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APPLICATIONS OF FUZZY SETS ON ALMOST INTERIOR IDEALS OF PARTIALLY ORDERED SEMIGROUPS

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Abstract. *Ideal theory plays an important role in studying in ring theory and semigroup theory. A partially ordered semigroup is one of generalizations of the classical semigroup. Many results in semigroups were extended to results in partially ordered semigroups. Many results in ideal theory of partially ordered semigroups were widely investigated. In this paper, we study two almostity of interior ideals of partially ordered semigroups and applications of fuzzy sets on both two almostity of interior ideals of partially ordered semigroups. We define almost interior ideals and weakly almost interior ideals of partially ordered semigroups. Every almost interior ideal of a partially ordered semigroup is clearly a weakly almost interior ideal but the converse is not true in general. The notions of both almost interior ideals and weakly almost interior ideals of partially ordered semigroups are generalizations of the notion of interior ideals. We investigate basic properties of both almost interior ideals and weakly almost interior ideals of partially ordered semigroups. Moreover, we introduce fuzzifications of almost interior ideals and weakly almost interior ideals of partially ordered semigroups. We investigate their properties. Finally, we give some relationship between almost interior ideals (weakly almost interior ideals) and their fuzzifications.*
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1. **Introduction.** In 1965, Zadeh [1] first introduced the notion of fuzzy subsets. Applications of fuzzy subsets have been developed in many fields. In this communication, we will focus on applications of fuzzy sets in algebraic structures. Rosenfeld applied fuzzy subsets on groups. He defined fuzzy subgroups of groups and investigated their remarkable properties in [2]. Applications of fuzzy subsets in semigroups were first considered by Kuroki [3]. Later, fuzzy subsets were studied in many algebraic structures. The definition of almost ideals of semigroups (or A-ideals) was first studied by Grosek and Satko [4]

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in 1980. Now, the notions of almost ideals in semigroups were extend to some generalizations of semigroups, for example, almost ideals in ternary semigroups [5], and almost hyperideals in semihypergroups [6]. Wattanatripop et al. [7, 8] first studied application of fuzzy subsets for almostity of ideals of semigroups and showed relationship between almost ideals and their fuzzifications of semigroups. Later, the applications of fuzzy sets for almost ideals were defined and studied in many algebraic structures, for example, in ternary semigroups [5], in Γ-semigroups [9], in semihypergroups [10], and in LA-semihypergroups [11]. Furthermore, Kaopusek et al. [12] using the concepts of interior ideals and almost ideals of semigroups, defined the notions of almost interior ideals and weakly almost interior ideals of semigroups. Moreover, they investigated their basic properties. Later, fuzzifications of almost ideals in [12] were studied in [13]. A partially ordered semigroup (shortly, a po-semigroup) is a generalization of the classical semigroup (see [14]). Many results in semigroups were extended to results in po-semigroups (for example, see [15]). Recently, almost ideals of a po-semigroup were first studied in [16, 17]. For the motivations of this paper, we will extend both almost interior ideals and weakly almost interior ideals and their fuzzifications in semigroups to that in po-semigroups. The results in [12, 13] will be special cases of the results of this paper. First, we will recall the basic definitions and notations in Section 2. For the purpose of this paper, we will define almost interior ideals and weakly almost interior ideals of po-semigroups in Section 3 and apply fuzzy sets to studying fuzzifications of both almost interior ideals and weakly almost interior ideals of po-semigroups in Section 4. We conclude significant results of this paper and suggest some future works in Section 5.

2. **Preliminaries.** Firstly, we recall the definitions and some notations of fuzzy sets. A fuzzy subset of a set S is a membership function from S into the closed unit interval $[0,1]$. Let S be any set and f and g be any two fuzzy subsets of S.

(1) The *intersection* of f and g (using the notation $f \cap g$) is a fuzzy subset of S defined as follows:
$$(f \cap g)(x) = \min\{f(x), g(x)\} \text{ for all } x \in S.$$
(2) The *union* of f and g (using the notation $f \cup g$) is a fuzzy subset of S defined as follows:
$$(f \cup g)(x) = \max\{f(x), g(x)\} \text{ for all } x \in S.$$
(3) $f \subseteq g$ if $f(x) \leq g(x)$ for all $x \in S$, and we say that f is a *fuzzy subset* of g.

