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OBSERVER-BASED CONTROLLER DESIGN FOR NETWORKED CONTROL SYSTEMS WITH INDUCED DELAYS AND DATA PACKET DROPOUTS

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Abstract. *This paper is concerned with the observer-based controller design problem for networked control systems (NCSs) with both network-induced delays and data packet dropouts which are caused by non-ideal network environment. The system states are assumed to be unavailable, then an observer is designed to estimate the states of the system and a sufficient condition for the observer-based controller design of NCSs is derived in the form of linear matrix inequalities (LMIs). Finally, a numerical example is given to illustrate the effectiveness of the proposed control method.*
Keywords: Networked control systems, Network-induced delays, Packet dropouts, Observer-based controller, Linear matrix inequality

1. **Introduction.** NCSs have many advantages when compared with traditional control systems, such as extensibility, easy maintenance, weight reduction, and high reliability [1, 2]. However, because of the complicated work environment and the introduction of communication networks, many challenges exist for the analysis of NCSs, such as network-induced delays and data packet dropouts. In practical systems, especially networked control systems, system states usually cannot be measured or partially cannot be measured. In order to solve this problem, output feedback control or state observer-based control is usually adopted. In recent years, output feedback controller design problem has been widely investigated. Compared with static output feedback control, observer-based output feedback controller design has less been studied and is more complicated. Therefore, it is very necessary to study the observer-based stabilization problem for NCSs with induced-delays and packet dropouts.

Due to the network environment of NCS, network-induced delays inevitably exist and may affect the system performance. Up to now, many achievements have been obtained. In [3], a kind of delay-dependent H_∞ controller design was studied for a class of uncertain networked control systems. A guaranteed cost controller was designed for uncertain time-delay network control systems in [4]. A new predictive control design scheme was proposed for a class of globally Lipschitz nonlinear continuous NCSs incorporating large time-varying transmission delays in [5]. Except for time delays, data packet dropout problem is also an important factor which can influence the system performance.

In recent years, data packet dropout problem has received more and more attentions and a lot of research productions have been obtained. In [6, 7], data packet dropout was modeled as Bernoulli process, and then data missing process was modeled as stochastic process with certain probability distributions. Except for Bernoulli process, Markovian process method was also utilized to model data packet dropouts process, such as in [8, 9]. In most of the above literature, system states can be measured while in practical systems,

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it is usually unattainable. So state observer-based control problem can be applied for NCSs with unmeasurable states.

Different from static state feedback control and static output feedback control, observer-based control problem is more difficult to study. In [10], observer-based controller design for NCS was studied in the presence of output quantisation and random communication delay. A design scheme was proposed in [11] for the observer-based output feedback controller to stabilize the closed-loop networked system with random sensor delays. In [12], observer-based control problem was investigated for a class of switched NCSs with missing data. However, all the above literature did not consider both induced delays and data packet dropouts.

Motivated by the discussions above, this paper mainly talks about the observer-based controller design problem for NCSs subject to induced delays and packet dropouts. In this paper, we model the data packet dropouts as Bernoulli process and the closed-loop system can be seen as a kind of stochastic system. Then, a sufficient condition is established for stochastic stability analysis and observer-based controller is obtained by Lyapunov-Krasovskii stability theory.

2. **Problem Statement and Preliminaries.** The structure of NCSs in this paper is shown in Figure 1. The plant to be controlled is a discrete-time system with the following form:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \tag{1}$$

where $x(k) \in R^n$, $u(k) \in R^m$, $y(k) \in R^p$ are the system states vector, control input signals, and measured output. A, B, C are known real matrices with appropriate dimensions. In this paper, the following assumptions are needed.

