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FORMATION CONTROL OF SECOND-ORDER NONLINEAR MULTI-AGENT SYSTEMS WITH SWITCHING TOPOLOGY AND HETEROGENEOUS LEADER

Ying Liu[1] and Yongming Li[2,*]

[1]Navigation College
Dalian Maritime University
No. 1, Linghai Road, Dalian 116026, P. R. China
lying94@dlmu.edu.cn

[2]College of Science
Liaoning University of Technology
No. 169, Shiying Street, Guta District, Jinzhou 121001, P. R. China
*Corresponding author: liyongming@lnut.edu.cn

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Abstract. *This paper investigated the formation control issue for multi-agent systems (MASs) with switching topology and heterogeneous leader. In order to lessen the impact of differences between leader and follower agents, heterogeneous information for leader in the system models is considered. Then, by developing a leader-follower formation control approach with switching topology, we can completely relax the fixed topology restriction which has the common assumption. Finally, the proposed formation control protocol ensures that all signals of considered system are semi-globally uniformly ultimately bounded (SGUUB).*

Keywords: Formation control, Switching topology, Heterogeneous, Multi-agent systems

1. **Introduction.** Formation control has attracted more attention in the field of MASs cooperative control, and has been widely used in many practical systems, such as multi-spacecraft, multi-ground robot, and multi-underwater vehicle. Therefore, formation control of MASs has become an important research topic in recent years, see [1-4]. For a class of stochastic MASs, [1] investigated the formation control issue with obstacle avoidance, and [2] studied an adaptive optimized formation control issue. Under a directed interconnection topology, [3] addressed the formation control obstacle avoidance issue of MASs. Furthermore, for heterogeneous MASs, [4] proposed an event-triggered time-varying formation control strategy.

Much research work has been focused on homogeneous MASs, without considering the differences between the leader and follower agents. For the heterogeneous longitudinal formations system, [5] designed an adaptive distributed decoupling controller that guarantees stability and string stability of the longitudinal platooning system. [6] investigated the flocking control of MASs with heterogeneous virtual leader. The authors in [6] not only considered the uncertain nonlinearity in the virtual leader information, but also assumed the weaker constraint on the velocity information measurements. However, it is worth mentioning that all the mentioned results in [1-6], did not consider the formation control issue with switching topology.

As is known to all, switching topology issue is important in consensus or formation control theory. Communication channels among agents are often disrupted by various factors, which can cause the communication structure to change with time. For second-order MASs with switching directed topology, [7] proposed the distributed consensus protocol.

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And the authors in [7] only considered position information and local velocity information. Furthermore, for second-order MASs with nonlinear dynamics, [8] considered the time-varying asymptotic velocity consensus issue, and [9] developed an adaptive leader-following state delays consensus issue. The above studies about switching topology issue are aimed at MASs consensus issue, and there is no formation control research for MASs with heterogeneous leader.

Although some progress has been made towards the consensus control with switching topology issue of nonlinear MASs. However, by far, there are no available results about the formation control with switching topology issue for second-order heterogeneous nonlinear MASs, which limits the validity of formation control for MASs. Based on the above discussion, we are inspired to study the formation control issue of nonlinear MASs with switching topology and heterogeneous leader.

The main features and contributions of this paper are summarized as follows: i) This paper is the first result on formation control for nonlinear MASs with switching topology and heterogeneous leader. A novel leader-follower formation control protocol is proposed. ii) The formation control of second-order MASs with switching topology can can be used to solve the vehicles or UAVs formation transformation problem.

2. **Problem Formulation.** Consider the second-order nonlinear MASs as follows:

$$\begin{aligned} \dot{\eta}_i(t) &= \nu_i(t), \\ \dot{\nu}_i(t) &= u_i(t) + f_i(\eta_i, \nu_i), \quad i = 1, \ldots, m, \end{aligned} \tag{1}$$

where $\eta_i(t) = (\eta_{i1}, \ldots, \eta_{in})^T \in R^n$ is the position vector, $\nu_i(t) = (\nu_{i1}, \ldots, \nu_{in})^T \in R^n$ is the velocity vector, $u_i(t) \in R^n$ is the control input, and $f_i(\cdot) \in R^n$ is the continuously differentiable vector-valued function with $f(0) = 0$.