For any fuzzy subset f of any set S, the *support* of f (using the notation $supp(f)$) is a subset of S defined by $supp(f) = \{x \in S \mid f(x) \neq 0\}$. We note that if $f \subseteq g$, then we have that $f \circ h \subseteq g \circ h$ and $supp(f) \subseteq supp(g)$ for any fuzzy subsets f, g, h.

For any subset A of any set S, the *characteristic mapping* C_A of A is a fuzzy subset of S defined by
$$C_A(x) = \begin{cases} 1 & x \in A, \\ 0 & x \notin A. \end{cases}$$

For any element x of any set S and a real number $t \in (0,1]$, a *fuzzy point* x_t ([18]) of S is a fuzzy subset of S defined by
$$x_t(y) = \begin{cases} t & y = x, \\ 0 & y \neq x. \end{cases}$$

A semigroup is a nonempty set S together with a binary operation $\cdot\colon S \times S \longrightarrow S$ that satisfies the associative property $(xy)z = x(yz)$ for all $x, y, z \in S$. The algebraic structure $(S, \cdot, \leq)$ is called a *partially ordered semigroup* or an *ordered semigroup* (shortly, a po-semigroup) if

(1) $(S, \cdot)$ is a semigroup,

(2) $(S, \leq)$ is a partially ordered set, and
(3) for all $x, y, z \in S$, if $x \leq y$, then $xz \leq yz$ and $zx \leq zy$.

Let S be a po-semigroup. For a nonempty subset A of S, we denote $(A] := \{x \in S \mid x \leq a \text{ for some } a \in A\}$. Let A and B be any two nonempty subsets of a po-semigroup S. The following statements hold.

(1) $A \subseteq (A]$.
(2) If $A \subseteq B$, then $(A] \subseteq (B]$.
(3) $(A \cap B] \subseteq (A] \cap (B]$ and $(A \cup B] = (A] \cup (B]$.

For nonempty subsets A and B of a po-semigroup S, we denote $AB := \{ab \mid a \in A \text{ and } b \in B\}$. A nonempty subset A of a po-semigroup S is called a *subsemigroup* of S if $AA \subseteq A$. A subsemigroup I of S is called an *interior ideal* of S if (1) $SIS \subseteq I$ and (2) if $x \in I$ and $s \in S$ such that $s \leq x$, then $s \in I$, that is, $(I] \subseteq I$.

Let $F(S)$ be the set of all fuzzy subsets of an ordered semigroup $(S, \cdot, \leq)$. For any $f, g \in F(S)$ and $x \in S$, we define the *product* of f and g by $f \circ g : S \longrightarrow [0, 1]$ such that

$$(f \circ g)(x) := \begin{cases} \sup\limits_{x \leq uv} \min\{f(u), g(v)\} & \text{if } x \leq uv \text{ where } u, v \in S, \\ 0 & \text{otherwise.} \end{cases}$$

Let f, g and h be fuzzy subsets of a po-semigroup $(S, \cdot, \leq)$. If $f \subseteq g$, then $f \circ h \subseteq g \circ h$. A fuzzy set f of S is called a *fuzzy subsemigroup* of S if $f(xy) \geq \min\{f(x), f(y)\}$ for all $x, y \in S$. A fuzzy subsemigroup f of S is called a *fuzzy interior ideal* of S if for all $x, y \in S$, (1) if $x \leq y$, then $f(x) \geq f(y)$ and (2) $(f \circ 1) \wedge (1 \circ f) \subseteq f$. For a fuzzy subset f of S, we defined $(f] : S \longrightarrow [0, 1]$ by

$$(f](x) = \sup_{x \leq y} f(y) \text{ for all } x \in S.$$

Let f, g and h be fuzzy subsets of an ordered semigroup S. The following statements hold.

(1) $f \subseteq (f]$.
(2) If $f \subseteq g$, then $(f] \subseteq (g]$.
(3) If $f \subseteq g$, then $(f \circ h] \subseteq (g \circ h]$ and $(h \circ f] \subseteq (h \circ g]$.

For a fuzzy subset f of S. The following statements are equivalent.

(1) If $x \leq y$, then $f(x) \geq f(y)$.
(2) $(f] = f$.

3. **Almost Interior Ideals.**

In this section, we give the definitions of almost interior ideals and weakly almost interior ideals of po-semigroups. Moreover, we investigate their properties.

