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FUZZY AUTOREGRESSIVE MODEL USING POSSIBILITY GRADE OF TIME SERIES AND ITS APPLICATIONS

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Abstract. *An autoregressive (AR) model with a fuzzy number of predictions was proposed. However, the difficulty of obtaining a prediction equation may hinder its use. Therefore, the interval AR model was combined with an interval regression model using possibility grades, and an interval AR was easily obtained. Compared with my previous fuzzy AR model, the proposed model requires only one LP to be solved and about 0.12 times less computation. In the model validation using the consumer price index, the multiple correlation coefficient of the proposed model is about 1.92 times that of the AR. In the model validation using the Nikkei Stock Average, the root mean squared errors of the proposed model are about 0.05 times those of the AR.*
Keywords: Interval regression, Interval autoregressive, Possibility grade, Consumer price index, Nikkei Stock Average

1. **Introduction.** Regression and time-series models have been used in various economic and management applications. However, there is a lack of data analysts, particularly in Japanese companies. Based on the concept of soft computing, data analysis methods have proposed models that are relatively easy to handle and that provide interval predictions that are easy to interpret [1, 2, 3, 4, 5, 6, 7, 10, 11, 16]. Among the four Box-Jenkins models, the autoregressive (AR) model is often used owing to its simple structure and ease of use.

There are several Box-Jenkins models [8, 9, 11, 12, 13, 15] based on soft computing. A fuzzy ARIMA model proposed by Tseng et al. [8] uses real-valued time series and expresses the vagueness of a time series system by using fuzzy regression coefficients. Because the time series model uses a fuzzy regression model, it is not only easy to handle, but also has good prediction accuracy. An AR model is not only simple in structure and easy to handle, but also has good prediction accuracy. There are two types of interval-type AR models that use fuzzy time series and fuzzy regression coefficients: the fuzzy AR model proposed by Ozawa introduced in [11] and the interval AR model proposed by Yabuuchi et al. [12, 13, 15]. Both models are easy to handle. Specifically, a fuzzy AR model uses an interval difference series and aims to fully illustrate the possibility of an analysis target of a difference series. In contrast, an interval AR model uses interval time series and a fuzzified Yule-Walker equation to illustrate the intrinsic trend of an analysis target. However, because interval-type time series are processed using fuzzy operations, the computational complexity of obtaining predictions is high. To solve this issue, this study combines an interval-type regression model using possibility grades with an interval-type AR model to significantly reduce computational complexity.

The remainder of this paper is organized as follows. As mentioned above, the main elements of this study were an interval AR model and an interval regression model with

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possibility grades. Section 2 briefly explains the interval AR model proposed by Yabuuchi et al. Section 3 briefly explains the interval regression model with the possibility grades. Section 4 explains the interval AR model with possibility grades for the time series proposed herein. In Section 5, the usefulness of the proposed method is verified using two numerical examples. Finally, Section 6 summarizes the study.

2. Interval Autoregressive Model.

The Box-Jenkins model has four time-series models, including an AR model and a moving average model. Among these models, the AR model is not only sufficiently flexible to be applied regardless of whether the subject of analysis is linear or nonlinear but also has good forecasting accuracy. Therefore, AR models are used in various applications. However, because the AR model uses stationary time series, it requires expertise in probability, statistics, etc. AR models are very useful time-series models; however, they may be described by users as time-series models with a high level of skill.

Yabuuchi et al. proposed an interval AR model based on soft computing [12]. This model uses an interval-valued time series $\boldsymbol{X}_t = \left[X_t^L, X_t^C, X_t^U\right]$. Here, L, C, and U denote the lower, central, and upper limits of the fuzzy numbers, respectively, and will be described in the same way in this study. The predictions of the next interval-type time series are written using the fuzzy autoregressive coefficients $\boldsymbol{b}_j = \left[b_j^L, b_j^C, b_j^U\right]$, $(j = 1, 2, \ldots, p)$ as follows:

$$\boldsymbol{X}_t = \boldsymbol{b}_1\boldsymbol{X}_{t-1} + \boldsymbol{b}_2\boldsymbol{X}_{t-2} + \cdots + \boldsymbol{b}_p\boldsymbol{X}_{t-p}. \tag{1}$$

Fuzzy autocovariance $\boldsymbol{l}_k = Cov\left[\boldsymbol{X}_t\boldsymbol{X}_{t-k}\right]$ and fuzzy autocorrelation $\boldsymbol{r}_k = \boldsymbol{l}_k/\boldsymbol{l}_0 = \left[r_k^L, r_k^C, r_k^U\right]$ are obtained using fuzzy operations. However, because the value range of the autocorrelation coefficient is $[-1, 1]$, the fuzzy autocorrelation coefficient is defined by alpha cutting such that $-1 \leq \boldsymbol{r} \leq 1$.

