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CONVENTIONAL SLIDING MODE CONTROL BASED ON REGULAR FORM USING DISTURBANCE OBSERVER

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Abstract. *In this paper, we examine conventional sliding mode control which can deal with unmatched disturbance. The design objective is to improve a system performance while handling unmatched disturbance. To this end, conventional sliding mode control is adopted instead of integral sliding mode control, in which the faster convergence can be expected. Moreover, the design effort can be alleviated since we only have to consider the design for a reduced order system. The basic idea is to introduce the term for disturbance compensation into the switching surface and to design the control law based on unit vector approach which can guarantee asymptotic stability. The well-known time-domain disturbance observer enable us to estimate disturbance. Using the disturbance estimate value effectively, we can design the conventional sliding mode control which is invulnerable to unmatched disturbance. The effectiveness of the proposed method is demonstrated through a numerical example.*

Keywords: Conventional sliding mode control, Regular form, Disturbance observer, Unmatched disturbance

1. **Introduction.** Sliding mode control (SMC), well-established robust control theory, has been capturing the attention because it can be applied to the wide range of control system such as linear/non-linear systems, time-invariant systems, and discrete-time systems [1].

The basic theory called conventional sliding mode control (CSMC) was first established in the 1970's [1]. CSMC has two different phases called reaching phase and sliding phase, respectively. The dynamics in these phases are determined by design procedures. One of design parameters is the so-called switching surface which determines the sliding mode dynamics. It should be designed so that sliding mode is able to have a desired dynamics such as fast convergence. Since the switching surface design is of importance, numerous studies have been carried out [1, 3, 4, 5, 6]. The effectiveness of the switching surface design based on the linear control theory was proved, and it was applied to some applications [8, 9, 10]. In addition, the most notable feature in sliding phase is an invariant property in which SMC is insensitive to any disturbances from the input channel. This powerful property is archived by an appropriate control law design.

Though it has such a powerful property, there are some potential drawbacks. For example, SMC cannot deal with unmatched uncertainty. By only using SMC, the complete robustness cannot be provided in some cases considering that there are unmatched uncertainty. However, since SMC itself has a limitation of dealing with unmatched uncertainty, the combined method has attracted much attention. H_∞ control based design was proposed in [11, 12], and the complete robustness was achieved. LMI-based control

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which provides more systematic design method has been considered by some researchers [13, 14, 15].

Except the methods mentioned above, there is another combined method which is disturbance observer based control. Disturbance observer enables us to use disturbance estimated value in the design; therefore, more intuitive design procedures can be expected. Non-linear disturbance observer based SMC was proposed in [16], and it was shown that the system was unaffected by the unmatched disturbance. Then many researchers have been tackled similar topics [17, 18]. On the other hand, linear disturbance observer based SMC was also proposed in [19, 20, 21, 22]. In [19], disturbance observer based integral sliding mode control (ISMC) was studied. Since ISMC is the method to show sliding mode at the initial point, the invariant property is satisfied during the entire time. However, it is widely known that there are some drawbacks in ISMC. First of all, ISMC cannot deal with unexpected large disturbance. If the large disturbance is added, then the system phase is changed to reaching phase. Moreover, ISMC degrades the property of fast convergence. In some applications, the faster convergence is more desirable. Hence, the motivation of this research is to achieve more faster convergence than the existing approach while dealing with unmatched disturbance.

In this paper, CSMC using linear disturbance observer will be proposed. The basic idea is to add a disturbance compensate term into switching function, and the stability analysis in sliding phase is given based on a modified switching function. It is expected that the proposed method can achieve faster convergence than the existing method. In addition, the merits of CSMC are inherited, which means that we just consider the design for reduced-order system by regular form and the initial condition of state is not required. The rest of this paper is organized as follows. The problem formulation is described in Section 2. The design procedures are given in Section 3. In order to show the effectiveness of the proposed method, Section 4 shows numerical example. Finally, we conclude this paper in Section 5.