Definition 3.1. *Let S be a po-semigroup. A nonempty subset I of S is called an* almost interior ideal *of S if $(aIb] \cap I \neq \emptyset$ for all $a, b \in S$.*

Theorem 3.1. *Let S be a po-semigroup. Every interior ideal of S is an almost interior ideal of S.*

Proof: Let I be any interior ideal of a po-semigroup S and let a, b be any two elements of S. Then $(aIb] \subseteq I$. This implies that $(aIb] \cap I \neq \emptyset$. We can conclude that I is an almost interior ideal of S. $\square$

We show that the converse of Theorem 3.1 is not generally true in Example 3.1.

Example 3.1. *We consider the po-semigroup $\mathbb{Z}_5$ under the addition with the partial order $\leq = id_{\mathbb{Z}_5}$. We have that $I = \{\overline{0}, \overline{2}, \overline{3}\}$ is an almost interior ideal of $\mathbb{Z}_5$. It is easy to see that I is not an interior ideal of $\mathbb{Z}_5$.*

Theorem 3.2. *Let I and J be any two nonempty subsets of a po-semigroup S such that $I \subseteq J$. If I is an almost interior ideal of S, then J is also an almost interior ideal of S.*

Proof: Let H be a subset of S containing I and let $a, b \in S$. Then $(aIb] \cap I \subseteq (aJb] \cap J$. We have $(aJb] \cap J \neq \emptyset$ because $(aIb] \cap I \neq \emptyset$. Hence, J is an almost interior ideal of S. □

From Theorem 3.2, we have the following corollary.

Corollary 3.1. *Let S be a po-semigroup and I_1, I_2 be any two almost interior ideals of a po-semigroup S. Thus, $I_1 \cup I_2$ is also an almost interior ideal of S.*

Proof: Since $I_1 \subseteq I_1 \cup I_2$, by Theorem 3.2, $I_1 \cup I_2$ is an almost interior ideal of S. □

Example 3.2. *We consider the po-semigroup $\mathbb{Z}_5$ under the addition with the partial order $\leq = id_{\mathbb{Z}_5}$. Let $I_1 = \{\overline{0}, \overline{2}, \overline{3}\}$ and $I_2 = \{\overline{1}, \overline{3}, \overline{4}\}$. It is easy to show that I_1 and I_2 are almost interior ideals of $\mathbb{Z}_5$. However, we have that $I_1 \cap I_2 = \{\overline{3}\}$ is not an almost interior ideal of $\mathbb{Z}_5$.*

From Example 3.2, we can note that the intersection of almost interior ideals of a po-semigroup need not be an almost interior ideal.

Definition 3.2. *Let S be a po-semigroup. A nonempty subset I of S is called a* weakly almost interior ideal *of S if $(aIa] \cap I \neq \emptyset$ for all $a \in S$.*

We note that every almost interior ideal of a po-semigroup S is a weakly almost interior ideal of S. However, the converse is not true by the following example.

Example 3.3. *We consider the po-semigroup $\mathbb{Z}_4$ under the addition with the partial order $\leq = id_{\mathbb{Z}_4}$. We see that $I = \{\overline{0}, \overline{2}\}$ is a weakly interior ideal but not an almost interior ideal of $\mathbb{Z}_4$.*

Theorem 3.3. *Let I and J be any two nonempty subsets of a po-semigroup S such that $I \subseteq J$. If I is a weakly almost interior ideal of S, then J is also a weakly almost interior ideal of S.*

Proof: Let a be any element of S. Since $I \subseteq J$, $(aIa] \cap I \subseteq (aJa] \cap J$. So $(aJa] \cap J \neq \emptyset$ because $(aIa] \cap I \neq \emptyset$. Therefore, J is a weakly almost interior ideal of S. □

From Theorem 3.3, the following corollary holds.