The desired reference signals are governed by

$$\begin{aligned} \dot{\eta}_l(t) &= \nu_l(t), \\ \dot{\nu}_l(t) &= f_l(\eta_l, \nu_l), \end{aligned} \tag{2}$$

where position reference $\eta_l \in R^n$, velocity reference $\nu_l \in R^n$, and $f_l(\cdot) \in R^n$ is a smooth bounded function.

The following assumptions are made on system (1).

Assumption 2.1. [8] *There exist constants $\rho_{1i} \geq 0$, $\rho_{2i} \geq 0$, such that*

$$\|f(\eta_i, \nu_i) - f(\eta_l, \nu_l)\| \leq \rho_{1i}\|\eta_i - \eta_l\| + \rho_{2i}\|\nu_i - \nu_l\|.$$

Assumption 2.2. [3] *There exist constants γ_1, γ_2, such that $\|\eta_l\| \leq \gamma_1$, $\|\nu_l\| \leq \gamma_2$.*

Algebraic Graph Theory. Define $G = \{G_m = (V, \varepsilon_m, A_m) | m \in \mathcal{M}\}$ as the collection of all possible digraphs. Then, the underlying graph can be denoted by $G_{\sigma(t)}$, and $\sigma(t) : [0, +\infty] \to \mathcal{M}$ represents the piecewise constant switching function. And $\sigma(t)$ represented that topology switches finite times in any bounded time interval. Define the switching topology $G_u = \left\{V, \bigcup_{m=1}^{M} \varepsilon_m, \sum_{m=1}^{M} A_m\right\}$ in a collection of digraphs $G = \{G_1, \ldots, G_M\}$. According to Lemma 2.4 in the later, the average matrix $-E_0 = -\frac{1}{M}\sum_{m=1}^{M} E_m$ is a Hurwitz matrix. Thus, the equation $E_0^T Q + QE_0 = I_{2(n-1)}$ has a positive definite solution Q. Denote $B = diag\{b_1, b_2, \ldots, b_n\}^T$ as the communication weights between follower agents and leader. And $b_1 + b_2 + \cdots + b_n > 0$ represent that at least one follower agent connects with leader.

Let

$$\sigma(t_0) = \arg\min \left\{h_0^T E_1 h_0, \ldots, h_0^T E_M h_0\right\},$$

where index arg min represents the minimum among $\mathcal{M}$, $E_m = E_m^T Q + QE_m$ and $h(t_0) = h_0$.

The switching instant is defined as follows:

$$t_{k+1} = \inf\left\{t > t_k : h^T(t)E_{\sigma(t_k)}h(t) > r_{\sigma(t_k)}h^T(t)h(t)\right\}$$
$$\sigma(t_{k+1}) = \arg\min\left\{h(t_{k+1})E_1h(t_{k+1}), \dots, h(t_{k+1})E_Mh(t_{k+1})\right\},$$

where $r_{\sigma(t)} \in (0, 1)$.

Lemma 2.1. [10] *Let irreducible matrix $L = [l_{ij}] \in R^{mm}$, $l_{ij} = l_{ji} \leq 0$ and $l_{ii} = \sum_{j=1}^{m} l_{ij}$. Then all the eigenvalues of* $\tilde{L} = \begin{pmatrix} l_{11}+d_1 & \cdots & l_{1m} \\ \vdots & \ddots & \vdots \\ l_{m1} & \cdots & l_{mm}+d_m \end{pmatrix}$ *are positive, and $d_1 + \dots + d_m > 0$, $d_i = \sum_{j=1}^{n} a_{ij}$.*

Lemma 2.2. [11] *If $Q_1(y) = Q_1^T(y)$ and $Q_2(y) = Q_2^T(y)$ can be satisfied, the matrix inequality that* $\begin{pmatrix} Q_1(y) & Q_3(y) \\ Q_3^T(y) & Q_2(y) \end{pmatrix} \geq 0$ *is equivalent to any one of the following two conditions:*

(a) $Q_1(y) > 0$, $Q_2(y) - Q_3^T(y)Q_1^{-1}(y)Q_3(y) > 0$;

(b) $Q_2(y) > 0$, $Q_1(y) - Q_3(y)Q_2^{-1}(y)Q_3^T(y) > 0$.