Notations

R	the set of real numbers.
A^T	transpose of A.
$A^\dagger$	pseudo inverse of A.
$A^\perp$	orthogonal complement of A.
$\Vert \{\cdot\} \Vert$	the Euclidean norm of $\{\cdot\}$.

2. **Problem Formulation.** Consider a linear time invariant system of the form

$$\dot{x}(t) = Ax(t) + B_d d(t) + Bu(t), \tag{1}$$

where $x(t) \in R^n$ is the state variable, $d(t) \in R^q$ is the disturbance, $u(t) \in R^m$ is the input, $A \in R^{n\times n}$, $B_d \in R^{n\times q}$ and $B \in R^{n\times m}$. In the system, the disturbance comes from a different channel as the control input, which is regarded as unmatched disturbance. The disturbance in practical application basically cannot satisfy the matching condition; therefore, considering this system is of importance.

Assume that (A, B) is controllable. According to [7], the disturbance term is decomposed as

$$B_d d(t) = BB^\dagger d(t) + B^\perp B^{\perp\dagger} B_d d(t), \tag{2}$$

where $B^\dagger$ is the pseudo inverse of B and $B^\perp$ is the orthogonal complement of B. The first term is matched disturbance and the second term is unmatched disturbance. Then the original system is rewritten as

$$\dot{x}(t) = Ax(t) + B_m d(t) + B_u d(t) + Bu(t), \tag{3}$$

where $B_m = BB^\dagger B_d$ and $B_u = B^\perp B^{\perp\dagger} B_d$.

The regular form given by a similarity transformation is as follows:

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}x_2(t) + B_{u1}d(t) \tag{4}$$

and

$$\dot{x}_2(t) = A_{21}x_1(t) + A_{22}x_2(t) + B_{m2}d(t) + B_{u2}d(t) + B_2u(t), \tag{5}$$

where $x_1(t) \in R^{n-m}$, $x_2(t) \in R^m$. It is assumed that B_{u1} has no eigenvalue on the origin, B_2 is of full rank, A_{12} is of full row rank and $m \geq (n-m)/2$. Note that (A_{11}, A_{12}) is controllable since (A, B) is assumed to be controllable.

The unmatched term B_{u1} in (4) can disturb the sliding mode system. Therefore, the sliding mode does not occur even if the switching surface is properly designed in a sense of CSMC.

The problem considered in this paper is to propose CSMC which can deal with unmatched disturbance using disturbance observer.

3. **Conventional Sliding Mode Control Using Disturbance Observer.** In this section, we will explain a design method for CSMC in order to compensate unmatched disturbance.

To deal with unmatched disturbance, time-domain disturbance observer is introduced to estimate the disturbance term which is used to compensate the occurrence of sliding mode. We adopt the disturbance observer written by

$$\begin{cases} \dot{z}(t) = L\hat{d}(t) + LB_{u1}(A_{11}x_1(t) + A_{12}x_2(t)) \\ \hat{d}(t) = z(t) - LB_{u1}x_1(t) \end{cases}, \tag{6}$$

where $z(t)$ is the state of disturbance observer, $\hat{d}(t)$ is the estimated disturbance and L is an observer gain matrix chosen so that L is Hurwitz matrix. Note that the eigenvalues of L should be large enough to estimate exactly.

A design method is divided into two parts: a switching function design and a control law design. First the switching function design is addressed. The switching function is defined as

$$\sigma(t) = S_1x_1(t) + x_2(t) + K_d\hat{d}(t), \tag{7}$$

where S_1 is the switching surface designed so that $(A_{11} - A_{12}S_1)$ is stable and K_d is the disturbance compensation matrix. Assuming $\sigma(t) = 0$, then we have

$$x_2(t) = -S_1x_1(t) - K_d\hat{d}(t). \tag{8}$$

By substituting (8) into (4), we have

$$\begin{aligned} \dot{x}_1(t) &= (A_{11} - A_{12}S_1)x_1(t) - A_{12}K_d\hat{d}(t) + B_{u1}d(t) \\ &= (A_{11} - A_{12}S_1)x_1(t) + (B_{u1} - A_{12}K_d)\hat{d}(t) + B_{u1}e_d(t), \end{aligned} \tag{9}$$

