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DEVELOPMENT OF SUPPORT SYSTEM FOR RATIONAL COMPREHENSIVE EVALUATION BASED ON MULTIPLE CRITERIA

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Abstract. *This study uses data envelopment analysis (DEA) and proposes a technique to allocate the efficiency of decision-making units (DMU) considered to act in a coordinated manner. It uses school grade evaluation as an example to derive a comprehensive evaluation from multiple criteria. We employed DEA and average cross-efficiency value using a cross-efficiency matrix obtained through the weights for each DMU. It also analyzes the characteristic function and the Shapley value, respectively. Finally, the study proposes a mechanism easily used on a visual program basing its processing on a server using Ajax communication.*
Keywords: DEA, Cross efficiency, Shapley value, Decision making, CPS

1. **Introduction.** With the development of information and communications technologies (ICT), all things in real space are now connected to the Internet and closely linked to virtual space, forming a new society on a global scale. Therefore, Japan has been actively working to welcome new social mechanisms, such as the concept of Society 5.0, which aims to solve social issues based on the relationship between information and the information obtained.

Since it is impossible to process a large amount of information in a virtual space by human labor, it is generally left to programs to automate the process. Therefore, programming knowledge and skills are necessary to handle information, but knowledge for learning a new language and skills is not immediately acquired. Visual programming languages were developed to provide an environment where programs could be easily created in a situation in which programming was a specialized field [1, 2]. Programs can be constructed as visual objects instead of text, which improves the unwieldiness of programming. We chose blocky as our visual programming environment because it is often used as a basis for professional analysis and research [3, 4]. In addition, DABlockly, a platform shown in Figure 1, was used to realize the unimplemented parts of Blockly, such as data processing [5].

The information processed and analyzed by the program can be utilized in real space. Moreover, when properly handling a large amount of information in a virtual space, it can be said that information has unlimited possibilities. In this context, data-driven selection by decision makers (DMs) enables them to make highly accurate decisions based on objective evidence.

Due to its capacity to identify efficient decision-making units (DMUs), data envelopment analysis (DEA) supports data-driven decisions. However, this evaluation requires

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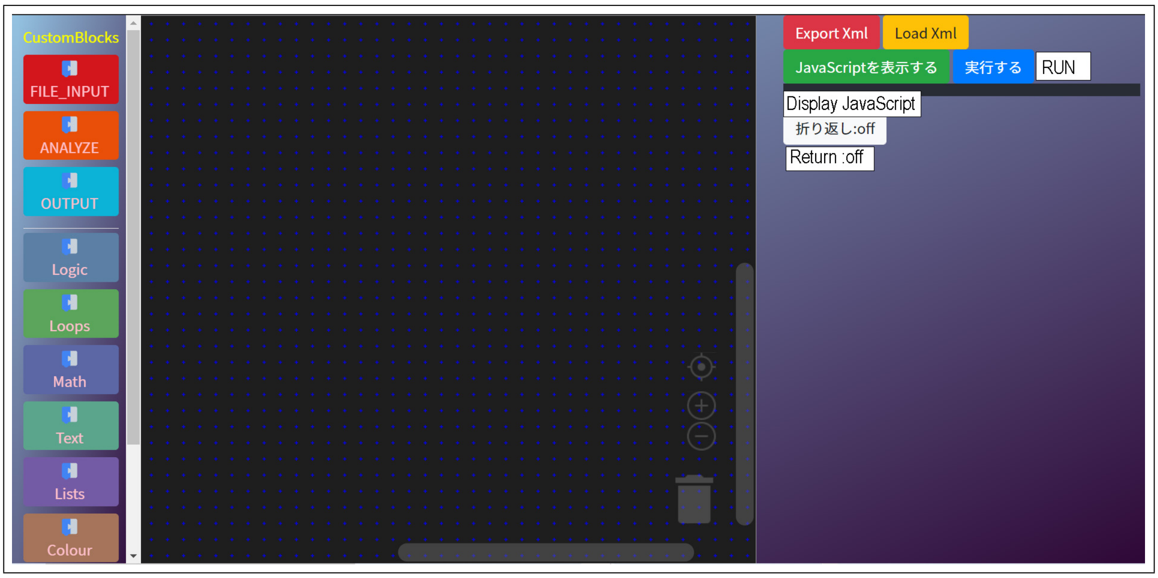