Lemma 2.3. [12] *For any two real vectors $x, y \in R^n$ and positive definite matrix $\Phi \in R^{nn}$, we have*

$$2x^Ty \geq x^T\Phi x + y^T\Phi^{-1}y.$$

Lemma 2.4. [13] *If Laplacian matrices $L_1, L_2, \dots, L_M$ are associated with the graphs $G_1, G_2, \dots, G_M$, respectively, then, $-\sum_{i=1}^{M} EL_iF$ is anti-stable if and only if node 0 is jointly globally reachable for these graphs.*

Definition 2.1. [3] *The leader-follower formation is achieved if the solutions of MASs satisfy*

$$\lim_{t\to\infty} \|\eta_i(t) - \eta_l(t) - p_i\| = 0, \quad \lim_{t\to\infty} \|\nu_i(t) - \nu_l(t)\| = 0,$$

where $p_i = (p_{i1}, \dots, p_{in})^T \in R^n$ describes the desired relative position between the reference signal and agent i.

The control objective of this article is to design a formation control protocol for the nonlinear MASs under switching topologies and heterogeneous leader, such that a) all closed-loop signals are SGUUB; b) the leader-follower formation can be achieved.

3. **Control Protocol Design.** In this section, the formation issue of MASs (1) is solved by designing the switching topology control strategy. To derive the switching topology formation protocol, the coordinate transformation is defined as follows:

$$h_{\eta i}(t) = \eta_i - \eta_l - p_i,$$
$$h_{\nu i}(t) = \nu_i - \nu_l, \quad i = 1, \dots, m. \tag{3}$$

Taking time derivative of (3) along (1) and (2), one has

$$\dot{h}_{\eta i}(t) = h_{\nu i},$$
$$\dot{h}_{\nu i}(t) = u_i + f_i(\eta_i, \nu_i) - f_l(\eta_l, \nu_l), \quad i = 1, \dots, m. \tag{4}$$

Rewrite the error dynamic (4) as follows:

$$\dot{h}(t) = \begin{pmatrix} h_{\nu i}(t) \\ u(t) + F(h) - f_l(\eta_l, \nu_l) \otimes 1_m \end{pmatrix}, \tag{5}$$

where $h(t) = \left[h_\eta^T, h_\nu^T\right]^T$, $h_\eta(t) = \left[h_{\eta 1}^T, \dots, h_{\eta m}^T\right]^T$, $h_\nu(t) = \left[h_{\nu 1}^T, \dots, h_{\nu m}^T\right]^T$, $u(t) = \left[u_1^T, \dots, u_m^T\right]^T$, $F(h) = \left[f_1^T, \dots, f_m^T\right]^T$.

The formation errors of position and velocity can be defined as follows:

$$e_{\eta i} = \sum_{j\in N_i} a_{ij}(\eta_i - p_i - \eta_j + p_j) + d_i(\eta_i - \eta_l - p_i),$$
$$e_{\nu i} = \sum_{j\in N_i} a_{ij}(\nu_i - \nu_j) + d_i(\nu_i - \nu_l), \quad i = 1,\dots,m, \tag{6}$$

where a_{ij} is the element of matrix A; d_i is the connection weight.

According to (3), one has

$$e_{\eta i} = \sum_{j\in N_i} a_{ij}(h_{\eta i} - h_{\eta j}) + d_i h_{\eta i},$$
$$e_{\nu i} = \sum_{j\in N_i} a_{ij}(h_{\nu i} - h_{\nu j}) + d_i h_{\nu i}, \quad i = 1,\dots,m, \tag{7}$$

where N_i is the collection of neighbors of agent i.

The formation control protocol can be designed as follows:

$$u_i = -\alpha_i e_{\eta i} - \alpha_i e_{\nu i}, \quad i = 1,\dots,m, \tag{8}$$

where $\alpha_i > 0$ is a constant designed later.

