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DISTRIBUTED ENERGY STORAGE SYSTEMS CONTROL FOR WIND FARM BASED ON PRESCRIBED-TIME CONSENSUS

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Abstract. *In this paper, a novel prescribed time multiagent system consensus is presented. The consensus is ensured to be established within the prescribed time. The corresponding proof is formulated by the Lyapunov method. This distributed method can be utilized to solve the energy dispatch problem of energy storage systems for a wind farm. The proposed method can ensure the energy dispatch problem is solved in a prescribed time. A numerical simulation is presented to validate the proposed method.*

Keywords: Distributed energy storage system, Prescribed-time control, Consensus, Dispatch problem

1. **Introduction.** Wind energy is a renewable energy source, yet its large-scale utilization involves great challenges, due to the wind power fluctuation. Energy storage attracts increasing attention as a promising approach to addressing this issue [1]. Multiple energy storage units are connected to coordinate the fluctuation. The corresponding energy dispatch problem is solved by a centralized or distributed scheme. However, a central controller needs the state of the entire system and has many performance limitations especially when the number of agents is large [2]. Therefore, this paper solves the dispatching method by a multiagent systems (MASs) consensus method.

Multiagent systems (MASs) consensus is a research focus in the last two decades [3]. As an important measurement for the control of multiagent systems, fast convergence is always pursued to achieve better performance [4]. However, the conventional multiagents consensus method only achieves the asymptotic stability. To achieve faster consensus, it is desired that the multiagent system enjoys the properties of finite-time stability (FNTS) [5]. The convergence time (settling time) of finite-time stability increases as the initial value of state increases, and has no uniform bounds [6]. To solve this problem, some new control methods are required.

The fixed-time stabilization (FTS) ensures the convergence time to be bounded by a constant, which is independent of the initial conditions [7]. The constant is determined by some controller parameters, so one should calculate these parameters according to an assigned constant [8]. The idea of the prescribed-time stability (PSTS) is inspired by some time-varying feedback functions that seem to go to infinity towards the terminal time, which is a constant parameter that can be arbitrarily assigned and is explicit in the controller [9].

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Multiagent system consensus with fixed-time [10] and prescribed-time [11] was studied in various aspects, for example, prescribed-time consensus of MASs with first-order and high-order dynamics, fixed-time output consensus of homogeneous MASs and heterogeneous MASs [12]. In this paper, we design a new prescribed-time controller to ensure the consensus happens before a selected time instant. The property of PSTS is very suitable for power electronic equipment. In fault detection and recovery schemes, failing to recover from the fault on time may lead to an unrecoverable mode. The rest of the paper is structured as follows. Section 2 provides some basic knowledge and definitions. Section 3 gives some important theoretical tools, based on which the main results are presented. In Section 4, simulations are provided to validate the proposed results. Section 5 concludes the paper.

2. **Preliminaries.** An undirected graph $G = (V, E)$, with $V = (v_1, v_2, \dots, v_n)$ being the set of nodes and $E \subseteq V \times V$ denotes the set of edges. $A = [a_{i,j}]_{n\times n}$ denotes the unweighted adjacency matrix such that $a_{i,j} = 1$, if $v_i v_j \in E$ and $a_{i,j} = 0$, otherwise. Define the in-degree of node as the ith row sum of A, i.e., $d_{in}(v_i) = \sum_{j=1}^{N} a_{i,j}$. Define the diagonal in-degree matrix $D = diag\{d_{in}(v_i)\}$ and the Laplacian matrix $L = D - A$.

Consider the following controlled system

$$\dot{x} = g(x, u) \tag{1}$$

Let us consider a time-varying control $u = w(x, t, \eta)$, i.e.,

$$\dot{x} = g(x, u, \eta), \quad x(t_0) = x_0 \tag{2}$$

Definition 2.1. *The origin of the system (2) is said to be prescribed-time stable (PSTS) if for any physically possible positive number T_f, there exists some $\eta \in R^l$ such that the settling time $T(x_0)$ can be prescribed and $T(x_0) \le T_f$, $\forall x_0 \in R^n$.*

3. **Main Results.**

Proposition 3.1. *Consider system (2) and let $D \subset R^n$ be a domain containing the equilibrium point $x = 0$. Assume that there exists a real-value differentiable function $V(t, x) : I \times D \to \mathbb{R}_{\ge 0}$ ($I = [t_0, t_f]$, $V(t, 0) = 0$, $V(t, x) > 0$, $\forall t \in I$, $x \in D\backslash 0$). If V satisfies*

$$\dot{V} \le -\frac{\eta\,(1 + V^2)\arctan(V)}{t_f - t} \tag{3}$$

where η is a positive real number, then the system in (2) is prescribed-time stable.