FIGURE 1. DABlockly

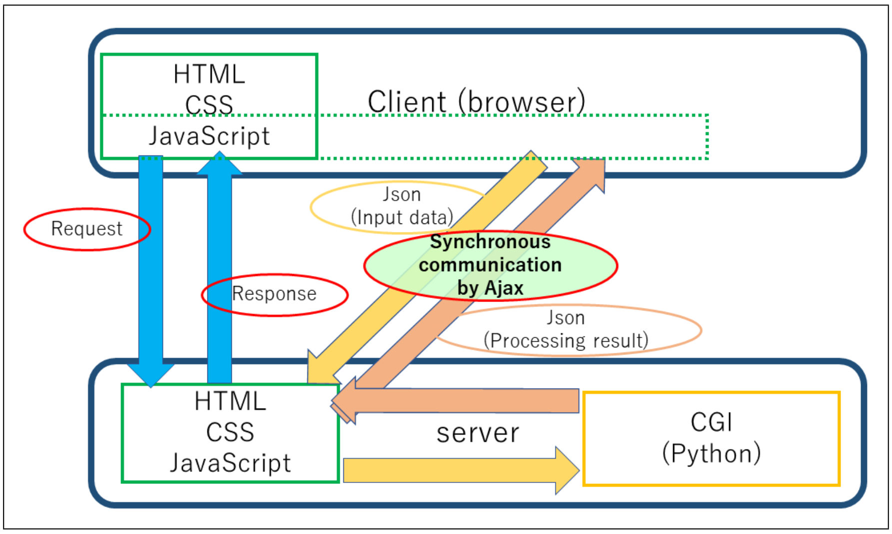

FIGURE 2. Data flow in the system of this study

the existence of weights observing joint agreement among all DMUs to avoid wrong decisions. Therefore, several evaluation methods are prepared based on several issues. When DMs have a clear ranking idea for some of the evaluators, DMs can select the appropriate method for each evaluation result to support the decision-making process made automatically for the other comprehensive evaluation results. Therefore, considering a model that enables fair evaluation is necessary, which is the focus of this study. The remainder of the paper is organized as follows. Section 2 describes the theory of multicriteria assessment for teaching and learning data, while Section 3 introduces the proposed model. Section 4 presents a numerical example and discussion, and Section 5 concludes the paper with pointers to future work.

2. Overall Evaluation from Multiple Evaluation Criteria for Teaching and Learning Data.

2.1. Various preprocessing for uncertain and indeterminate data. In this study, the target data is related to a group of students attending a practice class of English. First, students write their ID number and name on the handout and then fill in the table with a list of dates and their partner's student ID number through a checkmark in the column that matches the day's date during the training. Next, the teacher transcribes the printouts collected from each student after the exercise, with each student's number and table on Excel sheets. The resulting data on students' activity initiatives is used for analysis.

Let J be the total number of students in the class, with i the ID number of the student under evaluation, j the ID number of the person with whom the student spoke, N the total number of classes, n the number of classes, and d_{nij} a Boolean value equal to 1 if student i talked to student j in the nth class and 0, otherwise. The gender of student i is denoted by s_i as a string, m for a boy, f for a girl, and t for a teacher. In addition, a new variable r_{nij} was set to take the Boolean value whether student i spoke to student j in the nth class considering new data after preprocessing.

