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IMPROVED SPARROW SEARCH ALGORITHM BASED ON NONLINEAR DYNAMIC ADJUSTMENT AND CAUCHY MUTATION

Wei Wu and Xinyu Wang

Motion Control Motor Research Institute
Shenyang University of Technology
No. 111, Shenliao West Road, Shenyang 110870, P. R. China
wuwei@sut.edu.cn; 1264153073@qq.com

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Abstract. *Aiming at the problems that the sparrow search algorithm is easy to fall into local optimum and poorly stable, an improved sparrow search algorithm based on nonlinear dynamic adjustment strategy and Cauchy mutation is proposed. A nonlinear dynamic strategy is adopted to automatically adjust the number of early warning according to the change of the fitness value of sparrows and balance the capacity of local search and global search. Cauchy operator is used to disturb mutation and enhance the capacity of algorithm to jump out of local optimum. Simulation results on multiple test functions show that the proposed algorithm has strong optimization capacity and high stability, and can effectively overcome the premature convergence.*
Keywords: Sparrow search algorithm, Nonlinearity, Dynamic adjustment, Cauchy mutation

1. **Introduction.** The swarm intelligence optimization algorithm [1] is a random search algorithm inspired by group behavior or physical phenomena of natural organisms. It has features like easiness to operate and strong robustness. At present, the swarm intelligence optimization algorithm has got extensive attention and has become one of the research hotspots in the design and analysis of computer algorithms [2].

Sparrow search algorithm (SSA) is a novel swarm intelligence optimization algorithm proposed by Xue and Shen [3] in 2020 by simulating foraging behavior and anti-predation behavior of sparrows, with a higher solution accuracy and speed. However, there are defects like serious convergence in later iteration. In view of those defects, many scholars have proposed different improvement strategies of optimization formula and parameter selection. Wu et al. [4] brought in logistic chaotic mapping and linear decreasing weight method into SSA. Experiments show that the improved algorithm has less premature risk, faster speed and better stability than SSA. Mao and Zhang [5] applied Sin chaotic mapping, dynamic adaptive weights, Cauchy variance and backward learning strategies, got a better performing algorithm in finding the best solution than three basic algorithms SSA, GWO and MFO, and two improved sparrow algorithms ASSA and CASSA. Chen and Chen [6] proposed multiple swarm ant colony optimization (MACO) and quantum particle swarm optimization (QPSO) based on ACO and PSO to optimize the trajectory planning and positioning error of a five-degree-of-freedom manipulator, while the optimal route was verified by simulating the manipulator. However, further analysis found those existing improvement measures did not adjust the diversity of the population in the optimization process, for which it needs to be further improved.

Aiming at defects of SSA in convergence speed and accuracy, this paper proposes an improved sparrow search algorithm based on nonlinear dynamic adjustment and Cauchy

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mutation. Secondly, to verify the effectiveness of this new algorithm, 10 benchmark functions are simulated. Finally, the optimization process of ISSA is visualized.

2. **Sparrow Search Algorithm.** Sparrow population plays three roles in different periods: discoverer, participant and early warning. The location of discoverers is updated as follows:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^{t} \cdot \exp\left(-\frac{i}{\alpha \cdot iter_{\max}}\right) & \text{if } R_2 < ST \\ X_{i,j}^{t} + Q \cdot L & \text{if } R_2 \geq ST \end{cases} \tag{1}$$

where $iter_{\max}$ is the maximum of iterations. $\alpha \in (0,1]$ is a random number. R_2 ($R_2 \in [0,1]$) and ST ($ST \in [0.5,1]$) represent warning value and safety value, respectively. Q is a random number that follows a normal distribution. L represents a $1 \times d$ matrix, where all elements in the matrix are 1.