Autoregressive coefficients are obtained using the Yule-Walker, maximum likelihood, or least-squares methods. In this study, the fuzzified Yule-Walker method was used. In the fuzzified Yule-Walker method, the Yule-Walker equation consists of fuzzy numbers, and fuzzy autoregressive coefficients are obtained by solving a linear programming (LP) problem [14].

Using real-valued autocorrelation coefficients ρ_j $(j = 1, 2, \ldots, p)$ and autoregressive coefficients α_j, we obtain the following Yule-Walker equation:

$$\begin{bmatrix} 1 & \rho_1 & \cdots & \rho_{p-1} \\ \rho_1 & 1 & \cdots & \rho_{p-2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{p-1} & \rho_{p-2} & \cdots & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_p \end{bmatrix} = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_p \end{bmatrix}. \tag{2}$$

In a conventional AR model, autoregressive coefficients are obtained from Equation (2). However, in an interval AR model, the autocorrelation and autoregressive coefficients are both fuzzy numbers and are not equal, as shown in Equation (2). Therefore, the Yule-Walker equation using fuzzy numbers is defined as follows:

$$\begin{bmatrix} \boldsymbol{R}_1 \\ \boldsymbol{R}_2 \\ \vdots \\ \boldsymbol{R}_p \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{r}_1 & \cdots & \boldsymbol{r}_{p-1} \\ \boldsymbol{r}_1 & 1 & \cdots & \boldsymbol{r}_{p-2} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{r}_{p-1} & \boldsymbol{r}_{p-2} & \cdots & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \vdots \\ \boldsymbol{b}_p \end{bmatrix} \supseteq \begin{bmatrix} \boldsymbol{r}_1 \\ \boldsymbol{r}_2 \\ \vdots \\ \boldsymbol{r}_p \end{bmatrix}. \tag{3}$$

Here, the left-hand side of Equation (3) is written as $\boldsymbol{R}_j = \left[R_j^L, R_j^C, R_j^U\right]$ $(j = 1, 2, \ldots, p)$. In Equation (3), $\sum_{j=1}^{p}\left(R_j^U - R_j^L\right)$ can be interpreted as the vagueness of the interval AR. Because the autoregressive coefficients are not uniquely determined by Equation (3), they are attributed to the following LP problem to obtain the fuzzy autoregressive coefficients that minimize the vagueness of the interval AR.

$$\begin{aligned} &\min_{\boldsymbol{b}} \sum_{j=1}^{p} \left(R_j^U - R_j^L\right) \\ &\text{s.t.} \quad \boldsymbol{R}_j \supseteq \boldsymbol{r}_j,\ j = 1, 2, \ldots, p. \end{aligned} \tag{4}$$

The constraint $\boldsymbol{R}_j \supseteq \boldsymbol{r}_j$ in the above LP problem implies the following equation:

$$R_j^L \leq r_j^L,\ R_j^C = r_j^C,\ r_j^U \leq R_j^U. \tag{5}$$

Specifically, because an interval AR model uses fuzzy operations, the center of its interval AR coincides with that of the conventional AR.

3. **Interval Regression Model Using Possibility Grade.** The output of an interval-type fuzzy regression model is the interval value. In this regression model, the interval in which samples are observed is illustrated as the possibility of an analysis target. When the obtained samples are non-fuzzy numbers, such as $(\boldsymbol{x}_i, y_i)$, $\boldsymbol{x}_i = [x_{i1}, x_{i2}, \ldots, x_{ip}]$ $(i = 1, 2, \ldots, n)$, the predicted interval values are $\boldsymbol{Y}_i = \left[Y_i^L, Y_i^C, Y_i^U\right]$. If we want to obtain a fuzzy regression that includes all samples and illustrates the possibility of an analysis target, we have $Y_i^L \leq y_i \leq Y_i^U$. Although it is possible to include all the samples and illustrate the possibility of an analysis target, the trend of the estimated interval values may differ from that of y_i. In such cases, fuzzy regression can illustrate the trend and possibility of sample y_i by relaxing the restriction that fuzzy regression includes the samples.