2.2. **Evaluation of individual and group activities.** We took the teachers' opinions for each aspect and established four criteria for evaluating personal activities and one criterion for assessing group activities. The evaluation criteria in the group were based on the PageRank method, which captures the relationships among students in a network structure. The corresponding variables for the ith student's individual and group activities are as follows: α_i for how many students they talked to; β_i for whether the activities were evenly spread among boys and girls without bias; σ_i indicates situations where activities were evenly distributed and not biased toward any person; γ_i for actively involved with people who have less activity; μ_i for evaluation of activities among students. The number of students i talked to in the nth class is as follows.

$$x_{ni} = \sum_{j=1}^{J} r_{nij} \tag{1}$$

If $x_{ni} = 0$, student i is considered absent for the nth class. The number of classes missed by student i before the nth class is as follows.

$$\lambda_{ni} = \sum_{k=1}^{n} \prod_{j=1}^{J} (1 - r_{kij}) \tag{2}$$

Normalize to a number between 0.01 and 0.99 for $x_{ni} \neq 0$ except for absent students. However, $X_{ni} = 0$ when $x_{ni} = 0$.

$$X_{ni} = \begin{cases} 0.01 + 0.98 \dfrac{x_{ni} - \min\limits_{1 \leqq j \leqq J, x_{nj} \neq 0} x_{nj}}{\max\limits_{1 \leqq j \leqq J} x_{nj} - \min\limits_{1 \leqq j \leqq J, x_{nj} \neq 0} x_{nj}} & \max\limits_{1 \leqq j \leqq J} x_{nj} - \min\limits_{1 \leqq j \leqq J, x_{nj} \neq 0} x_{nj} \neq 0 \\ 0.5 & \max\limits_{1 \leqq j \leqq J} x_{nj} - \min\limits_{1 \leqq j \leqq J, x_{nj} \neq 0} x_{nj} = 0 \end{cases} \tag{3}$$

Calculate the average of the total number of students student i has spoken to by the nth class divided by the number of classes attended.

$$\pi_{ni} = \begin{cases} \dfrac{\sum_{k=1}^{n} X_{ki}}{n - \lambda_{ni}} & \lambda_{ni} \neq 0 \\ 0 & \lambda_{ni} = 0 \end{cases} \tag{4}$$

At the end of all classes, calculate the average of the total number of people i has talked to in the Nth class divided by the number of classes attended. Assuming that the number of students who missed all classes ($\lambda_{Ni} = N$) is 0, the evaluation item α_i is calculated as the following.

$$\alpha_i = \pi_{Ni} = \frac{\sum_{k=1}^{N} X_{ki}}{N - \lambda_{Ni}} \tag{5}$$

2.3. **Deriving an overall rating from multiple evaluation criteria.** Charnes, Cooper, and Rhodes proposed DEA in 1978 as an approach to evaluating the overall efficiency of DMUs [6, 7]. One can estimate the efficiency of each DMU using different weights, and the approach has several applications [8].

Let ϕ_o be the efficiency value of the oth DMU_o ($o = 1, \ldots, N$) with N DMUs and P and Q inputs and outputs, respectively, as shown in Table 1. Let x_{op}, w_p be the pth ($p = 1, \ldots, P$) input of DMU_o and its weights, and y_{oq}, u_q be the qth ($q = 1, \ldots, Q$) output of the DMU_o and their weights. The efficiency value ϕ_o of DMU_o in the Charnes, Cooper, and Rhodes (CCR) model is formulated as follows.

$$\phi_o = \max \left\{ \frac{\sum_{q=1}^{Q} u_q y_{oq}}{\sum_{p=1}^{P} w_p x_{op}} : \frac{\sum_{q=1}^{Q} u_q y_{o'q}}{\sum_{p=1}^{P} w_p x_{o'p}} \leqq 1 \;\; o' = 1, \ldots, N, \;\; w_p, u_q \geqq 0 \;\forall p, \forall q \right\} \tag{6}$$

The objective function in Equation (6) is an expression for determining the weights of the inputs and outputs to maximize the efficiency value of DMU_o. The constraints are the equations conditioning the maximum efficiency values of all DMUs to be less than or equal to 1 and the weights of the inputs and outputs to be nonnegative numbers.