The location of participants is updated as follows:

$$X_{i,j}^{t+1} = \begin{cases} Q \cdot \exp\left(-\frac{X_{worst} - X_{i,j}^{t}}{\alpha \cdot iter_{\max}}\right) & \text{if } i > n/2 \\ X_{p}^{t+1} + \left|X_{i,j}^{t} - X_{p}^{t+1}\right| \cdot A^{+} \cdot L & \text{otherwise} \end{cases} \tag{2}$$

where X_p represents the optimal position of the discoverer; X_{worst} represents the worst position in the whole world. A represents a $1 \times d$ matrix, and $A^{+} = A^{T}\left(AA^{T}\right)^{-1}$.

When the sparrow flock is foraging, some sparrows will be selected to be on guard. 10%~20% sparrows were randomly selected from the total for warning behavior. The location update formula is

$$X_{i,j}^{t+1} = \begin{cases} X_{best}^{t} + \beta \cdot \left|X_{i,j}^{t} - X_{best}^{t+1}\right| & \text{if } f_i > f_g \\ X_{i,j}^{t} + K \cdot \left(\frac{\left|X_{i,j}^{t} - X_{worst}^{t}\right|}{(f_i - f_w) + \epsilon}\right) & \text{if } f_i = f_g \end{cases} \tag{3}$$

where X_{best} is the current global optimal position. β is the step control parameter. $K \in [-1,1]$ is a random number, which represents the moving direction of the sparrow. f_i is the fitness value of the current sparrow individual. f_g and f_w are the current global best and worst fitness values, respectively.

3. **Improved Sparrow Search Algorithm.**

3.1. **Nonlinear dynamic change strategy of the number of early warning.** In SSA, the early warning update process is actually a secondary optimization of some individuals in the population and the number of early warning is constant, and the larger the number is set, the higher the global search capacity of the algorithm is got, but later it may cause the algorithm to fall into local extreme.

Consider a 5th order Butterworth filter amplitude-frequency response curve model exhibiting excellent transitions between linear and nonlinear behavior [7], a strategy for nonlinear dynamic change in the number of early warning based on Butterworth filter is proposed, according to the fitness value of sparrows after each iteration [8], the proportion of individuals undergoing secondary optimization gets adjusted by adjusting the number of early warning. Mathematical model is as follows:

$$SD = \frac{0.1}{1 + \left(\frac{f_g}{f_i}\right)^{10}} + 0.1 \tag{4}$$

The number of early warning can be automatically adjusted in a range of 10%~20% with the change of fitness value, which enhances the intelligence of sparrows in the process of optimization and keeps the balance between global search and local development capacity.

3.2. **Cauchy mutation strategy.** Cauchy mutation [8] originates from Cauchy distribution, longer distributions at both ends of the Cauchy density function give individuals a higher probability of jumping out of the local optimum, and the mutation produces a large variability between offspring and parent. Cauchy mutation operation is introduced, which not only keeps the population diversity when sparrows are obviously clustered at later algorithm, but also enables the algorithm to jump out of the local optimum. The specific steps of Cauchy mutation are as follows.

1) Judge whether the algorithm falls into premature convergence according to the variance of fitness value of sparrow population. The formula is as follows:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{f_i - f_{avg}}{f}\right)^2 \tag{5}$$

where $f = \max(1, \max|f_i - f_{avg}|)$, f_{avg} is the fitness value of all the current sparrows on average. The size of σ^2 reflects the convergence state of the sparrow population, the greater the value of σ^2, the better the diversity of the sparrow population possesses; on the contrary, the population diversity is poor and the algorithm tends to converge. In this paper, the threshold value of σ^2 is set to 0.1, and when σ^2 is less than this threshold value, the algorithm falls into a premature state.