Substituting (8) into (4), the following result can be obtained

$$\dot{h}_{\eta i}(t) = h_{\nu i}, \quad i = 1,\dots,m,$$
$$\dot{h}_{\nu i}(t) = -\alpha_i e_{\eta i} - \alpha_i e_{\nu i} + f_i(\eta_i,\nu_i) - f_l(\eta_l,\nu_l). \tag{9}$$

Transform (9) to compact form as

$$\begin{aligned}\dot{h}(t) &= \left(-\begin{pmatrix} 0_{n\times n} & -I_n \\ \Delta\tilde{L} & \Delta\tilde{L}\end{pmatrix} \otimes I_m\right) h(t) + \begin{pmatrix} 0_{nm} \\ F(h) - f_l(\eta_l,\nu_l)\end{pmatrix} \\ &= (-E\otimes I_m)h(t) + \bar{F}, \end{aligned}\tag{10}$$

where $\Delta = diag\{\alpha_1,\dots,\alpha_n\}$, $\tilde{L} = L + D$, $D = diag\{d_1,\dots,d_n\}$.

Theorem 3.1. *Consider the MASs (1) with reference signals (2) under the switching topology. The formation control (8) can achieve the control objective if the design parameters α_i and κ satisfy the following conditions*

$$\begin{aligned}\alpha_i &= \kappa\delta_i, \\ \kappa &\geq \frac{\max_{l\leq i\leq n}\{4\rho_{i1} + 3\rho_{i2}\}}{2r\lambda_{\min}(rI_n + 2\Lambda D)} + 2,\end{aligned}\tag{11}$$

where $r = \min\{r_1, r_2, \dots, r_M\}$, $\delta = (\delta_1,\delta_2,\dots,\delta_n)^T$, $\lambda_{\min}(rI_n + 2\Lambda D)$ is the minimum eigenvalue of symmetrical matrix $rI_n + 2\Lambda D$, $\Lambda = diag\{\delta_1,\delta_2,\dots,\delta_n\}$.

Proof: Choose the Lyapunov function as follows:

$$V(t) = \frac{1}{2}h^T(t)(Q\otimes I_m)h(t),$$

where

$$Q = \begin{pmatrix} \kappa(\Theta + 2\Lambda D) & I_n \\ I_n & I_n\end{pmatrix}, \quad \Theta = L^T\Lambda + \Lambda L.$$

From (11), reexpress $Q = \begin{pmatrix} \tilde{L}^T\Delta + \Delta\tilde{L} & I_n \\ I_n & I_n\end{pmatrix}$ with $\alpha_i = \kappa\delta_i$.

The time derivative of $V(t)$ is

$$\dot{V}(t) = -\frac{1}{2}h^T(t)\left(\left(E^TQ + QE\right)\otimes I_m\right)h(t) + h^T(t)(Q\otimes I_m)\bar{F}. \tag{12}$$

According to (12), we have that for $t \in (t_k, t_{k+1})$

$$-h^T\left(\left(E_{\sigma(t_k)}^T Q + QE_{\sigma(t_k)}\right)\otimes I_m\right)h \le -\begin{bmatrix} \kappa\left(r_{\sigma(t_k)}I_n + 2\Lambda D\right) & 0_{n\times n} \\ 0_{n\times n} & \kappa\left(r_{\sigma(t_k)}I_n + 2\Lambda D\right) - 2I_n \end{bmatrix} h^T h.$$

By applying matrix theory, one gets

$$H_1 = E_{\sigma(t_k)}^T Q + QE_{\sigma(t_k)} = \begin{bmatrix} \kappa\left(r_{\sigma(t_k)}I_n + 2\Lambda D\right) & 0_{n\times n} \\ 0_{n\times n} & \kappa\left(r_{\sigma(t_k)}I_n + 2\Lambda D\right) - 2I_n \end{bmatrix}. \tag{13}$$

Substituting (13) into (12), yields

$$\dot{V}(t) = -\frac{1}{2}h^T(t)(H_1\otimes I_m)h(t) + h^T(t)(Q\otimes I_m)\bar{F}. \tag{14}$$

Using the following fact

$$h^T(t)(Q\otimes I_m)\bar{F} = \left(h_\eta^T(t) + h_\nu^T(t)\right)(F(h) - f_l(\eta_l,\nu_l)),$$