Table 1. Values of each DMU for each assessment item

	x_1	$\cdots$	x_P	y_1	$\cdots$	y_Q
DMU_1	x_{11}	$\cdots$	x_{1P}	y_{11}	$\cdots$	y_{1Q}
DMU_2	x_{21}	$\cdots$	x_{2P}	y_{21}	$\cdots$	y_{2Q}
DMU_3	x_{31}	$\cdots$	x_{3P}	y_{31}	$\cdots$	y_{3Q}
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
DMU_N	x_{N1}	$\cdots$	x_{NP}	y_{N1}	$\cdots$	y_{NQ}

3. Proposed Method.

3.1. **Average cross efficiency value based on cross efficiency analysis.** In this study, we conduct an evaluation analysis using cross-efficiency analysis [9], an extension of the conventional DEA used in Subsection 3.3. Let ϕ_o be the efficiency value of DMU_o (oth DMU) with N DMUs and P and Q input and output items, respectively. Let x_{op} be the pth ($p = 1, \ldots, P$) input of DMU_o ($o = 1, \ldots, N$) and y_{oq} be the qth ($q = 1, \ldots, Q$) output of DMU_o ($o = 1, \ldots, N$).

To obtain a cross-efficiency value from the weights of each DMU, a new variable is set up with w^*_{op} as the weight for x_{op} and u^*_{oq} as the weight for y_{oq}.

Then, using the cross-efficiency matrix created by the cross-efficiency values $\theta_{oo'}$ obtained from the mutual evaluation, the average cross-efficiency values are derived by considering the common weights (C.W.) of each $\theta_{oo'}$ as equivalent. In this case, w_i, the C.W. of the ith student, becomes

$$w_i = \frac{1}{N} \tag{7}$$

Thus, when the number of students N is, for example, 32 C.W. is as shown in Table 2. Since C.W. is equivalent, the average cross-efficiency value can be obtained as a weighted average as follows:

$$\bar{\theta_o} = \frac{1}{N} \sum_{o'=1}^{N} \theta_{o'o} \tag{8}$$

Table 3 summarizes the derived cross-efficiency value $\theta_{oo'}$, the average cross-efficiency value $\bar{\theta}_o$, and C.W. Next, each variable is defined to create a cross-efficiency matrix among the students. Under the conditions of Subsection 3.3, to distinguish the weights for each DMU, we denote the weight of α in DMU_i by u^*_{i1} and the weight of μ by u^*_{i2}, and in the output, we represent the weight of β by w^*_{i1}, the weight of γ by w^*_{i2} and the weight of σ by w^*_{i3}. Then the FP in Subsection 3.3 becomes

$$\phi_i = \max\left\{\frac{u^*_{i1}\alpha_i + u^*_{i2}\mu_i}{w^*_{i1}\beta_i + w^*_{i2}\sigma_i + w^*_{i3}\gamma_i} : \frac{u^*_{i1}\alpha_j + u^*_{i2}\mu_j}{w^*_{i1}\beta_j + w^*_{i2}\sigma_j + w^*_{i3}\gamma_j} \leqq 1 \quad j = 1, \ldots, N, \right. \\ \left. u^*_{i1}, u^*_{i2}, w^*_{i1}, w^*_{i2}, w^*_{i3} \geqq 0\right\} \tag{9}$$

Table 2. Equivalent C.W.

	DMU under evaluation				
	DMU_1	DMU_2	DMU_3	…	DMU_N
Common weight	0.03125	0.03125	0.03125	…	0.03125

Table 3. Cross-efficiency matrix and equivalent C.W.

Evaluation	DMU under evaluation					C.W.
DMU	DMU_1	DMU_2	DMU_3	…	DMU_N	
DMU_1	θ_{11}	θ_{12}	θ_{13}	$\cdots$	θ_{1N}	$1/N$
DMU_2	θ_{21}	θ_{22}	θ_{23}	$\cdots$	θ_{2N}	$1/N$
DMU_3	θ_{31}	θ_{32}	θ_{33}	$\cdots$	θ_{3N}	$1/N$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
DMU_N	θ_{N1}	θ_{N2}	θ_{N3}	$\cdots$	θ_{NN}	$1/N$
Average	$\bar{\theta}_1$	$\bar{\theta}_2$	$\bar{\theta}_3$	$\cdots$	$\bar{\theta}_N$	

The FP is converted to an LP as follows.