2) When the algorithm fell into premature state, Cauchy perturbation was performed on 70% of sparrows. The Cauchy distribution random variable generating function is introduced as follows:

$$Cauchy(0,1) = \tan((rand - 0.5) * \pi) \tag{6}$$

And the Cauchy mutation was carried out on selected sparrows to obtain

$$X_{i,j}^{t+1} = X_{i,j}^{t}(1 + 0.7 * Cauchy(0,1)) \tag{7}$$

3) Although Cauchy perturbation strategy can enhance the capacity of the algorithm to jump out of local extremum, it cannot be determined that the new position obtained after perturbation mutation is better than the fitness value of the original position, therefore, the greedy rule is introduced, by comparing the fitness values of the old and new locations, whether to update the location is determined. The rules of greed are as follows:

$$X_{best} = \begin{cases} X_{i,j}^{t+1}, & f\left(X_{i,j}^{t+1}\right) < f(X_{best}) \\ X_{best}, & f\left(X_{i,j}^{t+1}\right) \geq f(X_{best}) \end{cases} \tag{8}$$

The pseudo-code of the improved sparrow optimization algorithm is as follows:

Input pop, $iter$, PD, SD, RD, ST
Establish an objective function $F(x)$, where variable $x = (x_1, x_2, \ldots, x_d)$.
Initialize the sparrows population x_i and v_i.
Rank the fitness values and find the current best individual and the current worst individual.
For $i = 1 : iter$
    $R_2 = rand(1)$
    For $j = 1 : PD$
        Use Equation (1) to update the sparrow's location;
    end for
    For $j = (PD + 1) : pop$
        Use Equation (2) to update the sparrow's location;
    end for
    For $j = 1 : SD$
        Use Equation (3) to update the sparrow's location;
    end for

IF variance of the fitness values $c_1 < 0.1$
For $j = 1 : RD$
Use Equation (7) to update the sparrow's location;
end for
end if
The number of the sparrows who perceive the danger nonlinear dynamic change according to Equation (4).
Get the current new location;
If the new location is better than before, update it;
$t = t + 1$;
end for
Return X_{best}, f_g

4. **Simulation Experiment and Result Analysis.** To further prove the effectiveness of ISSA that was proposed, PSO, BA, GWO and SSA are used for comparison, and ten test functions commonly used are selected to test the convergence accuracy, convergence speed and stability of the proposed algorithm. In addition, three two-dimensional functions are selected to visualize the movement track of sparrow population. For avoiding the bias of single results, 30 experiments were conducted independently for each optimization problem in 30 and 70 dimensions for each of the five algorithms.

4.1. **Basic test functions.** In order to simulate difficulties of searching space in practice, 10 kinds of single-peak and multi-peak test functions were selected according to their dimensions for numerical experiments. The specific function expressions and the range of values are shown in Table 1.

Table 1. Test functions

Function name	Expression	Initial range
Schwefel's 2.22	$f_1(x) = \sum_{i=1}^{n} \lvert x_i\rvert + \prod_{i=1}^{n} \lvert x_i\rvert$	$[-10, 10]$
Schwefel's 2.21	$f_2(x) = \max\{\lvert x_i\rvert, 1 \le i \le n\}$	$[-100, 100]$
Rosenbrock's	$f_3(x) = \sum_{i=1}^{n-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right]$	$[-30, 30]$
Step	$f_4(x) = \sum_{i=1}^{n} ([x_i + 0.5])^2$	$[-100, 100]$
Quartic	$f_5(x) = \sum_{i=1}^{n} i x_i^4 + random[0, 1)$	$[-1.28, 1.28]$
Schwefel's 2.26	$f_6(x) = \sum_{i=1}^{n} -x_i \sin\left(\sqrt{\lvert x_i\rvert}\right)$	$[-500, 500]$
Rastrigin's	$f_7(x) = \sum_{i=1}^{n} \left[x_i^2 - 10\cos(2\pi x_i) + 10\right]$	$[-5.12, 5.12]$
Ackley's	$f_8(x) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$	$[-32, 32]$
Griewank's	$f_9(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	$[-600, 600]$
Peneralized	$f_{10}(x) = \frac{\pi}{n}\left\{10\sin(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2\left[1 + 10\sin^2(\pi y_{i+1})\right]\right\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$	$[-50, 50]$

4.2. **Algorithm parameter setting.** For verifying the effectiveness of ISSA fairy, the simulation experiment runs in the same running environment, MATLAB 2020b is used to complete this simulation. The population number of each algorithm is 100, the maximum of iteration is 100, and other parameter algorithms as shown in Table 2.