Corollary 3.2. *Let I_1 and I_2 be any weakly almost interior ideals of a po-semigroup S. Then $I_1 \cup I_2$ is also a weakly almost interior ideal of S.*

Proof: Since $I_1 \subseteq I_1 \cup I_2$, it follows from Theorem 3.3 that $I_1 \cup I_2$ is a weakly almost interior ideal of S. □

Example 3.4. *We consider the po-semigroup $\mathbb{Z}_4$ under the addition with the partial order $\leq = id_{\mathbb{Z}_4}$. Let $I_1 = \{\overline{0}, \overline{2}\}$ and $I_2 = \{\overline{1}, \overline{3}\}$. We see that both I_1 and I_2 are weakly almost interior ideals of $\mathbb{Z}_4$. However, we find that $I_1 \cap I_2 = \emptyset$ is not a weakly almost interior ideal of $\mathbb{Z}_4$.*

From Example 3.4, we can note that if I_1 and I_2 are any two weakly almost interior ideals of a po-semigroup, then $I_1 \cap I_2$ need not be a weakly almost interior ideal.

In the last theorem of this section, we give the necessary and sufficient condition for po-semigroups having no proper weakly almost interior ideals.

Theorem 3.4. *A po-semigroup S has no proper weakly almost interior ideals if and only if for each element a of S, there exists b_a such that $(b_a(S \setminus \{a\})b_a] = \{a\}$.*

Proof: Assume that a po-semigroup S has no proper weakly almost interior ideals. Let a be any fixed element of S. So $S \setminus \{a\}$ is not a weakly almost interior ideal of S. This implies that there exists an element $b_a \in S$ such that $(b_a(S \setminus \{a\})b_a] \cap (S \setminus \{a\}) = \emptyset$. Therefore, $(b_a(S \setminus \{a\})b_a] = \{a\}$.

Conversely, we let A be any proper subset of S. Thus, $A \subseteq S \setminus \{a\}$ for some $a \in S$. By assumption, there exists an element $b_a \in S$ such that $(b_a(S \setminus \{a\})b_a] = \{a\}$. Then $(b_a A b_a] \cap A \subseteq (b_a(S \setminus \{a\})b_a] \cap S \setminus \{a\} = \{a\} \cap S \setminus \{a\} = \emptyset$, this implies that $(b_a A b_a] \cap A = \emptyset$. Therefore, A is not a weakly almost interior ideal of S. □

4. **Fuzzy Almost Interior Ideals.** In this section, we focus on applying fuzzy set theory to studying fuzzifications of both almost interior ideals and weakly almost interior ideals of po-semigroups. Moreover, we investigate their basic properties and show the relationship between their fuzzification and them.

Now, we define fuzzifications of almost interior ideals of po-semigroups as follows.

Definition 4.1. *A fuzzy subset f of a po-semigroup S is said to be a* fuzzy almost interior ideal *of S if for all fuzzy points x_{t_1}, y_{t_2} of S, we have $(x_{t_1} \circ f \circ y_{t_2}] \cap f \neq 0$.*

Example 4.1. *We consider the po-semigroup $\mathbb{Z}_5$ under the addition with the partial order $\leq = id_{\mathbb{Z}_5}$. Define a fuzzy subset f of $\mathbb{Z}_5$ by*

$$f\left(\overline{0}\right) = 0,\ f\left(\overline{1}\right) = 0.6,\ f\left(\overline{2}\right) = 0,\ f\left(\overline{3}\right) = 0.1,\ f\left(\overline{4}\right) = 0.2.$$

It is easy to see that f is a fuzzy almost interior ideal of $\mathbb{Z}_5$.

Next, fuzzification of weakly almost interior ideals of po-semigroups is defined as follows.

Definition 4.2. *A fuzzy subset f of a po-semigroup S is said to be a* fuzzy weakly almost interior ideal *of S if for all fuzzy points x_{t_1}, x_{t_2} of S, we have $(x_{t_1} \circ f \circ x_{t_2}] \cap f \neq 0$.*

Furthermore, we note that every fuzzy almost interior ideal of a po-semigroup S is a fuzzy weakly almost interior ideal of S.