A general fuzzy regression model uses the regression coefficients $\boldsymbol{A} = [\boldsymbol{a}, \boldsymbol{c}]$ for triangular fuzzy numbers, multiplied by

$$\boldsymbol{Y}_i = \boldsymbol{A}\boldsymbol{x}_i = [\boldsymbol{a}, \boldsymbol{c}]\boldsymbol{x}_i = [\boldsymbol{a}\boldsymbol{x}_i - \boldsymbol{c}|\boldsymbol{x}|_i, \boldsymbol{a}\boldsymbol{x}_i, \boldsymbol{a}\boldsymbol{x}_i + \boldsymbol{c}|\boldsymbol{x}|_i]. \tag{6}$$

In regression coefficient $\boldsymbol{A}$, $\boldsymbol{a} = [a_1, a_2, \ldots, a_p]$ is the center and $\boldsymbol{c} = [c_1, c_2, \ldots, c_p]$ $(\boldsymbol{c} \geq \boldsymbol{0})$ is the coefficient of its width. Therefore, the fuzzy regression model can be rewritten as the following LP problem:

$$\begin{aligned} &\min_{\boldsymbol{a},\boldsymbol{c}} \ \boldsymbol{c} \\ &\text{s.t.} \quad \boldsymbol{Y}_i \supseteq y_i,\ i = 1, 2, \ldots, n. \end{aligned} \tag{7}$$

Because this fuzzy regression model includes all samples in its intervals, the possibility of an analysis target illustrated by the obtained regression is distorted depending on the data distribution. Therefore, fuzzy robust regression models that reduce the distortion caused by the shape of the data distribution were investigated. Among the proposed models is a regression model that does not use the inclusion relation between the sample and regression as constraints for an LP problem [15]. A regression model was obtained by maximizing the possibility grade obtained from the predicted and observed values.

The possibility grade μ derived from samples and predicted values are as follows:

$$\mu_i = \max\left\{0, 1 - \frac{|\boldsymbol{a}\boldsymbol{x}_i - y_i|}{\boldsymbol{c}|\boldsymbol{x}_i|}\right\}. \tag{8}$$

The objective function used in the LP problem to obtain the regression was $\sum_{i=1}^{n} \mu_i/\boldsymbol{c}|\boldsymbol{x}_i|$, which is the possibility grade divided by the width of the predictions. The possibility grade is the degree to which the interval regression illustrates the possibility of samples, and the value of the possibility grade is at its maximum when the center of the interval regression coincides with the sample values. The width of the predictive value of interval regression is the vagueness of the obtained interval regression. In particular, when the value of $\mu/\boldsymbol{c}|\boldsymbol{x}|$ is large, the sample is close to the center of the predictions, $\boldsymbol{a}\boldsymbol{x}$, and the width of the predictions is small. Additionally, an interval regression model was defined to include the data distribution. Hence, the prediction accuracy of interval regression varies significantly depending on the data distribution. Specifically, samples near the periphery of the data distribution distort the possibility of what is being analyzed. In a curve-fitting method, such as regression, the predictions must illustrate the trend being analyzed. Therefore, we manage the sample included in the interval regression to obtain the appropriate regression coefficients. The LP problem can be expressed as follows:

$$\begin{aligned} \max_{\boldsymbol{a},\boldsymbol{c}} \quad & \sum_{i=1}^{n} \frac{\mu_i}{\boldsymbol{c}|\boldsymbol{x}_i|} \\ \text{s.t.} \quad & s \le s_{\max}. \end{aligned} \tag{9}$$

Parameter s is the number of samples inside the interval predictor $\boldsymbol{Y}$, and $s_{\max}$ is an upper bound for s. Because the conventional model has a constraint equation for the inclusion relationship between samples and regression, the size of the LP problem increases when the number of samples is large, making it difficult to obtain a solution. However, in the fuzzy robust regression model with this possibility grade, the size of the LP problem did not change with the number of samples. Therefore, it can be used for time series analysis.