TABLE 2. Parameter setting table

Algorithm	Parameter setting
PSO	$c_1 = c_2 = 1.49$
BA	$R_0 = 0.7$, $A_f = 0.9$, $R_f = 0.9$, $F \in [0, 2]$
GWO	Linearly decreasing from A 2 to 0, $r_1, r_2 \in [0, 1]$
SSA	$PD = 70\%$, $R_2 = 0.6$, $SD = 20\%$
ISSA	$PD = 70\%$, $R_2 = 0.6$, $SD = 20\%$, $RD = 70\%$

4.3. **Results analysis.** Table 3 shows results of five algorithms on 10 test functions, it is shown both in 30 dimensions and in 70 dimensions, and the proposed ISSA has different magnitudes of improvement in convergence accuracy, where the optimal and average values of functions f_1, f_2, f_6 to f_9 all reach the theoretical optimum. Comparing the experimental results in 30 and 70 dimensions, it is shown the convergence accuracy of the proposed algorithm does not decrease significantly with the increase of dimensions, that is, sparrows can jump out of the local optimum. The data of standard deviation shows the standard deviation of ISSA is smaller than that of the other four algorithms under 10 standard test functions, which indicates that the stability and robustness of ISSA are obviously better than those of the other five algorithms.

In order to show the optimization speed and precision of ISSA algorithm more intuitively, draw the convergence curves of ISSA algorithm and the other four comparison algorithms on 10 test functions in 70 dimensions, are drawn and shown in following Figures 1 and 2, where the abscissa is the iteration number and the ordinate is the optimal logarithmic fitness value.

ISSA convergence curve always shows inflection point first, which indicates that the convergence speed of ISSA is better than other four algorithms, and the convergence speed advantage of ISSA is more obvious in f_3, f_6 and f_{10}.

In order to show ISSA's trajectory more intuitively, the reduced wave function, Rasstrigin function and Ackley function are selected for optimization search. The darkest position in the figure is the global optimum. Figure 3 clearly shows that most sparrows can gather at or near the global optimum, and a small number of sparrows are trapped in the local optimum, but still have the ability to move to $(0, 0)$.

The simulation results show that ISSA has higher accuracy and faster convergence speed. The results of numerical experiments also show that the global search ability and local development ability of the algorithm are dynamically balanced and more stable. Therefore, ISSA's optimization performance is relatively stronger and smarter, and it is a more efficient algorithm.

5. **Conclusion.** Aiming at the defects of SSA, an improved sparrow search algorithm based on nonlinear dynamic adjustment and Cauchy mutation is proposed. Based on the nonlinear dynamic change of Butterworth filter and sparrow population fitness value, the number of vigilants is changed dynamically, which effectively balances the global development and local search ability of the algorithm; the Cauchy variation strategy is integrated to improve the probability of the algorithm to jump out of the local extremes and improve the global exploration performance. 10 test functions are selected to simulate