Assumption 2.1. *Due to the network environment, $x(k)$ is usually difficult to measure, so in this paper, we assume that all the system states cannot be measured.*

Assumption 2.2. *As is shown in Figure 1, networked-induced delays and data packet dropouts exist in both sensor-to-observer channel and controller-to-actuator channel.*

Assumption 2.3. [13] *The sensor is time-driven, and the controller and actuator are event-driven.*

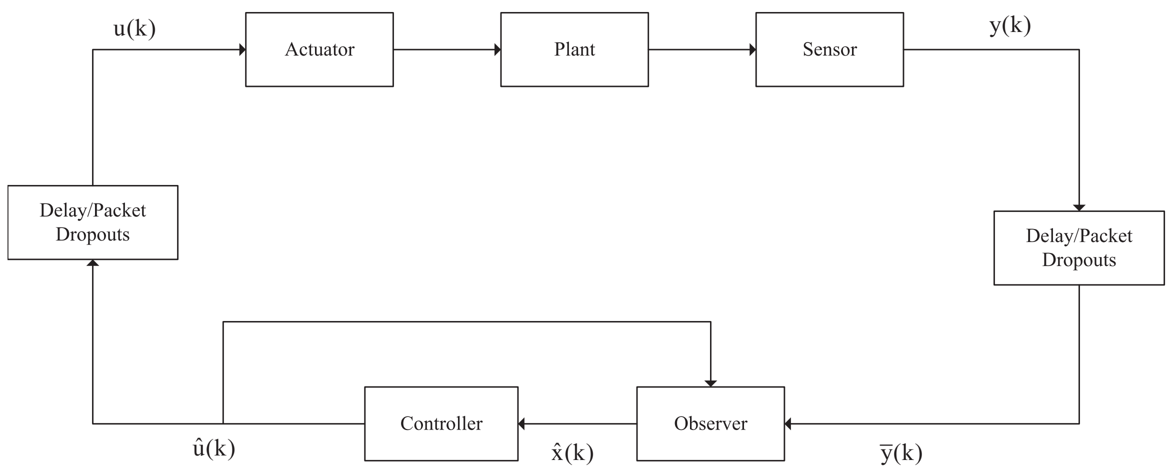

FIGURE 1. NCSs model

In this paper, network-induced delay in sensor-to-observer channel is denoted as $d(k)$ and the delay in controller-to-actuator is denoted as $\tau(k)$. Networked-induced delays are time varying and bounded satisfying: $0 \le d(k) \le \bar{d}$, $0 \le \tau(k) \le \bar{\tau}$, where $\bar{d}$ and $\bar{\tau}$ are known positive integers.

Data packet dropout process is assumed to be random in this manuscript. Normally, in some existing papers, Bernoulli process is introduced to represent the random packet dropout [10, 14]. Bernoulli processes $\alpha(k)$ and $\beta(k)$ denote the packet dropout from sensor to the observer and controller to the actuator, respectively. $\alpha(k)$ takes values in $\{0, 1\}$ and satisfies [6].

$$\begin{aligned} &\text{prob}\{\alpha(k)=1\} = E\{\alpha(k)\} = \bar{\alpha},\ 0 \le \bar{\alpha} \le 1,\ \text{prob}\{\alpha(k)=0\} = 1 - E\{\alpha(k)\} = 1 - \bar{\alpha} \\ &\text{var}\{\alpha(k)\} = E\left\{(\alpha(k)-\bar{\alpha})^2\right\} = (1-\bar{\alpha})\bar{\alpha} = \alpha_1^2 \end{aligned} \tag{2}$$

Similarly, $\beta(k)$ represents the data packet dropout from controller to actuator and satisfies [6].

$$\begin{aligned} &\text{prob}\{\beta(k)=1\} = E\{\beta(k)\} = \bar{\beta},\ 0 \le \bar{\beta} \le 1,\ \text{prob}\{\beta(k)=0\} = 1 - E\{\beta(k)\} = 1 - \bar{\beta} \\ &\text{var}\{\beta(k)\} = E\left\{\left(\beta(k)-\bar{\beta}\right)^2\right\} = \left(1-\bar{\beta}\right)\bar{\beta} = \beta_1^2 \end{aligned} \tag{3}$$

When $\alpha(k) = 1$, $\beta(k) = 1$, we mean that the delayed signals are transmitted successfully. On the contrary, $\alpha(k) = 0$, $\beta(k) = 0$ means the data is missing. Thus the measurement $\bar{y}(k)$ with induced delay and packet dropout can be expressed as

$$\bar{y}(k) = \alpha(k)y(k - d(k)) \tag{4}$$

The control input $u(k)$ can be obtained as

$$u(k) = \beta(k)\hat{u}(k - \tau(k)) \tag{5}$$

According to Assumption 2.1, in this paper, a state observer is designed in the following:

$$\begin{aligned} &\hat{x}(k+1) = A\hat{x}(k) + B\hat{u}(k) + L(\bar{y}(k) - \hat{y}(k)) \\ &\hat{y}(k) = C\hat{x}(k) \end{aligned} \tag{6}$$

Then the observer-based controller can be designed as follows:

$$\hat{u}(k) = K\hat{x}(k) \tag{7}$$

where K and L are the gain matrices to be designed later.