Proof: It is easy to obtain $\dot{V} \le 0$, and then V is bounded and x for system (2) is uniform stable. Solve the differential equation $\frac{\mathrm{d}w}{\mathrm{d}t} = -\frac{\eta(1+w^2)\arctan(w)}{t_f - t}$. We have $w = \tan(C(t_f - t)^\eta)$, where $C = \frac{\arctan(w_0)}{(t_f - t_0)^\eta}$. As t tends to t_f, w tends to zero. According to the comparison lemma [13], the conclusion is established.

Lemma 3.1. *(Jenson inequality). Suppose function $f(x)$ is concave on the interval I. For any $x_1, x_2, \dots, x_n \in I$, we have the following inequality:*

$$\frac{\sum_{i=1}^{n} f(x_i)}{n} \le f\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) \tag{4}$$

Consider the communication topology G of the multiagent system is undirected and connected. Let $x \in \mathbb{R}^n$ be the states of n agents that have single-integrator dynamics, that is

$$\dot{x} = u; \quad x(t_0) = x_0 \tag{5}$$

where $x \in \mathbb{R}^n$ is a time-varying control.

Definition 3.1. *The MAS is said to attain consensus if each agent belongs to the set* $C_s = \{x \in \mathbb{R}^n | x_i = x_j,\, i, j = 1, 2, \ldots, n\}$.

Theorem 3.1. *For the MAS (5) with undirected communication topology under the control given by*

$$u = \begin{cases} -\dfrac{\eta}{t_f - t} P_1 P_2 \mathbf{1}_n, & t < t_f \\ \mathbf{0}, & t_f \le t \end{cases} \tag{6}$$

will attain prescribed-time consensus with t_f*. Especially,* $\mathcal{L}$ *is the graph Laplacian and* η *is a positive constant. Denote the* i*th item of the vector* $\mathcal{L}x$ *as* $(\mathcal{L}x)_i$*, the matrices* P_1 *and* P_2 *are*

$$P_1 = \left[I_n + diag(\mathcal{L}x)^2\right] = \begin{pmatrix} 1 + (\mathcal{L}x)_1^2 & 0 & \cdots & 0 \\ 0 & 1 + (\mathcal{L}x)_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 + (\mathcal{L}x)_n^2 \end{pmatrix} \tag{7}$$

$$P_2 = diag[\arctan(\mathcal{L}x)] = \begin{pmatrix} \arctan(\mathcal{L}x)_1 & 0 & \cdots & 0 \\ 0 & \arctan(\mathcal{L}x)_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \arctan(\mathcal{L}x)_n \end{pmatrix} \tag{8}$$

Proof: Let the Lyapunov function be $V = x^T \mathcal{L} x$. To utilize Proposition 3.1, we will first establish Equation (3). The derivative of the Lyapunov function is

$$\dot{V} = -\frac{2\eta}{t_f - t} (\mathcal{L}x)^T P_1 P_2 \mathbf{1}_n \le -\frac{2\eta n}{t_f - t} \left|\frac{\mathcal{L}x}{n}\right| \left(1 + \left|\frac{\mathcal{L}x}{n}\right|^2\right) \arctan\left|\frac{\mathcal{L}x}{n}\right| \tag{9}$$

The above inequality utilized the result of Lemma 3.1. Recall Holder inequality

$$\left|\mathbf{w}^T \mathbf{v}\right| \le ||\mathbf{w}||_p \cdot ||\mathbf{v}||_q, \quad \frac{1}{p} + \frac{1}{q} = 1. \quad \forall \mathbf{w} \in \mathbb{R}^n,\, \mathbf{v} \in \mathbb{R}^n \tag{10}$$

Therefore,

$$V = \left|x^T \mathcal{L} x\right| \le ||x||_\infty \cdot |\mathcal{L}x| \tag{11}$$

For a vector $\mathbf{v}$, we have $||\mathbf{v}||_1 \ge ||\mathbf{v}||_2 \ge ||\mathbf{v}||_\infty$. So $V \le ||x||_2 \cdot |\mathcal{L}x|$. On the other hand, $\lambda_2(\mathcal{L}) x^T x \le x^T \mathcal{L} x$ [14]. We have

$$\lambda_2(\mathcal{L}) ||x||_2^2 \le x^T \mathcal{L} x \le |x^T \mathcal{L} x| \le ||x||_2 \cdot |\mathcal{L}x| \tag{12}$$