$$\theta_i = \max\left\{u^*_{i1}\alpha_i + u^*_{i2}\mu_i : w^*_{i1}\beta_i + w^*_{i2}\sigma_i + w^*_{i3}\gamma_i = 0, \right. \\ \left(u^*_{i1}\alpha_j + u^*_{i2}\mu_j\right) - \left(w^*_{i1}\beta_j + w^*_{i2}\sigma_j + w^*_{i3}\gamma_j\right) \leqq 0 \quad j = 1, \ldots, N, \\ \left. u^*_{i1}, u^*_{i2}, w^*_{i1}, w^*_{i2}, w^*_{i3} \geqq 0\right\} \tag{10}$$

Next, the derived weights for each DMU assessment item are used to obtain the student's cross-efficiency value, where θ_{ij} is the cross-efficiency value rated by student j with the weights of the items rated by student i.

$$\theta_{ij} = \frac{u^*_{i1}\alpha_j + u^*_{i2}\mu_j}{w^*_{i1}\beta_j + w^*_{i2}\sigma_j + w^*_{i3}\gamma_j} \tag{11}$$

Furthermore, the average cross-efficiency value $\bar{\theta}_i$ of student i, equivalent to C.W. from the cross-efficiency value, is given as follows.

$$\bar{\theta}_i = \frac{1}{N}\sum_{j=1}^{N}\theta_{ji} \tag{12}$$

The average cross-efficiency value in (12) represents the value of the student's evaluation.

3.2. **DEA game and characteristic function, Shapley value.** The average cross-efficiency value could not be reasonable for the evaluation of the DMU. Thus, we consider a mechanism using a cross-efficiency matrix based on a cooperative game.

The DEA game [10, 11] represents a game between the evaluated DMU_o, which determines the evaluation vector of the input-output items of the DEA, and the evaluator of the entire DMU, which determines the evaluation vector of the DMU [7]. In addition, solving for the characteristic function created by the cooperative game yields the Shapley value, which is an individual benefit [12]. First, to bring in the DEA game, normalize each row by dividing the cross-efficiency value of the cross-efficiency matrix by the row sum. For example, consider the normalized cross-efficiency value to be θ'_{ij}. Then this value can be obtained using Equation (13) as follows.

$$\theta'_{ij} = \frac{\theta_{ij}}{\sum_{i=1}^{N} \theta_{ij}} \tag{13}$$

Here, the C.W. of DMU_i is found through Equation (14).

$$w_i = w_i^j \tag{14}$$

While one can find the characteristic function C of j using Equation (15).

$$C(j) = \max \left\{ \sum_{i=1}^{N} w_i^j \theta'_{ij} : \sum_{i=1}^{J} w_i^j = 1,\ w_i^j \geqq 0 \ \ \forall i \right\} \tag{15}$$

$\theta'_i(S)$ represents the sum of θ'_{ij} after normalization by i and j and is given through Equation (16) as follows.

$$\theta'_i(S) = \sum_{j \in S} \theta'_{ij} \tag{16}$$

Since the dual form can obtain the same solution, the value of the characteristic function D when the number of elements in the coordination of j is 1.

$$D(j) = \min \left\{ \sum_{i=1}^{N} w_i^j \theta'_{ij} : \sum_{i=1}^{N} w_i^j = 1,\ w_i^j \geqq 0 \ \ \forall i \right\} \tag{17}$$

Here, C.W. when considering the cooperation of DMU_i is given as follows.

$$w_i = w_i^S \tag{18}$$

Therefore, the characteristic function at the time of cooperation S is as follows.

$$D(S) = \min \left\{ \sum_{i=1}^{N} w_i^S \theta'_i(S) : \sum_{i=1}^{N} w_i^S = 1,\ w_i^S \geqq 0 \ \ \forall i \right\} \tag{19}$$

In this study, the characteristic function is the following LP.

$$\begin{aligned} D(k) = \text{Minimize} \quad & u_1\alpha_d + u_2\mu_d \\ \text{Subject to} \quad & w_1\beta_d + w_2\sigma_d + w_3\gamma_d = 0 \\ & (u_1\alpha_{d'} + u_2\mu_{d'}) - (w_1\beta_{d'} + w_2\sigma_{d'} + w_3\gamma_{d'}) \leqq 0 \qquad \forall d' \\ & u_1, u_2, w_1, w_2, w_3 \geqq 0 \end{aligned} \tag{20}$$