Equation (14) can become

$$\dot{V}(t) = -\frac{1}{2}h^T(t)(H_1\otimes I_m)h(t) + \left(h_\eta^T(t) + h_\nu^T(t)\right)(F(h) - f_l(\eta_l,\nu_l)). \tag{15}$$

By using Young's inequality and Cauchy-Schwarz inequality, one gets

$$\begin{aligned} h_\eta^T(t)(F(h) - f_l(\eta_l,\nu_l)) &\le \sum_{i=1}^n (\|h_{\eta i}\|\|F(h) - f_l(\eta_l,\nu_l)\|) \\ &\le \sum_{i=1}^n (\|h_{\eta i}\|(\rho_{1i}\|h_{\eta i}\| + \rho_{2i}\|h_{\nu i}\| + \rho_{1i}\|p_i\|)) \\ &\le \sum_{i=1}^n \left(\frac{3\rho_{1i}+\rho_{2i}}{2}\|h_{\eta i}\|^2 + \frac{\rho_{2i}}{2}\|h_{\nu i}\|^2 + \frac{\rho_{1i}}{2}\|p_i\|^2\right). \end{aligned} \tag{16}$$

Similarly, one has

$$\begin{aligned} h_\nu^T(t)(F(h) - f_l(\eta_l,\nu_l)) &\le \sum_{i=1}^n (\|h_{\nu i}\|\|F(h) - f_l(\eta_l,\nu_l)\|) \\ &\le \sum_{i=1}^n (\|h_{\eta i}\|(\rho_{1i}\|h_{\eta i}\| + \rho_{2i}\|h_{\nu i}\| + \rho_{1i}\|p_i\|)) \\ &\le \sum_{i=1}^n \left(\frac{\rho_{1i}}{2}\|h_{\eta i}\|^2 + (\rho_{1i}+\rho_{2i})\|h_{\nu i}\|^2 + \frac{\rho_{1i}}{2}\|p_i\|^2\right). \end{aligned} \tag{17}$$

Substituting (16) and (17) into (15), one gets

$$\dot{V}(t) \le -\frac{1}{2}h^T(t)(H_2\otimes I_m)h(t) + \sum_{i=1}^n \rho_{1i}\|p_i\|^2, \tag{18}$$

where

$$H_2 = \begin{bmatrix} \kappa\left(r_{\sigma(t_k)}I_n + 2\Lambda D\right) - N_1 & 0_{n\times n} \\ 0_{n\times n} & \kappa\left(r_{\sigma(t_k)}I_n + 2\Lambda D\right) - N_2 \end{bmatrix},$$

$$N_1 = \begin{pmatrix} 4\rho_{11}+\rho_{21} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 4\rho_{1n}+\rho_{2n} \end{pmatrix},$$

$$N_2 = \begin{pmatrix} 2\rho_{11} + 3\rho_{21} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 2\rho_{1n} + 3\rho_{2n} \end{pmatrix}.$$

Together with (11), rewrite Inequality (18) as follows:

$$\dot{V}(t) \leq -\frac{\lambda_{\min}(H_2)}{\lambda_{\max}(Q)} V(t) + \sum_{i=1}^{n} \rho_{1i} \|p_i\|^2 \leq -c_1 V(t) + c_2, \tag{19}$$

where $c_1 = \frac{\lambda_{\min}(H_2)}{\lambda_{\max}(Q)}$, $c_2 = \sum_{i=1}^{n} \rho_{1i} \|p_i\|^2$.

By (19), one gets

$$V(t) \leq e^{-c_1 t} V(0) + \frac{c_2}{c_1} \left(1 - e^{-c_1 t}\right). \tag{20}$$

The proof of Theorem 3.1 is completed.

4. **Conclusion.** This article has introduced the formation control issue for the nonlinear MASs with switching topology and heterogeneous leader. Based on the switching topology design principle and the formation control design theory, a new heterogeneous formation control scheme has been developed. The proposed formation control method can guarantee that all the signals of the second-order heterogeneous MASs are SGUUB and achieve the formation control objective. Our future work will be directed at the formation control for heterogeneous nonlinear MASs with quantized control inputs. In addition, finite-time formation control [14], is an interesting work in the future.

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