TABLE 3. Comparison of test function optimization results

F	Algorithm	Best	Ave	Std	Best	Ave	Std
		$d = 30$			$d = 70$		
f_1	PSO	4.594E-04	5.416E-03	1.487E-02	3.549E-04	6.264E-03	1.769E-02
	BA	2.073E-06	3.407E-03	1.487E-02	1.663E-04	9.658E-03	2.278E-02
	GWO	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	SSA	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	ISSA	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
f_2	PSO	1.375E-04	5.692E-03	1.157E-02	2.989E-04	3.909E-03	8.843E-03
	BA	2.015E-05	1.414E-04	1.499E-04	3.636E-06	1.313E-04	2.576E-04
	GWO	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	SSA	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	ISSA	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
f_3	PSO	1.001E+01	4.564E+01	4.747E+01	1.131E+01	4.573E+01	8.506E+01
	BA	1.007E-04	1.148E+01	1.435E+01	1.088E-04	2.732E+01	3.415E+01
	GWO	8.134E-04	7.550E+00	1.356E+01	9.424E-01	2.065E+01	2.886E+01
	SSA	6.715E-06	1.697E-04	3.093E-04	1.238E-04	1.689E-03	3.612E-03
	ISSA	1.523E-08	3.638E-06	4.773E-06	1.752E-07	2.024E-05	4.469E-05
f_4	PSO	1.171E-05	1.870E-04	7.611E-04	1.290E-06	1.233E-04	1.092E-03
	BA	8.687E-08	1.938E-06	5.171E-06	1.160E-07	4.649E-06	1.098E-05
	GWO	3.244E-03	1.645E-01	2.307E-01	1.320E-01	5.837E+00	7.241E+00
	SSA	1.065E-07	2.854E-06	1.311E-05	1.318E-07	7.038E-06	5.082E-05
	ISSA	8.007E-12	2.806E-09	4.789E-09	2.839E-11	3.563E-08	4.823E-08
f_5	PSO	2.227E-04	2.576E-03	6.138E-03	1.794E-04	3.270E-03	1.545E-03
	BA	3.368E-04	2.815E-03	5.912E-03	1.039E-04	3.539E-03	8.463E-03
	GWO	1.040E-04	2.368E-03	1.876E-02	1.015E-04	4.095E-03	2.396E-02
	SSA	1.024E-04	3.140E-04	4.332E-04	1.029E-04	2.947E-04	2.377E-04
	ISSA	7.693E-07	3.571E-05	4.599E-05	2.192E-06	3.297E-05	4.050E-05
f_6	PSO	−8.380E+03	−8.308E+03	1.250E+02	−8.380E+03	−7.748E+03	1.185E+03
	BA	−1.257E+04	−1.257E+04	1.897E-07	−2.933E+04	−2.933E+04	4.622E-07
	GWO	−1.257E+04	−1.242E+04	5.280E+02	−2.933E+04	−2.897E+04	7.453E+02
	SSA	−1.257E+04	−1.257E+00	1.219E+00	−2.933E+04	−2.933E+04	8.147E+00
	ISSA	−1.257E+04	−1.257E+04	5.324E+00	−2.933E+04	−2.933E+04	5.855E+00
f_7	PSO	1.770E-02	3.953E+00	2.954E+00	1.598E+00	5.922E+00	3.535E+00
	BA	6.674E-08	1.257E-04	4.709E-04	2.097E-07	9.195E-04	7.054E-03
	GWO	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	SSA	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	ISSA	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
f_8	PSO	1.649E+00	1.651E+00	2.950E-03	1.689E+00	1.691E+00	4.520E-03
	BA	4.074E-06	2.584E-04	4.222E-04	6.310E-06	5.018E-03	1.010E-03
	GWO	8.882E-16	8.882E-16	0.000E+00	8.882E-16	8.882E-16	0.000E+00
	SSA	8.882E-16	8.882E-16	0.000E+00	8.882E-16	8.882E-16	0.000E+00
	ISSA	8.882E-16	8.882E-16	0.000E+00	8.882E-16	8.882E-16	0.000E+00
f_9	PSO	1.056E-07	2.375E-04	1.172E-07	1.140E-07	1.559E-03	1.172E-03
	BA	5.875E-09	2.500E-03	5.697E-07	1.176E-10	4.872E-08	2.430E-08
	GWO	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	SSA	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	ISSA	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
f_{10}	PSO	1.053E-07	2.079E-06	1.718E-06	2.134E-07	5.658E-06	3.260E-05
	BA	9.238E-08	4.224E-08	3.388E-08	1.02E-08	4.575E-08	1.278E-07
	GWO	3.826E-04	4.297E-02	7.391E-02	9.969E-06	4.410E-02	2.725E-01
	SSA	2.216E-07	3.235E-07	3.340E-07	2.279E-08	5.163E-07	1.468E-06
	ISSA	2.484E-12	3.025E-10	4.956E-10	1.185E-10	5.067E-09	9.099E-09