where $e_d(t)$ is defined by

$$e_d(t) = d(t) - \hat{d}(t). \tag{10}$$

In order to compensate the unmatched disturbance, from (9), we need to satisfy

$$\lim_{t\to\infty} B_{u1}e_d(t) = 0 \tag{11}$$

and

$$\lim_{t\to\infty} (B_{u1} - A_{12}K_d)\hat{d}(t) = 0. \tag{12}$$

Equation (11) is satisfied by appropriately chosen observer gain L. From the assumption that A_{12} is of full row rank and $m \geq (n-m)/2$, when K_d is chosen as

$$K_d = A_{12}^{\dagger}B_{u1}, \tag{13}$$

then (12) holds true. Since the control system is designed for the reduced order system, the design is easier than the existing approach. Such advantages in CSMC are inherited to the proposed method.

Next we explain the control law. The proper control law is required to show sliding mode even if there exists unmatched disturbance. The control input is chosen as

$$\begin{aligned} u(t) = &-B_2^{-1}\left\{S_1\left(A_{11}x_1(t) + A_{12}x_2(t) + B_{u1}\hat{d}(t)\right) + A_{21}x_1(t) + A_{22}x_2(t) + B_{m2}\hat{d}(t)\right. \\ &\left. + B_{u2}\hat{d}(t)\right\} - \rho(t)B_2^{-1}\frac{\sigma(t)}{\|\sigma(t)\|}, \end{aligned} \tag{14}$$

where $\rho(t)$ is a parameter which is needed to be satisfied of the following theorem.

Theorem 3.1. *If the control input $u(t)$ in (14) is used and the parameter $\rho(t)$ satisfies*

$$\rho(t) > \|(S_1B_{u1} + B_{m2} + B_{u2} - K_dLB_{u1})\, e_d(t)\|, \tag{15}$$

then the system (4) and (5) is asymptotically stable.

Proof: From (7), $\dot{\sigma}(t)$ with (14) is derived by

$$\begin{aligned} \dot{\sigma}(t) &= S_1\dot{x}_1(t) + \dot{x}_2(t) + K_d\dot{\hat{d}}(t) \\ &= S_1(A_{11}x_1(t) + A_{12}x_2(t) + B_{u1}d(t)) + A_{21}x_1(t) + A_{22}x_2(t) + B_{m2}d(t) + B_{u2}d(t) \\ &\quad + B_2u(t) - K_dLB_{u1}e_d(t) \\ &= (S_1B_{u1} + B_{m2} + B_{u2} - K_dLB_{u1})e_d(t) - \rho(t)\frac{\sigma(t)}{\|\sigma(t)\|}. \end{aligned} \tag{16}$$

The candidate of Lyapunov function is set as

$$V(t) = \frac{1}{2}\sigma^T(t)\sigma(t), \tag{17}$$

and then the time-derivative of Lyapunov function is calculated as

$$\begin{aligned} \dot{V}(t) &= \sigma^T(t)\dot{\sigma}(t) \\ &= \sigma^T(t)\left\{(S_1B_{u1} + B_{m2} + B_{u2} - LB_{u1})\, e_d(t) - \rho(t)\frac{\sigma(t)}{\|\sigma(t)\|}\right\} \\ &\le \left\|\sigma^T(t)\right\| \|(S_1B_{u1} + B_{m2} + B_{u2} - LB_{u1})e_d(t) - \rho(t)\|. \end{aligned} \tag{18}$$

Since the switching function asymptotically tends to zero by selecting the parameter $\rho(t)$ satisfying (15), the system is asymptotically stable.

By using the switching function (7) and the control input (14) which Theorem 3.1 can be satisfied, the system (4) cannot be affected by unmatched disturbance.

4. Numerical Example.

In this section, we apply the proposed method to a linear time invariant system in (3) and compare it with the existing approach in [19].

