix R of the system are unit matrices. The optimal state feedback control quantity $u(k) = u(k|k)$ of each sampling period can be directly obtained according to LMI constraint optimization.

Figure 1 shows a unit-step z-input tracking response of state variable. The dashed line is the system response curve with a downward disturbance of 20%. The long adjustment time and large overshoot are due to the smaller internal space of cavitation caused by the downward disturbance of the cavitation radius, and the vehicle is more likely to contact the cavitation wall. When the cavitation radius is disturbed, the state of the vehicle can quickly track the command signal. Therefore, the designed controller has strong robustness.

From the control variable and planing force response curve of the vehicle in Figure 2, it can be seen that the deflection angle of the cavitator and the rudder are obviously smaller than the control input constraint set by the system. The planing force disappears in a short time, and the vehicle navigates stably inside the cavitation bubble, reflecting that the controller can effectively suppress the planing force. Therefore, the designed controller has good control performance. The simulation results show that when the bubble radius is perturbed by external disturbances with input constraints, the Quasi-Min-Max predictive control ensures the progressive stability of the system state, the control input satisfying the constraints, and the planing force gradually disappears.

5. **Conclusions.** Aiming at the strong nonlinear problem of planing force, the LPV time-delay model of supercavitating vehicle is obtained by decomposing and transforming them. Considering the model uncertainty caused by time-varying parameters, the predictive model is equivalent to a linear time-varying uncertain system described by polytope.