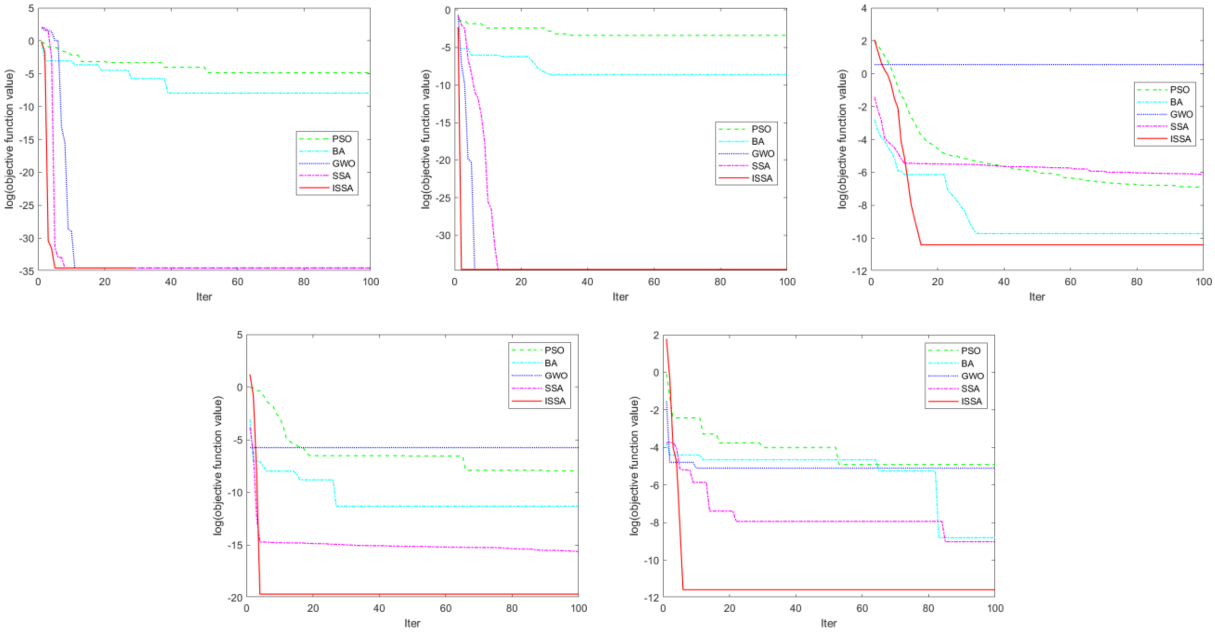

FIGURE 1. Convergence curves of single-peak test functions of 5 algorithms

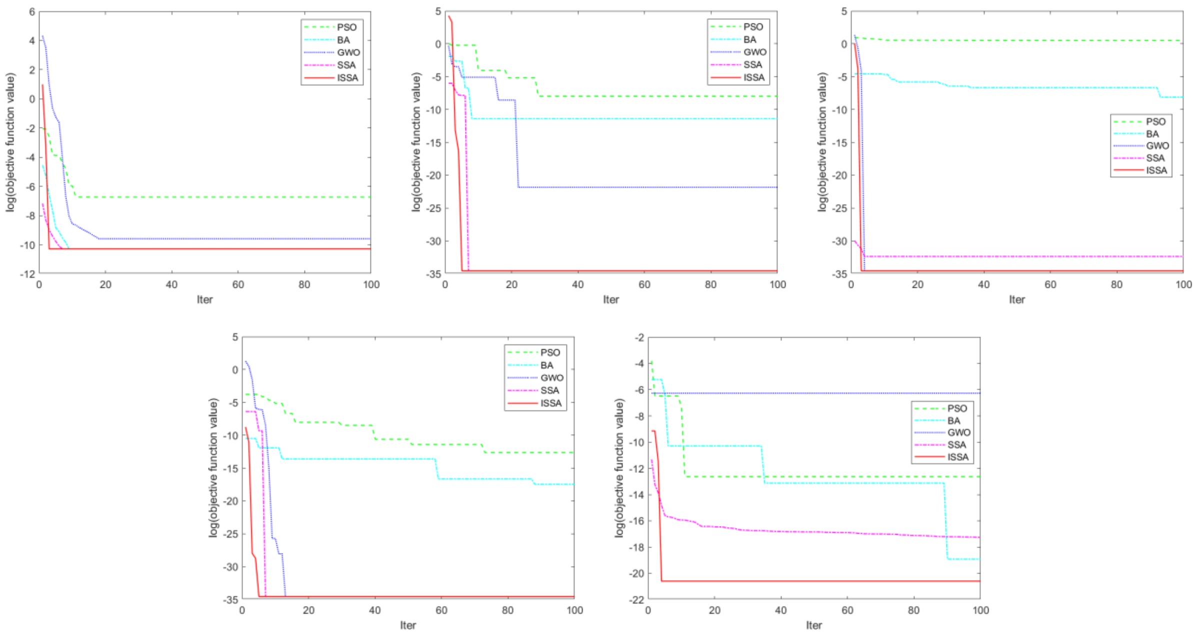

FIGURE 2. Convergence curves of multi-peak test functions of 5 algorithms

the ISSA proposed in this paper with similar algorithms in different dimensions, and the results show that ISSA has better overall performance, better convergence speed and accuracy, and good