Example 4.2. *We consider the po-semigroup $\mathbb{Z}_6$ under the addition with the partial order $\leq = id_{\mathbb{Z}_6}$. Define a fuzzy subset f of $\mathbb{Z}_6$ by $f\left(\overline{0}\right) = 0.6$, $f\left(\overline{1}\right) = 0$, $f\left(\overline{2}\right) = 0$, $f\left(\overline{3}\right) = 0$, $f\left(\overline{4}\right) = 0.3$ and $f\left(\overline{5}\right) = 0$. We have that f is a fuzzy weakly almost interior ideal of $\mathbb{Z}_6$ but not a fuzzy almost interior ideal of $\mathbb{Z}_6$.*

Theorem 4.1. *Let f and g be any two fuzzy subsets of a po-semigroup S such that $f \subseteq g$. Suppose that f is a fuzzy almost interior ideal of S. Then g is also a fuzzy almost interior ideal of S.*

Proof: Let x_{t_1} and y_{t_2} be any two fuzzy points of S. We note that x and y are elements in S and t_1 and t_2 are real numbers where $t_1, t_2 \in (0, 1]$. Since f is a fuzzy almost interior ideal of S, $(x_{t_1} \circ f \circ y_{t_2}] \cap f \neq 0$. Since $f \subseteq g$, $(x_{t_1} \circ f \circ y_{t_2}] \cap f \subseteq (x_{t_1} \circ g \circ y_{t_2}] \cap g$. This implies that $(x_{t_1} \circ g \circ y_{t_2}] \cap g \neq 0$. We can conclude that g is a fuzzy almost interior ideal of S. □

From Theorem 4.1, the following corollary holds.

Corollary 4.1. *Let f and g be any two fuzzy almost interior ideals of a po-semigroup S. Thus, $f \cup g$ is also a fuzzy almost interior ideal of S.*

Proof: Since $f \subseteq f \cup g$, by Theorem 4.1, $f \cup g$ is a fuzzy almost interior ideal of S. □

Theorem 4.2. *Let f and g be any two fuzzy subsets of a po-semigroup S such that $f \subseteq g$. Assume that f is a fuzzy weakly almost interior ideal of S. Then g is also a fuzzy weakly almost interior ideal of S.*

Proof: The proof of this theorem is similar to the proof of Theorem 4.1. □

We have the following corollary holds by Theorem 4.2.

Corollary 4.2. *Let f and g be any two fuzzy weakly almost interior ideals of a po-semigroup S. Thus, $f \cup g$ is also a fuzzy weakly almost interior ideal of S.*

Proof: The proof of this corollary is similar to the proof of Corollary 4.1. □

Example 4.3. *We consider the po-semigroup $\mathbb{Z}_5$ under the addition with the partial order $\leq = id_{\mathbb{Z}_5}$. Define fuzzy subsets f and g of $\mathbb{Z}_5$ by $f(\overline{0}) = 0.1$, $f(\overline{1}) = 0$, $f(\overline{2}) = 0.1$, $f(\overline{3}) = 0.4$, $f(\overline{4}) = 0$ and $g(\overline{0}) = 0$, $g(\overline{1}) = 0.3$, $g(\overline{2}) = 0$, $g(\overline{3}) = 0.4$, $g(\overline{4}) = 0.5$. We have that both f and g are fuzzy almost interior ideals of $\mathbb{Z}_5$ but $f \cap g$ is not a fuzzy almost interior ideal of $\mathbb{Z}_5$.*

By Example 4.3, we can note that if f and g are any two fuzzy almost interior ideals of po-semigroups, then $f \cap g$ need not be a fuzzy almost interior ideal.

Example 4.4. *We consider the po-semigroup $\mathbb{Z}_6$ under the addition with the partial order $\leq = id_{\mathbb{Z}_6}$. Define fuzzy subsets f and g of $\mathbb{Z}_6$ by $f(\overline{0}) = 0.1$, $f(\overline{1}) = 0$, $f(\overline{2}) = 0.3$, $f(\overline{3}) = 0$, $f(\overline{4}) = 0$, $f(\overline{5}) = 0$ and $g(\overline{0}) = 0.5$, $g(\overline{1}) = 0$, $g(\overline{2}) = 0$, $g(\overline{3}) = 0$, $g(\overline{4}) = 0.8$, $g(\overline{5}) = 0$. We have that f and g are fuzzy weakly almost interior ideals of $\mathbb{Z}_6$ but $f \cap g$ is not a fuzzy weakly almost interior ideal of $\mathbb{Z}_6$.*

Likewise, we can conclude that if f and g are any two fuzzy weakly almost interior ideals of po-semigroups, then $f \cap g$ need not be a fuzzy weakly almost interior ideal.