By applying the characteristic function obtained to Equation (21), the Shapley value for each DMU is then computed.

$$\phi_i = \sum_{j \in S \subset N} \frac{(s-1)!(n-s)!}{n!} \{D(S) - D(S \setminus \{i\})\} \tag{21}$$

3.3. **Development of a comprehensive evaluation system applying common weights.** In this subsection, we develop a comprehensive evaluation system applying common weights. The least-squares calculation determines the C.W. of the DMUs derived from the Shapley values. Here, the weights derived are shown in Table 4. For the average cross efficiency values, the values of C.W. were the same, while the values of C.W. varied with the contribution level.

TABLE 4. Sharpley value and common weights

	DMU under evaluation				
	DMU_1	DMU_2	DMU_3	...	DMU_N
Shapley value	0.000330174	0.0005206707	0.0008710043	$\cdots$	0.0000951163
Common weight	0.0	0.0	0.0086897253	$\cdots$	0.0

Once C.W. is determined by the least-squares method, the efficiency value $\tilde{\theta}_i$ is obtained by the following equation.

$$\tilde{\theta}_i = \sum_{j=1}^{N} w_j^N \theta'_{ji} \tag{22}$$

4. **Numerical Experiments and Discussion.** Analysis is performed by assembling analytical blocks on Blockly that reflect the developed system, using data from an elementary school English class. Figure 3 shows a combination of those blocks using the created blocks. Figure 3 shows a block that outputs the results of the analysis by the three methods in a radar chart.

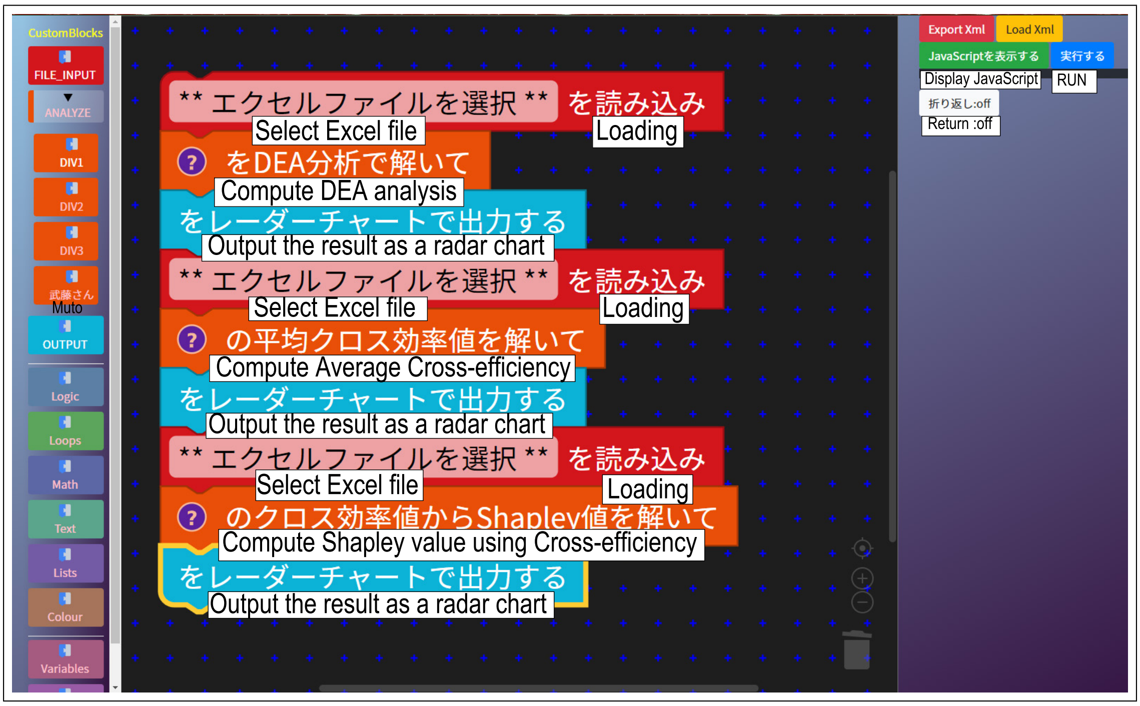

FIGURE 3. Output radar chart with the created system

The data for this experiment consisted of 32 DMUs. Using the data, the values of α and μ, β, γ, and σ for each evaluation item were derived, and the results are shown in Table 5.