4. **Interval Autoregressive Model Using Possibility Grade of Time Series.** In the interval AR model, an autocorrelation coefficient is calculated from fuzzy numbers, and autoregressive coefficients are obtained from the Yule-Walker equation constructed using fuzzy numbers, which complicates the task. Therefore, rather than using the method of moments to obtain the autoregressive coefficients, it is more efficient to apply the model-building method of an interval regression model that maximizes the possibility grade, as shown in Equation (9). In this study, an interval AR model obtained by maximizing the sum of the possibility grades was proposed.

The interval AR model proposed in this study obtains the autoregression shown in Equation (1). The autoregressive coefficients were obtained using the LP shown in Equation (9). The calculation procedure is as follows.

Step 1: Determination of the AR equation
Using the stationary time series, we determined the AR formula using autocorrelation coefficients.

Step 2: Set $s_{\max}$
In the proposed model, $s_{\max}$ has a significant impact on the prediction accuracy of this model. Not only is fine-tuning impossible, but the small values of parameter $s_{\max}$ increase the vagueness of the time series. Therefore, $s_{\max}$ should be set to an appropriate value based on the number and distribution of data used to construct this model. In this study, however, this value is set heuristically.

Step 3: Obtain regression coefficients
Regression coefficients are obtained using the LP of Equation (9).

The proposed model uses only fuzzy operations in model construction and, in some cases, in the conversion of original series to predictions. When using fuzzy time series, fuzzify a time series, compute a fuzzy factorial series, calculate the alpha cut value of a fuzzy autocorrelation with LP, derive the regression coefficient with LP, and compute the predictive value. Using Excel, to calculate autocorrelations up to lag 24, approximately 99 columns are calculated and two LP problems are solved. In the case of the proposed model, only 12 columns of calculations and one LP problem are required. Thus, the number of columns to calculate and LP are reduced by a factor of 0.12 and 0.5, respectively.

5. **Analysis of the Nikkei Stock Average and Consumer Price Index.** The interval AR model with possibility grades for the time series proposed in this study is applied to the Japanese consumer price index (CPI) and the Nikkei Stock Average. The CPI does not fluctuate significantly over a short period. In contrast, the Nikkei Stock Average is

an average of specific stocks, and its stock price changes relatively widely. Moreover, the Nikkei Stock Average tends to change widely over a short period. In this section, the effectiveness of the proposed model for these time series is discussed.

The CPI used in this section is based on monthly data from January 1970 to September 2013, spanning 43 years. The Nikkei Stock Average is based on monthly data from January 1970 to December 2000, spanning 30 years. In both datasets, the last two years of data were used for model validation, whereas the other data were used for model building. In particular, the CPI was used for model building from January 1970 to September 2011, and the Nikkei Stock Average was used from January 1970 to December 1998.

5.1. **Analysis of consumer price index.** For the consumer price index, the following factorial series was used to obtain a time series.

$$x_t = \Delta_6 \Delta_3 y_t = (y_t - y_{t-3}) - (y_{t-6} - y_{t-9}). \tag{10}$$

Table 1 shows the autocorrelation coefficients for the time series x_t. Owing to the large values of the autocorrelation coefficients ρ_1 and ρ_6 for lags 1 and 6, shown in Table 1, the autoregression can be obtained as follows:

$$x_t = b_{1,1} x_{t-1} + b_{1,2} x_{t-6},$$
$$\boldsymbol{X}_t = \boldsymbol{A}_{1,1} \boldsymbol{X}_{t-1} + \boldsymbol{A}_{1,2} \boldsymbol{X}_{t-6}.$$

Here, the obtained conventional AR is denoted as AR1, and the obtained interval AR is denoted as IAR1. The coefficients of IAR1 are $\boldsymbol{A}_{1,j} = [a_{1,j}, c_{1,j}]$, $j = 1, 2$, where a_j is the center of the coefficient and c_j is the width. As mentioned above, the number of samples in the time series used to build a model was 486; therefore, the interval autoregressive coefficients were obtained using $s_{1,\max} = 243$. The regression coefficients for AR1 and IAR1 are presented in Table 2. The original series and predictions obtained using AR1 and IAR1 are shown in Figure 1. Figure 1 shows that the predictions based on AR1 were more variable than those of the original series. In contrast, the IAR1 predictions did not fluctuate as much as the AR1 predictions and predicted the original series with good accuracy. Table 3 shows the correlation coefficients and root mean squared errors (RMSE) between the predictions of the two ARs and the original series. The correlation coefficients were almost unity for both the ARs. The correlation coefficient between AR1 and the original series was 0.2966, whereas the correlation coefficient between IAR1 and the original series was 0.5691 in the model validation. These correlation coefficients may be due to the small sample size of 24 during the validation period.