Dividing $||x||_2$ on both sides, then $|\mathcal{L}x| \ge \lambda_2(\mathcal{L}) ||x||_2 \ge \lambda_2(\mathcal{L}) ||x||_\infty$. It is evident that $||x||_\infty \le \frac{|\mathcal{L}x|}{\lambda_2(\mathcal{L})}$. Combining Equation (11), we have $V \le ||x||_\infty \cdot |\mathcal{L}x| \le \frac{|\mathcal{L}x|^2}{\lambda_2(\mathcal{L})}$. It can be inferred that $\frac{\sqrt{\lambda_2(\mathcal{L})V}}{n} \le \left|\frac{\mathcal{L}x}{n}\right|$. Let us assume $\xi = \frac{\sqrt{\lambda_2(\mathcal{L})V}}{n}$, and its derivative is

$$\begin{aligned} \dot{\xi} &= \frac{\sqrt{\lambda_2(\mathcal{L})}}{2n} \frac{\dot{V}}{\sqrt{V}} \le \frac{-\eta\sqrt{\lambda_2(\mathcal{L})}}{t_f - t} \frac{1}{\sqrt{V}} \frac{\sqrt{\lambda_2(\mathcal{L})V}}{n} \left(1 + \xi^2\right) \arctan\xi \\ &= \frac{-\eta_1}{t_f - t} \left(1 + \xi^2\right) \arctan\xi \end{aligned} \tag{13}$$

where $\eta_1 = \frac{\eta\lambda_2(\mathcal{L})}{n}$. It is clear that ξ will converge to zero before or at the instant $t = t_f$, and so does V. Because $V = x^T \mathcal{L} x = \frac{1}{2} \sum_{i,j=1}^n a_{i,j} (x_i - x_j)^2$, we have $x_1 = x_2 = \cdots = x_n$, and prescribed-time consensus.

4. **Simulation.** The numerical example is operated by MATLAB 2018a, the simulation step is set as the fixed-step with 1×10^{-4} s. The sampling time is chosen according to the normal sampling time of power electronic equipment.

Consider an energy storage system given in Figure 1. Each storage is taken as an agent, and their energy storage or other physical quantities can be considered as system states.

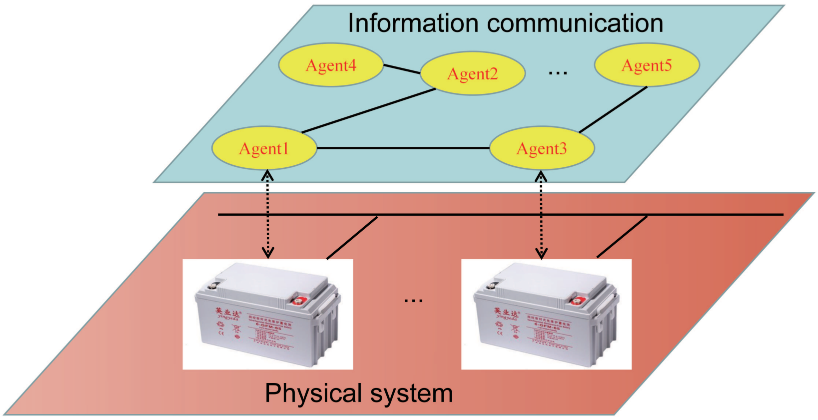

FIGURE 1. Distributed energy storage system

Suppose the system communication topology graph is given in Figure 2. Each agent has two dimensions (x, y). The control objective is to obtain the prescribed-time consensus with the controller in Equation (6).