In this paper, the state observer error is defined as $e(k) = x(k) - \hat{x}(k)$, considering (1), (4), (5) and (6), we have

$$\begin{aligned} e(k+1) &= x(k+1) - \hat{x}(k+1) = Ax(k) + Bu(k) - A\hat{x}(k) - B\hat{u}(k) - L(\bar{y}(k) - \hat{y}(k)) \\ &= (A + BK - LC)e(k) + (LC - BK)x(k) + \beta(k)BKx(k - \tau(k)) \\ &\quad - \alpha(k)LCx(k - d(k)) - \beta(k)BKe(k - \tau(k)) \end{aligned} \tag{8}$$

For the sake of analysis in the following section, we have the following conversion:

$$\alpha(k) = \bar{\alpha} - (\bar{\alpha} - \alpha(k)),\quad \beta(k) = \bar{\beta} - \left(\bar{\beta} - \beta(k)\right) \tag{9}$$

Then (8) can be rewritten as

$$\begin{aligned} e(k+1) &= (A + BK - LC)e(k) + (LC - BK)x(k) + \left(\bar{\beta} - \left(\bar{\beta} - \beta(k)\right)\right)BKx(k - \tau(k)) \\ &\quad - (\bar{\alpha} - (\bar{\alpha} - \alpha(k)))LCx(k - d(k)) - \left(\bar{\beta} - \left(\bar{\beta} - \beta(k)\right)\right)BKe(k - \tau(k)) \end{aligned} \tag{10}$$

Similarly, the closed-loop system can be obtained as

$$x(k+1) = Ax(k) + \left(\bar{\beta} - \left(\bar{\beta} - \beta(k)\right)\right)BK(x(k - \tau(k)) - e(k - \tau(k))) \tag{11}$$

Based on the above discussions, we have the following augmented system

$$\eta(k+1) = \bar{A}\eta(k) + \bar{A}_1\eta(k - \tau(k)) + \bar{A}_2\eta(k - d(k)) \tag{12}$$

where

$$\eta(k) = \begin{bmatrix} x^T(k) & e^T(k) \end{bmatrix}^T,\quad \bar{A} = \begin{bmatrix} A & 0 \\ LC - BK & A + BK - LC \end{bmatrix}$$

$$\bar{A}_1 = \beta(k) \begin{bmatrix} BK & -BK \\ BK & -BK \end{bmatrix}, \quad \bar{A}_2 = \alpha(k) \begin{bmatrix} 0 & 0 \\ -LC & 0 \end{bmatrix}$$

The following definition and lemmas are introduced to develop our main results.

Definition 2.1. [10] *The closed-loop system (12) is stochastically stable if for every initial condition $x_0 \in R^n$, there exists a scalar $\upsilon > 0$ such that:*

$$E\left\{\sum_{k=0}^{\infty} \|\eta(k)\|^2\right\} \le \upsilon E\left\{\|\eta(0)\|^2\right\}$$

Lemma 2.1. [10] *Let $V(\eta(k))$ be a Lyapunov functional. If there exist real scalars $\gamma_1 > 0$, $\gamma_2 > 0$, $\psi_1 < 0$ and $0 < \psi_2 < 1$ such that*

$$\gamma_1\|\eta(k)\|^2 \le V(\eta(k)) \le \gamma_2\|\eta(k)\|^2$$

and

$$E\{V(\eta(k+1)) \mid V(\eta(k))\} - V(\eta(k)) \le \psi_1 - \psi_2 V(\eta(k))$$

then the sequence $\eta(k)$ satisfies

$$E\left\{\|\eta(k)\|^2\right\} \le \frac{\gamma_2}{\gamma_1}\|\eta(0)\|^2 (1-\psi_2)^k + \frac{\psi_1}{\eta_1\psi_2}$$