Table 1. Autocorrelation coefficients of the consumer price index

Lag	0	1	2	3	4	5	6	7
COR	1	0.5740	0.1679	−0.0220	−0.0769	−0.3328	−0.6085	−0.2738
Lag	8	9	10	11	12	13	14	15
COR	0.0364	0.1226	0.0572	0.1867	0.3184	0.0920	−0.1079	−0.0629

Note: COR denotes a correlation coefficient.

Table 2. Regression coefficients of the consumer price index

j		1	2
AR1	$b_{1,j}$	1.3770	−1.3989
IAR1	$a_{1,j}$	0.1667	−0.6271
	$c_{1,j}$	0	0.7063

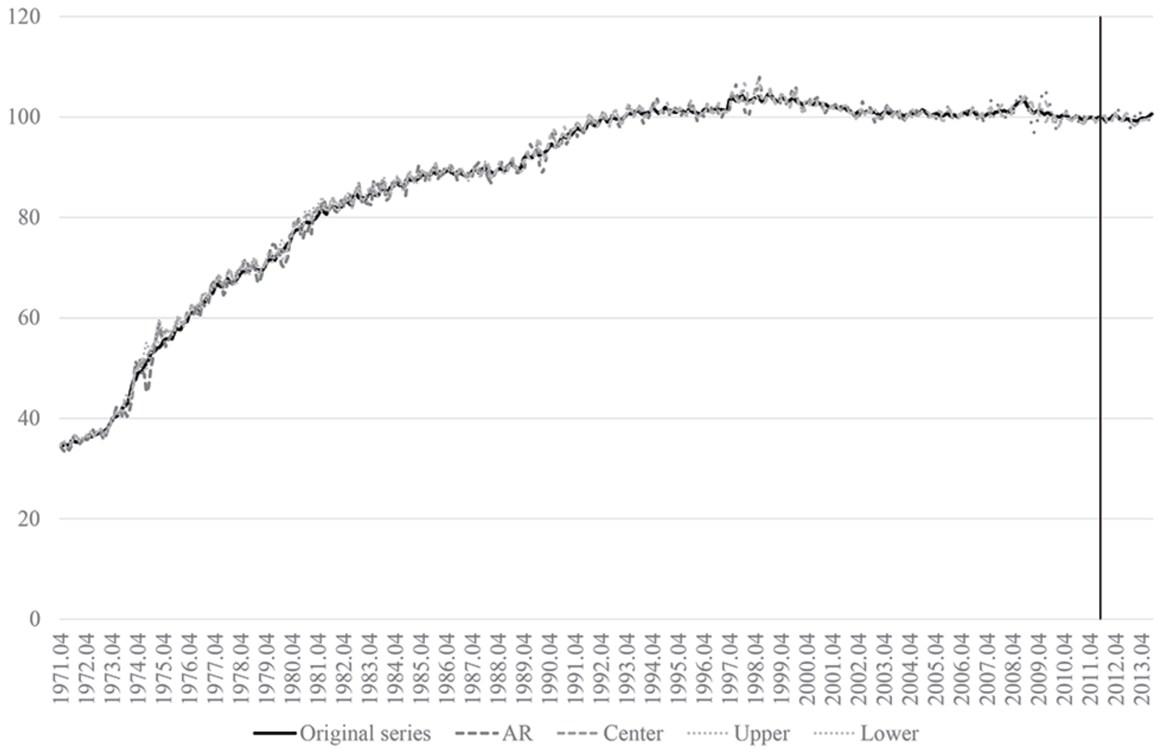

FIGURE 1. Original series of the consumer price index and forecasted values by AR1 and IAR1

TABLE 3. RMSE and correlation coefficients between the original series, AR1, and IAR1

Used samples		1970.04-2013.09	1970.01-2011.09	2011.10-2013.09
COR	AR1	0.9973	0.9974	0.2966
	IAR1	0.9993	0.9993	0.5691
RMSE	AR1	1.9041×10^3	1.9037×10^3	1.8740×10^3
	IAR1	1.9043×10^3	1.9039×10^3	1.8739×10^3