The state-space matrices are as follows:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -2 & 0 \\ 2 & -1 & 0 & 2 \end{bmatrix},\ B_d = \begin{bmatrix} 0 & 0.5 \\ 1 & 0 \\ 0 & -0.5 \\ 1 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{19}$$

Calculating (2) provides

$$B_m = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -0.5 \\ 1 & 0 \end{bmatrix},\ B_u = \begin{bmatrix} 0 & 0.5 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{20}$$

By applying a similarity transformation to (19), we have

$$A_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\ A_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ A_{21} = \begin{bmatrix} -1 & 0 \\ 2 & -1 \end{bmatrix},\ A_{22} = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix} \tag{21}$$

$$B_{m2} = \begin{bmatrix} 0 & -0.5 \\ 1 & 0 \end{bmatrix},\ B_{u1} = \begin{bmatrix} 0 & 0.5 \\ 1 & 0 \end{bmatrix},\ B_{u2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{22}$$

$$x_1(t) = \begin{bmatrix} x_{11}(t) \\ x_{12}(t) \end{bmatrix},\ x_2(t) = \begin{bmatrix} x_{21}(t) \\ x_{22}(t) \end{bmatrix}. \tag{23}$$

First we design the switching surface using pole placement method. We choose -5 and -10 as poles, and then we have

$$S_1 = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}. \tag{24}$$

Observer gain matrix L is chosen as

$$L = \begin{bmatrix} -40 & 0 \\ 0 & -60 \end{bmatrix} \tag{25}$$

so that the eigenvalues are -40 and -60. The disturbance compensation matrix and the sliding mode controller are chosen in (13) and (14) where $\rho(t) = 2\|(S_1B_{u1} + B_{m2} + B_{u2} - LB_{u1})e_d(t)\|$.

For a comparison study, we will design a control system by following the method presented at Section III-B in [19]. The same state-space equation as the proposed method is used. However, since the method is based on output information, we consider the following output equation:

$$y(t) = Cx(t), \tag{26}$$

where

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{27}$$

The disturbance observer is designed as

$$\begin{cases} \dot{z}(t) = L\hat{d}(t) + LB_d^{\dagger}(Ax(t) + Bu(t)) \\ \hat{d}(t) = z(t) - LB_d^{\dagger}x(t) \end{cases}, \tag{28}$$

where L is observer gain matrix chosen as

$$L = \begin{bmatrix} -40 & 0 \\ 0 & -60 \end{bmatrix} \tag{29}$$

so that the eigenvalues are -40 and -60. The switching function is defined as

$$\sigma(t) = B^{\dagger}\left(x(t) - x(0) - \int_0^t \left(Ax(s) + Bu_1(s) + B_d\hat{d}(s)\right) ds\right), \tag{30}$$

where $u_1(t) = -Kx(t) - K_d\hat{d}(t)$. K is designed so that $(A - BK)$ has the eigenvalues at $-3, -5, -7$ and -10; therefore,

$$K = \begin{bmatrix} 20 & 0 & 8 & 0 \\ 2 & 49 & 0 & 17 \end{bmatrix}. \tag{31}$$

K_d is the disturbance rejection matrix designed by $K_d = \left(C(A - BK)^{-1}B\right)^{-1}C(A - BK)^{-1}B_d$:

$$K_d = \begin{bmatrix} 0 & 4.5 \\ 16 & 0 \end{bmatrix}. \tag{32}$$

The integral sliding mode controller is designed as

$$u(t) = u_1(t) + B^{\dagger} B_d u_N(t) \tag{33}$$

where

$$u_N(t) = -\rho \frac{B_d^T B^{\dagger T} \sigma(t)}{\| B_d^T B^{\dagger T} \sigma(t) \|}. \tag{34}$$

From the Lyapunov stability analysis, the design parameter is chosen as $\rho = 2$.

When the disturbance is given by

$$d_1(t) = \begin{cases} 10 & \text{if } 1 \le t \le 1.5 \\ 0 & \text{if } 0 \le t < 1,\ t > 1.5 \end{cases},\ d_2(t) = 0 \tag{35}$$

and the initial conditions of $x_1(t)$ and $x_2(t)$ are

$$x_1(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ x_2(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{36}$$

the response of $x_1(t)$ and that of $x_2(t)$ are shown in Figure 1 and Figure 2, respectively.