stability and robustness. There are still many problems for SSA research, which can try to introduce reverse learning into the algorithm and also combine the new ideas of this paper with other swarm intelligence algorithms.

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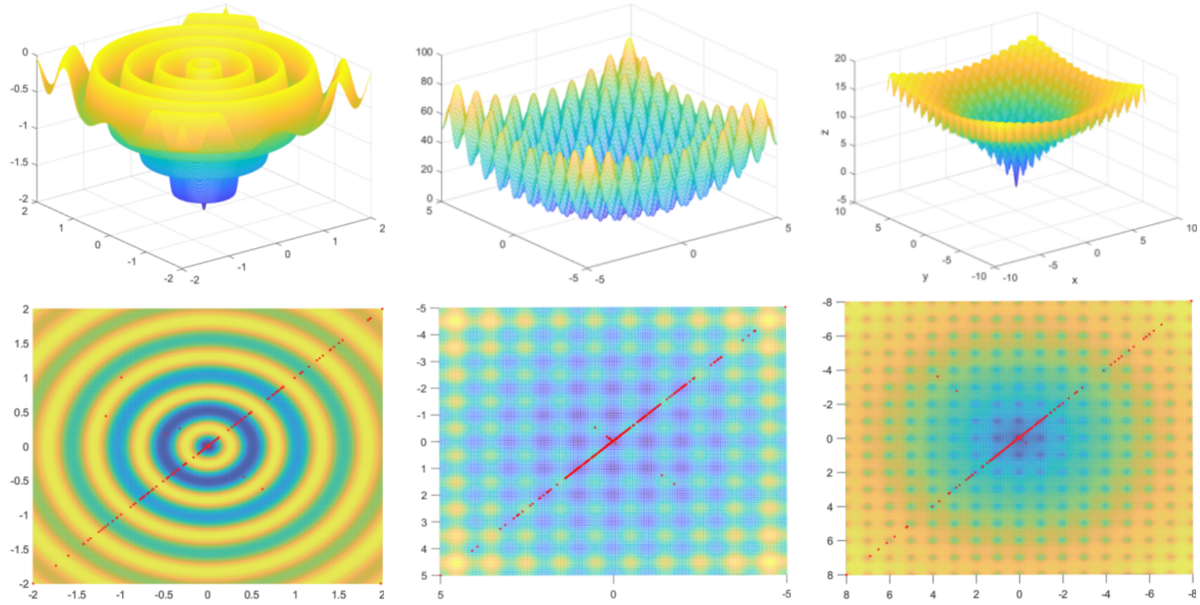

FIGURE 3. (color online) Optimization trajectory of ISSA

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