Theorem 4.3. *Let I be a nonempty subset of a po-semigroup S. Then I is an almost interior ideal of S if and only if C_I is a fuzzy almost interior ideal of S.*

Proof: Assume that I is any almost interior ideal of a po-semigroup S and let x_{t_1} and y_{t_2} be any two fuzzy points of S. We have $(xIy] \cap I \neq \emptyset$. So there exists an element $z \in S$ such that $z \in (xIy]$ and $z \in I$. Thus, $z \leq xay$ for some $a \in I$. So $(x_{t_1} \circ C_I \circ y_{t_2}](z) \neq 0$ and $C_I(z) = 1$. Hence, $(x_{t_1} \circ C_I \circ y_{t_2}] \cap C_I \neq 0$. We can conclude that C_I is a fuzzy almost interior ideal of S.

Conversely, suppose that C_I is a fuzzy almost interior ideal of S. Let x, y be any two elements of S. Then $(x_{t_1} \circ C_I \circ y_{t_2}] \cap C_I \neq 0$ for all real numbers $t_1, t_2 \in (0, 1]$. Then there exists $z \in S$ such that $[(x_{t_1} \circ C_I \circ y_{t_2}] \cap C_I](z) \neq 0$. Hence, $z \in (xIy] \cap I$. So $(xIy] \cap I \neq \emptyset$. Consequently, I is an almost interior ideal of S. □

The proof of this following theorem is similar to the proof of Theorem 4.3.

Theorem 4.4. *Let I be a nonempty subset of a po-semigroup S. We have that I is a weakly almost interior ideal of S if and only if C_I is a fuzzy weakly almost interior ideal of S.*

Theorem 4.5. *Let f be a fuzzy subset of a po-semigroup S. Then f is a fuzzy almost interior ideal of S if and only if $supp(f)$ is an almost interior ideal of S.*

Proof: Let f be any fuzzy almost interior ideal of a po-semigroup S. Let x_{t_1} and y_{t_2} be any two fuzzy points of S. Thus, $(x_{t_1} \circ f \circ y_{t_2}] \cap f \neq 0$. Hence, there exists an element $a \in S$ such that $[(x_{t_1} \circ f \circ y_{t_2}] \cap f](a) \neq 0$. So $f(a) \neq 0$ and there exists an element $b \in S$ such that $a \leq xby$ and $f(b) \neq 0$. That is $a, b \in supp(f)$. Thus, $\left(x_{t_1} \circ C_{supp(f)} \circ y_{t_2}\right](a) \neq 0$ and $C_{supp(f)}(a) \neq 0$. Therefore, $\left(x_{t_1} \circ C_{supp(f)} \circ y_{t_2}\right] \cap C_{supp(f)} \neq 0$. Hence, $C_{supp(f)}$ is a fuzzy almost interior ideal of S. By Theorem 4.3, we can conclude that $supp(f)$ is an almost interior ideal of S.

To prove the converse, we suppose that $supp(f)$ is an almost interior ideal of S. By Theorem 4.3, we have that $C_{supp(f)}$ is a fuzzy almost interior ideal of S. Let x_{t_1} and y_{t_2} be any two fuzzy points of S. Then $(x_{t_1} \circ C_{supp(f)} \circ y_{t_2}] \cap C_{supp(f)} \neq 0$. Therefore, there exists an element $a \in S$ such that $\left[\left(x_{t_1} \circ C_{supp(f)} \circ y_{t_2}\right] \cap C_{supp(f)}\right](a) \neq 0$. Hence, $\left(x_{t_1} \circ C_{supp(f)} \circ y_{t_2}\right](a) \neq 0$ and $C_{supp(f)}(a) \neq 0$. Then $f(a) \neq 0$ and there exists an element $b \in S$ such that $a \leq xby$ and $f(b) \neq 0$. This means that $(x_{t_1} \circ f \circ y_{t_2}] \cap f \neq 0$. Therefore, f is a fuzzy almost interior ideal of S. □

Theorem 4.6. *Let f be any fuzzy subset of a po-semigroup S. Then f is a fuzzy weakly almost interior ideal of S if and only if $supp(f)$ is a weakly almost interior ideal of S.*

Proof: The proof of this theorem is similar to the proof of Theorem 4.5. □

Next, we define the minimality of fuzzy almost interior ideals in po-semigroups and give some relationship between minimality of almost interior ideals and minimality of fuzzy almost interior ideals of po-semigroups.