5.2. **Analysis of the Nikkei Stock Average.** For the Nikkei Stock Average, the original series y_t is transformed as follows and used in the analysis.

$$x_t = (y_t - y_{t-9})/y_{t-9}.$$

Table 4 shows the autocorrelation coefficients for the time series x_t. Table 4 shows that the correlations between lags 1 and 5 are large, leading to the following autoregressive equation:

$$x_t = b_{2,1}x_{t-1} + b_{2,2}x_{t-2} + b_{2,3}x_{t-3} + b_{2,4}x_{t-4} + b_{2,5}x_{t-5},$$
$$\boldsymbol{X}_t = \boldsymbol{A}_{2,1}\boldsymbol{X}_{t-1} + \boldsymbol{A}_{2,2}\boldsymbol{X}_{t-2} + \boldsymbol{A}_{2,3}\boldsymbol{X}_{t-3} + \boldsymbol{A}_{2,4}\boldsymbol{X}_{t-4} + \boldsymbol{A}_{2,5}\boldsymbol{X}_{t-5}.$$

Here, a conventional AR is denoted as AR2, and an interval AR is denoted as IAR2; the coefficients of IAR2 are $\boldsymbol{A}_{2,j} = [a_{2,j}, c_{2,j}]$, $j = 1, 2, 3, 4, 5$, where $a_{2,j}$ is the center of a coefficient and $c_{2,j}$ is its width. The number of samples in the time series used to build the model was 334, and the interval AR was obtained with $s_{2,\max} = 184$. The obtained autoregressive coefficients obtained for AR2 and IAR2 are listed in Table 5. The predicted values based on the original series, AR2, and IAR2 are shown in Figure 2. The results of the model validation are shown to the right of the vertical line for January 1989 in Figure 2. The correlation coefficients and RMSE between the predictions of the two ARs and the

Table 4. Autocorrelation coefficients of the Nikkei Stock Average

Lag	0	1	2	3	4	5	6	7
COR	1	0.9078	0.8335	0.7595	0.6734	0.5873	0.5006	0.4282
Lag	8	9	10	11	12	13	14	15
COR	0.3563	0.2710	0.2730	0.2647	0.2517	0.2377	0.2358	0.2203

Note: COR denotes a correlation coefficient.

Table 5. Regression coefficients of the consumer price index

j		1	2	3	4	5
AR2	$b_{2,j}$	0.8504	0.0921	0.0704	−0.0551	−0.0640
IAR2	$a_{2,j}$	1.1225	0.0384	−0.0101	−0.2084	−0.3024
	$c_{2,j}$	0	0	0.3185	0.1254	0

Figure 2. Original series of the Nikkei Stock Average and forecasted values by AR2 and IAR2

original series are listed in Table 6. During the model validation period, the correlation coefficients for AR2 and IAR2 were −0.3392 and −0.0269, respectively. Both the ARs were uncorrelated, as illustrated in Figure 2. The small sample size (24) may be related to the small correlation coefficient values.

Comparing AR2 and IAR2 again in Figure 2, it appears that the predictions of AR2 are more accurate than those of IAR2 for the period up to December 1988, when the model was being constructed. Conversely, IAR2 had a better prediction accuracy from January 1999 onward.

6. **Conclusions.** The interval AR model proposed by the author uses an interval time series that has a heavy workload. This study introduced a method for constructing an interval regression model to reduce the workload of interval AR. The proposed approach

Table 6. RMSE and correlation coefficients between the original series, AR2, and IAR2

Used samples		1970.03-2000.12	1970.01-1998.12	1999.01-2000.12
COR	AR2	0.9833	0.9887	-0.3392
	IAR2	0.9787	0.9811	-0.0269
RMSE	AR2	1.7853×10^3	1.7311×10^3	2.4158×10^3
	IAR2	1.1733×10^2	1.1675×10^2	1.1675×10^2

significantly reduced the workload. We also confirmed that the prediction accuracy was comparable to that of the conventional AR, even for time series with significant behavior change. We confirmed that the proposed method is useful not only for time series with moderate behavior change but also for time series with significant changes in behavior, such as stock prices.

This study reduced the computational complexity. However, in Figures 1 and 2, the predicted values show different behavior from the original series. The research work in this study is to improve the accuracy of the predictions.

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