Lemma 2.2. [15] *For a given matrix $W = W^T > 0 \in R^{n\times n}$, two positive integers $\bar{\gamma}_1$, $\bar{\gamma}_2$ satisfy $\bar{\gamma}_2 > \bar{\gamma}_1 \ge 1$ and a vector function $x(i) \to R^n$, then the following inequality holds:*

$$-(\bar{\gamma}_2 - \bar{\gamma}_1 + 1)\sum_{i=\bar{\gamma}_1}^{\bar{\gamma}_2} x^T(i)Wx(i) \le \left[\sum_{i=\bar{\gamma}_1}^{\bar{\gamma}_2} x(i)\right]^T W \left[\sum_{i=\bar{\gamma}_1}^{\bar{\gamma}_2} x(i)\right]$$

Lemma 2.3. [16] *Given the symmetric matrix $\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{12}^T & \Theta_{22} \end{bmatrix}$, the following statements are equivalent:*

1) $\Theta < 0$
2) $\Theta_{11} < 0,\ \Theta_{22} - \Theta_{12}^T\Theta_{11}^{-1}\Theta_{12} < 0$
3) $\Theta_{22} < 0,\ \Theta_{11} - \Theta_{12}\Theta_{22}^{-1}\Theta_{12}^T < 0$

3. **Main Results.** In this section, by using Lyapunov-Krasovskii theory and LMIs technology, an observer-based controller is designed to ensure the stability of the closed-loop system.

Theorem 3.1. *For the given scalars $0 \le \bar{\alpha} \le 1$, $0 \le \bar{\beta} \le 1$, $\varepsilon_i > 0$, $i = 1,2,3,4,5,6$, the closed-loop system is stochastically stable by the observer-based controller (6) and (7) if there exist matrices $P_1 > 0$, $P_2 > 0$, $Q_1 > 0$, $Q_2 > 0$, $R > 0$, $S_1 > 0$, $S_2 > 0$, $T > 0$, K, L satisfying the following linear matrix inequality:*

$$\begin{bmatrix} \bar{\Phi} & \Omega_1 & \Omega_2 & \Omega_3 & \Omega_4 & \Omega_5 & \Omega_6 & \Omega_7 & \Omega_8 \\ * & \Theta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Theta_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Theta_3 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Theta_4 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Theta_5 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Theta_5 & 0 & 0 \\ * & * & * & * & * & * & * & \Theta_6 & 0 \\ * & * & * & * & * & * & * & * & \Theta_6 \end{bmatrix} < 0 \tag{13}$$

where

$$\bar{\Phi}=\begin{bmatrix} \bar{\Phi}_{11} & \bar{\Phi}_{12} & \bar{\Phi}_{13} & \bar{\Phi}_{14} & \bar{\Phi}_{15} & \bar{\Phi}_{16} & \bar{\Phi}_{17} & \bar{\Phi}_{18} \\ * & \bar{\Phi}_{22} & \bar{\Phi}_{23} & \bar{\Phi}_{24} & \bar{\Phi}_{25} & \bar{\Phi}_{26} & \bar{\Phi}_{27} & \bar{\Phi}_{28} \\ * & * & \bar{\Phi}_{33} & \bar{\Phi}_{34} & \bar{\Phi}_{35} & \bar{\Phi}_{36} & \bar{\Phi}_{37} & \bar{\Phi}_{38} \\ * & * & * & \bar{\Phi}_{44} & \bar{\Phi}_{45} & \bar{\Phi}_{46} & \bar{\Phi}_{47} & \bar{\Phi}_{48} \\ * & * & * & * & \bar{\Phi}_{55} & \bar{\Phi}_{56} & \bar{\Phi}_{57} & \bar{\Phi}_{58} \\ * & * & * & * & * & \bar{\Phi}_{66} & \bar{\Phi}_{67} & \bar{\Phi}_{68} \\ * & * & * & * & * & * & \bar{\Phi}_{77} & \bar{\Phi}_{78} \\ * & * & * & * & * & * & * & \bar{\Phi}_{88} \end{bmatrix}$$