TABLE 5. Value of each DMU's evaluation item

Evaluation DMU	α	μ	β	σ	γ
DMU_1	0.137458333	0.2525	0.413507372	0.17998677	0.107256512
DMU_2	0.134638889	0.15125	0.213876063	0.332998699	0.461305556
DMU_3	0.552666667	0.284166667	0.230902871	0.747628751	0.344623599
⋮	⋮	⋮	⋮	⋮	⋮
DMU_{32}	0.0541	0.0325	0.84841068	0.114410915	0.2501

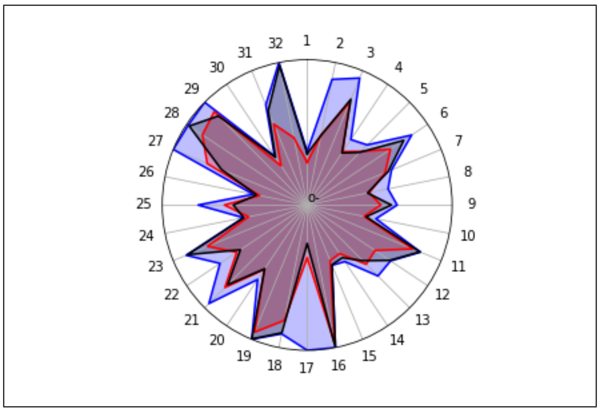

FIGURE 4. (color online) Results of all numerical experiments

TABLE 6. Three efficiency values and rankings

Student ID	DEA	Rank	Average cross efficiency value	Rank	Efficiency value considering Shapley value	Rank
1	0.368882095	32	0.296544279	32	0.288704804	32
2	0.999999997	4	0.499420417	21	0.664747636	11
3	0.945580943	9	0.78532013	5	0.75176131	7
4	0.542037829	25	0.453969747	23	0.428708134	23
5	0.584905719	23	0.524783716	19	0.517517134	21
6	0.781622979	15	0.611824796	11	0.59202568	14
⋮	⋮	⋮	⋮	⋮	⋮	⋮
27	1.000000016	1	0.746251849	8	0.749595665	8
28	0.996058988	8	0.851305423	4	0.859795521	4
29	0.999999999	3	0.890855375	3	0.896377321	3
30	0.404856112	30	0.309779421	31	0.323301568	30
31	0.745912219	16	0.601057882	12	0.568809393	16
32	0.999999988	5	0.43494452	24	0.713239178	10

Figure 4 and Table 6 summarize the results of the experiment. The blue graph shows the overall evaluation results using DEA described in Subsection 3.3. The red graph shows the overall assessment results using the average cross-efficiency values from the DEA, and the black graph shows the general assessment results after setting the weight for each DMU to estimate the allocation for the cross-efficiency values. The evaluation results by DEA show that the overall efficiency value was high, and the difference between the efficiency values for each student was small. In the evaluation by the average cross-efficiency value, only one student had the most significant efficiency value, and it varied more than in DEA. The assessment with the contribution factor showed that students with the most significant efficiency values were sparse and varied. From the above, it is considered that the proposed system can support decision-making by automatically performing other comprehensive evaluations when DM has a clear ranking of some of the evaluated students.

5. **Conclusions.** This study proposed a new allocation method using a DEA game in which DMUs are regarded as capable of cooperative behavior and developed a system that allows this mechanism to be used in visual programs. In order to derive a multiple criteria evaluation using school grades as an example, the evaluation values were obtained through the ordinary DEA. The average cross efficiency value was evaluated using a cross efficiency matrix whose elements were the weights for each DMU. We developed a mechanism that allows easy use of this mechanism on a visual program by processing by a server using Ajax communication.

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