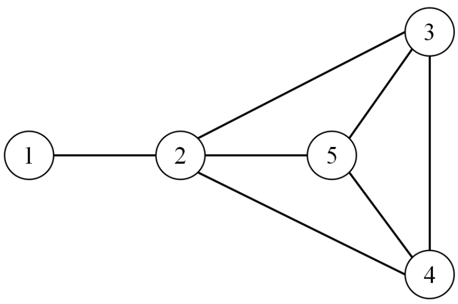

FIGURE 2. Communication topology

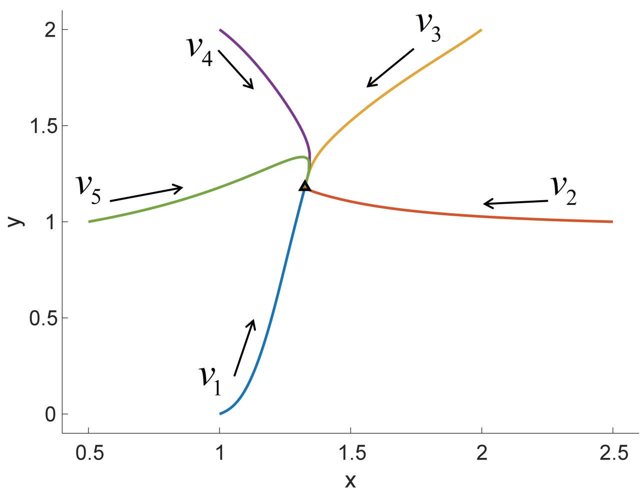

FIGURE 3. The trajectories of each agent

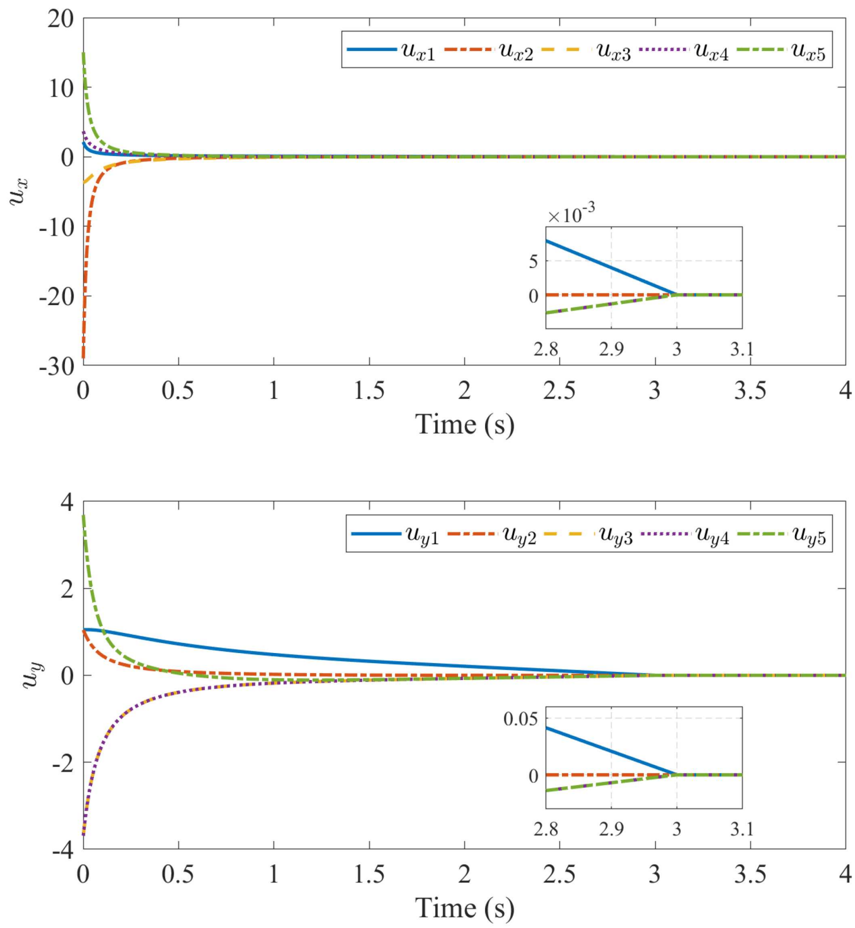

Figure 4. The controls of prescribed-time consensus

The initial states of the five agents are chosen as $x(0) = [1, 2.5, 2, 1, 0.5]^T$, $y(0) = [0, 1, 2, 2, 1]^T$. The parameter is set as $\eta = 2$. The prescribed time is set to be $t_f = 3$ s.

As demonstrated in Figure 3, the five agents converge to the black triangle point from their initial location. As shown in Figure 4, the corresponding control also goes to zero at the prescribed time $t = 3$ s. Hence, the conclusion is drawn that our proposed controller (6) established the prescribed-time consensus of the multiagent system (5).

5. **Conclusions.** In this paper, we presented a new prescribed-time consensus method, which is utilized to solve the dispatch problem of energy storage systems for a wind farm. The distributed energy storage system is considered to have an undirected communication topology. As demonstrated in simulations, the energy storage system states consensus is established within the prescribed time.

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