$\bar{\Phi}_{11}=-P_1+Q_1+Q_2-S_1-S_2,\ \bar{\Phi}_{12}=0,\ \bar{\Phi}_{13}=S_1,\ \bar{\Phi}_{14}=S_2$

$\bar{\Phi}_{15}=\bar{\Phi}_{16}=\bar{\Phi}_{17}=\bar{\Phi}_{18}=0,\ \bar{\Phi}_{22}=-P_2+R-T$

$\bar{\Phi}_{23}=\bar{\Phi}_{24}=\bar{\Phi}_{25}=\bar{\Phi}_{26}=\bar{\Phi}_{27}=\bar{\Phi}_{28}=0,\ \bar{\Phi}_{33}=-2S_1,\ \bar{\Phi}_{34}=\bar{\Phi}_{35}=0,\ \bar{\Phi}_{36}=S_1$

$\bar{\Phi}_{37}=\bar{\Phi}_{38}=0,\ \bar{\Phi}_{44}=-2S_2,\ \bar{\Phi}_{45}=\bar{\Phi}_{46}=0,\ \bar{\Phi}_{47}=S_2,\ \bar{\Phi}_{48}=0,\ \bar{\Phi}_{55}=-2T$

$\bar{\Phi}_{56}=\bar{\Phi}_{57}=0,\ \bar{\Phi}_{58}=T,\ \bar{\Phi}_{66}=-Q_1-S_1,\ \bar{\Phi}_{67}=\bar{\Phi}_{68}=0,\ \bar{\Phi}_{77}=-Q_2-S_2,\ \bar{\Phi}_{78}=0$

$\bar{\Phi}_{88}=-R-T,\ \Omega_1=\begin{bmatrix} A & 0 & 0 & \bar{\beta}BK & -\bar{\beta}BK & 0 & 0 & 0 \end{bmatrix}$

$\Omega_2=\begin{bmatrix} \Gamma_1 & \Gamma_2 & -\bar{\alpha}LC & \bar{\beta}BK & -\bar{\beta}BK & 0 & 0 & 0 \end{bmatrix}$

$\Omega_3=\begin{bmatrix} A-I & 0 & 0 & \bar{\beta}BK & -\bar{\beta}BK & 0 & 0 & 0 \end{bmatrix}$

$\Omega_4=\begin{bmatrix} \Gamma_1 & \Gamma_2-I & -\bar{\alpha}LC & \bar{\beta}BK & -\bar{\beta}BK & 0 & 0 & 0 \end{bmatrix}$

$\Omega_5=\begin{bmatrix} 0 & 0 & \alpha_1 LC & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\ \Omega_6=\begin{bmatrix} 0 & 0 & -\bar{\alpha}LC & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\Omega_7=\begin{bmatrix} 0 & 0 & 0 & \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 \end{bmatrix}$

$\Omega_8=\begin{bmatrix} 0 & 0 & 0 & \bar{\beta}BK & -\bar{\beta}BK & 0 & 0 & 0 \end{bmatrix}$

$\Theta_1=-2\varepsilon_1 I+\varepsilon_1^2 P_1,\ \Theta_2=-2\varepsilon_2 I+\varepsilon_2^2 P_1,\ \Theta_3=-2\varepsilon_3 I+\varepsilon_3^2\left(\vec{d}^2 S_1+\bar{\tau}^2 S_2\right)$

$\Theta_4=-2\varepsilon_4 I+\varepsilon_4^2\bar{\tau}^2 S_2,\ \Theta_5=-2\varepsilon_5 I+\varepsilon_5^2\left(P+\bar{\tau}^2 S_2\right)$

$\Theta_6=-2\varepsilon_6 I+\varepsilon_6^2\left(P_1+P_2+\vec{d}^2 S_1+\bar{\tau}^2 S_2+\bar{\tau}^2 T\right)$