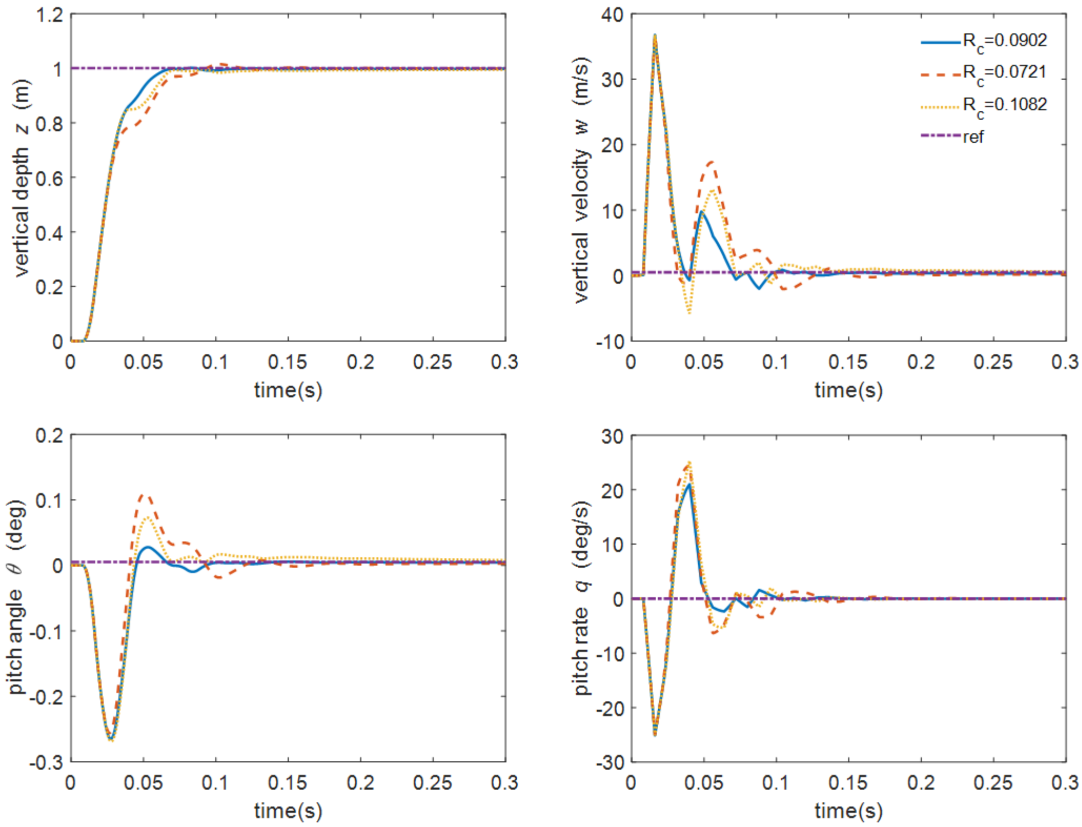

FIGURE 1. Z-step input tracking response curve of state variable

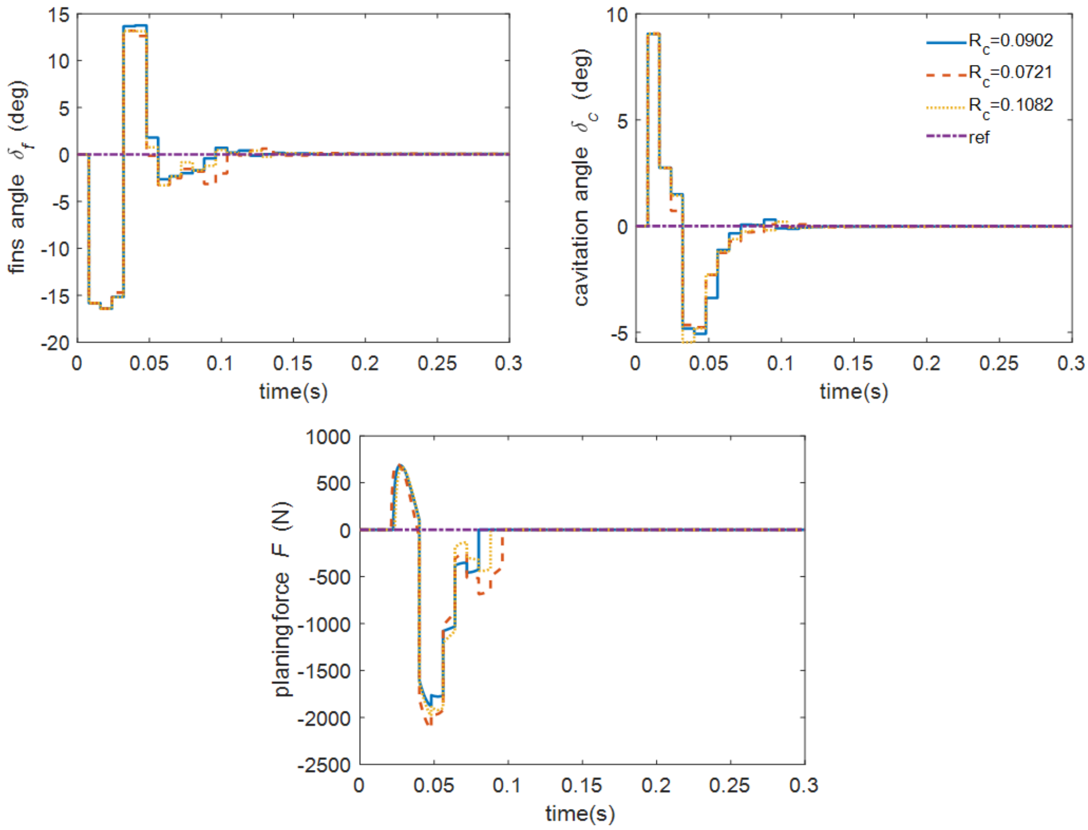

FIGURE 2. Z-step input tracking response curve of control variable and planing force

Based on this model, a Quasi-Min-Max robust predictive controller is designed. The simulation results show that the algorithm has good robustness to solve the problems of system parameter uncertainty, nonlinear planing force, time-delay and actuator saturation.

In the future, we plan to design a predictive controller with better control performance considering the anti-interference ability and input delay of supercavitating vehicle. Moreover, we will design the corresponding off-line algorithm to reduce the online workload of predictive control.

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