From Figure 1 and Figure 2, it can be clearly seen that the state response in the proposed method reaches the origin faster than the existing approach. Moreover, the

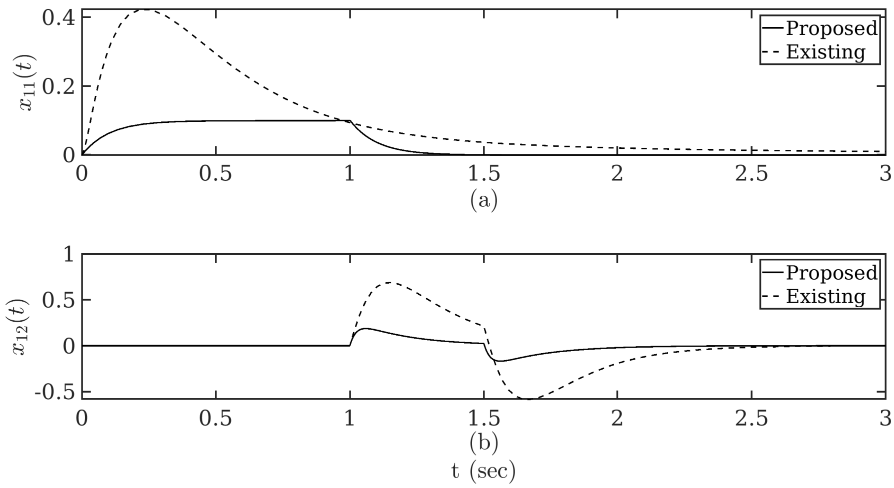

FIGURE 1. Response of the state variables $x_1(t)$: (a) $x_{11}(t)$ and (b) $x_{12}(t)$

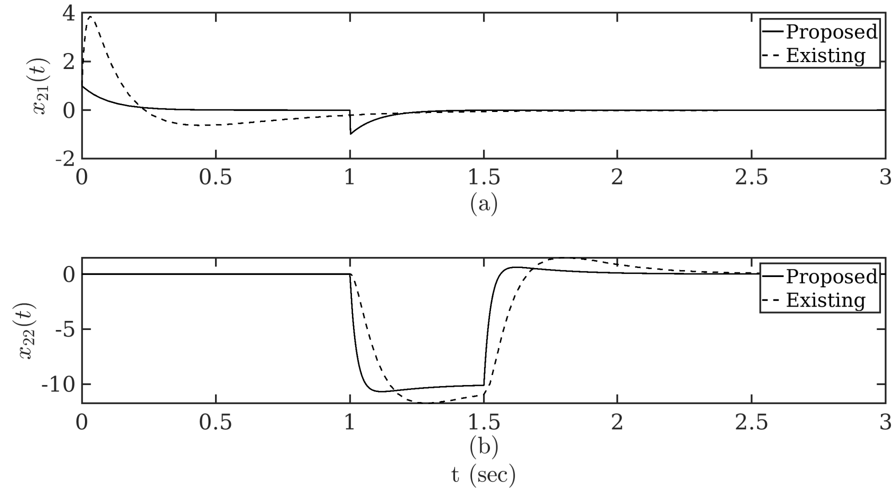

FIGURE 2. Response of the state variables $x_2(t)$: (a) $x_{21}(t)$ and (b) $x_{22}(t)$

proposed method does not require the output information. Apparently, the design is easier than the existing approach.

5. **Conclusions.** In this paper, we have proposed CSMC using disturbance observer for unmatched disturbance compensation. The switching surface including disturbance compensation term is given. The disturbance observer is designed for reduced order system by regular form; therefore, the design is easier than the existing approach. Moreover, the state variables reach the origin faster than the existing approach. The numerical example demonstrated that the proposed method asymptotically stabilized the system with the unmatched disturbance.

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