$\Gamma_1=LC-BK,\ \Gamma_2=A+BK-LC$

Proof: Let $s(k)=x(k+1)-x(k)$ and $z(k)=e(k+1)-e(k)$, according to (10) and (11), we have

$$\begin{aligned} s(k) &= (A-I)x(k)-\left(\bar{\beta}-\left(\bar{\beta}-\beta(k)\right)\right)BK(x(k-\tau(k))-e(k-\tau(k))) \\ z(k) &= (\Gamma_2-I)\,e(k)-\left(\bar{\beta}-\left(\bar{\beta}-\beta(k)\right)\right)BKe(k-\tau(k))+\Gamma_1 x(k) \\ &\quad +\left(\bar{\beta}-\left(\bar{\beta}-\beta(k)\right)\right)BKx(k-\tau(k))-(\bar{\alpha}-(\bar{\alpha}-\alpha(k)))LCx(k-d(k)) \end{aligned} \tag{14}$$

Then, we construct the Lyapunov-Krasovskii functional candidate as

$$V(k)=V_1(k)+V_2(k)+V_3(k)+V_4(k)+V_5(k)$$

where

$$V_1(k)=x^T(k)P_1x(k)+e^T(k)P_2e(k)$$

$$V_2(k)=\sum_{i=k-\bar{d}}^{k-1}x^T(i)Q_1x(i)+\sum_{i=k-\bar{\tau}}^{k-1}x^T(i)Q_2x(i)$$

$$V_3(k)=\sum_{i=k-\bar{\tau}}^{k-1}e^T(i)Re(i)$$

$$V_4(k)=\sum_{i=-\bar{d}}^{-1}\sum_{j=k+i}^{k-1}\bar{d}s^T(j)S_1s(j)+\sum_{i=-\bar{\tau}}^{-1}\sum_{j=k+i}^{k-1}\bar{\tau}s^T(j)S_2s(j)$$

$$V_5(k) = \sum_{i=-\bar{\tau}}^{-1} \sum_{j=k+i}^{k-1} \bar{\tau} z^T(j) T z(j)$$

Calculating the forward difference of $V(k)$ and taking the mathematical expectation $E\{\Delta V_i(k)\} = E\{V_i(k+1)\} - V_i(k)$, we obtain

$$\begin{aligned} E\{\Delta V_1(k)\} &= E\left\{x^T(k+1)P_1x(k+1) + e^T(k+1)P_2e(k+1)\right\} - x^T(k)P_1x(k) \\ &\quad - e^T(k)P_2e(k) \end{aligned} \tag{15}$$

Then, it can be obtained that

$$\begin{aligned} E\{\Delta V_2(k)\} &= x^T(k)Q_1x(k) - x^T\left(k-\bar{d}\right)Q_1x\left(k-\bar{d}\right) + x^T(k)Q_2x(k) \\ &\quad - x^T(k-\bar{\tau})Q_2x(k-\bar{\tau}) \\ E\{\Delta V_3(k)\} &= e^T(k)Q_2e(k) - e^T(k-\bar{\tau})Q_2e(k-\bar{\tau}) \end{aligned} \tag{16}$$

and

$$\begin{aligned} E\{\Delta V_4(k)\} &\le E\left\{\bar{d}^2 s^T(k)S_1s(k)\right\} - \sum_{i=k-\bar{d}}^{k-1} \bar{d} s^T(i)S_1s(i) + E\left\{\bar{\tau}^2 s^T(k)S_2s(k)\right\} \\ &\quad - \sum_{i=k-\bar{d}}^{k-1} \bar{\tau} s^T(i)S_2s(i) \\ E\{\Delta V_5(k)\} &\le E\left\{\bar{\tau}^2 z^T(k)Tz(k)\right\} - \sum_{k=1}^{k-1} \bar{\tau} z^T(k)Tz(k) \end{aligned} \tag{17}$$

Obviously we have

$$\begin{aligned} -\sum_{i=k-\bar{d}}^{k-1} \bar{d} s^T(i)S_1s(i) &= -\sum_{k-d(k)}^{k-1} \bar{d} s^T(i)S_1s(i) - \sum_{i=k-\bar{d}}^{k-d(k)-1} \bar{d} s^T(i)S_1s(i) \\ &\le -\sum_{k-d(k)}^{k-1} d(k)s^T(i)S_1s(i) - \sum_{i=k-\bar{d}}^{k-d(k)-1} \left(\bar{d}-d(k)\right) s^T(i)S_1s(i) \end{aligned} \tag{18}$$