Definition 4.3. *A fuzzy almost interior ideal f of a po-semigroup S is called* minimal *if $supp(g) = supp(f)$ for every fuzzy almost interior ideal g of S such that $g \subseteq f$.*

Theorem 4.7. *A nonempty subset I of a po-semigroup S is a minimal almost interior ideal of S if and only if C_I is a minimal fuzzy almost interior ideal of S.*

Proof: Let I be a minimal almost interior ideal of S. By Theorem 4.3, we have that C_I is a fuzzy almost interior ideal of S. Let g be any fuzzy almost interior ideal of S such that $g \subseteq C_I$. By Theorem 4.5, we have that $supp(g)$ is an almost interior ideal of S. Moreover, we have that $supp(g) \subseteq supp\,(C_I) = I$. Since I is minimal, $supp(g) = I = supp\,(C_I)$. We can conclude that C_I is minimal.

Conversely, we assume that C_I is a minimal fuzzy almost interior ideal of S. Let I' be any almost interior ideal of S where $I' \subseteq I$. This implies that $C_{I'}$ is a fuzzy almost interior ideal of S such that $C_{I'} \subseteq C_I$. Hence, $I' = supp\,(C_{I'}) = supp\,(C_I) = I$. Therefore, I is minimal. □

We give the necessary and sufficient condition for po-semigroups having no proper almost interior ideals in the following corollary.

Corollary 4.3. *Let S be a po-semigroup. Then S has no proper almost interior ideal if and only if $supp(f) = S$ for every fuzzy almost interior ideal f of S.*

Proof: Let f be a fuzzy almost interior ideal of S. By Theorem 4.5, we have that $supp(f)$ is an almost interior ideal of S. Since S has no proper almost interior ideal, $supp(f) = S$.

Conversely, suppose that I' is a proper almost interior ideal of S. By Theorem 4.7, we have that C_I' is a fuzzy almost interior ideal of S. Thus, $supp\,(C_I') = I' \neq S$, which is a contradiction. Hence, S has no proper almost interior ideal. □

Next, we will study the minimality of fuzzy weakly almost interior ideals of po-semigroups.

Definition 4.4. *A fuzzy weakly almost interior ideal f of a po-semigroup S is called* minimal *if for each fuzzy weakly almost interior ideal g of S such that $g \subseteq f$, we have $supp(g) = supp(f)$.*

Theorem 4.8. *A nonempty subset I of a po-semigroup A is a minimal weakly almost interior ideal of S if and only if C_I is a minimal fuzzy weakly almost interior ideal of S.*

Proof: The proof of this theorem is similar to the proof of Theorem 4.7. □

From Theorem 4.8, we have the following corollary. We give the necessary and sufficient condition for po-semigroups having no proper almost interior ideals in this corollary.

Corollary 4.4. *Let S be a po-semigroup. Then S has no proper weakly almost interior ideals if and only if for all fuzzy weakly almost interior ideals f of S, $supp(f) = S$.*

Proof: The proof of this corollary is similar to the proof of Corollary 4.3. □

5. **Conclusions.** In this paper, we define some novel ideals and fuzzy ideals of po-semigroups and investigate their remarkable properties. In Section 3, we define almost interior ideals and weakly almost interior ideals of po-semigroups. We show that the union of two (weakly) almost interior ideals is also a (weakly) almost interior ideal but the intersection of them need not be a (weakly) almost interior ideal in po-semigroups. Moreover, we investigate the necessary and sufficient condition of po-semigroups having

no proper weakly almost interior ideals in Theorem 3.4. In Section 4, we apply fuzzy set theory on fuzzifications of both almost interior ideals and weakly almost interior ideals of po-semigroups. We show that the union of two fuzzy (weakly) almost interior ideals is also a fuzzy (weakly) almost interior ideal but the intersection of them need not be a fuzzy (weakly) almost interior ideal in po-semigroups. We give some relationship between almost interior ideals and their fuzzification as shown in Theorems 4.3-4.8. We give the necessary and sufficient condition of po-semigroups having no proper (weakly) almost interior ideals by checking all (weakly) almost interior ideals in Corollaries 4.3 and 4.4.

In the future work, we can study other kinds of almost ideals and fuzzifications in po-semigroups or almost ideals and fuzzifications in other algebraic structures.

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