According to Lemma 2.2, we have

$$\begin{aligned} &-\sum_{k-d(k)}^{k-1} d(k)s^T(i)S_1s(i) - \sum_{i=k-\bar{d}}^{k-d(k)-1} \left(\bar{d}-d(k)\right) s^T(i)S_1s(i) \\ &\le -[x(k)-x(k-d(k))]^T S_1[x(k)-x(k-d(k))] \\ &\quad -\left[x(k-d(k))-x\left(k-\bar{d}\right)\right]^T S_1\left[x(k-d(k))-x\left(k-\bar{d}\right)\right] \end{aligned}$$

Furthermore we have

$$\begin{aligned} -\sum_{i=k-\bar{d}}^{k-1} \bar{d} s^T(i)S_1s(i) &\le -[x(k)-x(k-d(k))]^T S_1[x(k)-x(k-d(k))] \\ &\quad -\left[x(k-d(k))-x\left(k-\bar{d}\right)\right]^T S_1\left[x(k-d(k))-x\left(k-\bar{d}\right)\right] \end{aligned} \tag{19}$$

Similarly, we have

$$\begin{aligned} -\sum_{i=k-\bar{\tau}}^{k-1} \bar{\tau} s^T(i)S_2s(i) &\le -[x(k)-x(k-\tau(k))]^T S_2[x(k)-x(k-\tau(k))] \\ &\quad -[x(k-\tau(k))-x(k-\bar{\tau})]^T S_2[x(k-\tau(k))-x(k-\bar{\tau})] \end{aligned} \tag{20}$$

and

$$-\sum_{i=k-\bar{\tau}}^{k-1}\bar{\tau}z^T(i)Tz(i) \leq -[e(k)-e(k-\tau(k))]^T T[e(k)-e(k-\tau(k))] \\ -[e(k-\tau(k))-e(k-\bar{\tau})]^T T[e(k-\tau(k))-e(k-\bar{\tau})] \quad (21)$$

Then based on the above analysis, (17) can be rewritten as

$$\begin{aligned} E\{\Delta V_4(k)\} \leq\ & E\left\{\bar{d}^2 s^T(k)S_1 s(k)\right\} - [x(k)-x(k-d(k))]^T S_1[x(k)-x(k-d(k))] \\ & - \left[x(k-d(k)) - x\left(k-\bar{d}\right)\right]^T S_1\left[x(k-d(k)) - x\left(k-\bar{d}\right)\right] \\ & + E\left\{\bar{\tau}^2 s^T(k)S_2 s(k)\right\} - [x(k)-x(k-\tau(k))]^T S_2[x(k)-x(k-\tau(k))] \\ & - [x(k-\tau(k))-x(k-\bar{\tau})]^T S_2[x(k-\tau(k))-x(k-\bar{\tau})] \end{aligned} \quad (22)$$

and

$$\begin{aligned} E\{\Delta V_5(k)\} \leq\ & E\left\{\bar{\tau}^2 z^T(k)Tz(k)\right\} - [e(k)-e(k-\tau(k))]^T T[e(k)-e(k-\tau(k))] \\ & -[e(k-\tau(k))-e(k-\bar{\tau})]^T T[e(k-\tau(k))-e(k-\bar{\tau})] \end{aligned} \quad (23)$$

Denote the augmented state vector as

$$\xi^T(k) = \left[\ x^T(k) \quad e^T(k) \quad x^T(k-d(k)) \quad x^T(k-\tau(k)) \quad e^T(k-\tau(k)) \right. \\ \left. x^T\left(k-\bar{d}\right) \quad x^T(k-\bar{\tau}) \quad e^T(k-\bar{\tau})\ \right]$$

According to (15), (16), (22) and (23), we have

$$E\{\Delta V(k)\} \leq \xi^T(k)\Phi\xi(k) \quad (24)$$

where

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} & \Phi_{17} & \Phi_{18} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} & \Phi_{27} & \Phi_{28} \\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36} & \Phi_{37} & \Phi_{38} \\ * & * & * & \Phi_{44} & \Phi_{45} & \Phi_{46} & \Phi_{47} & \Phi_{48} \\ * & * & * & * & \Phi_{55} & \Phi_{56} & \Phi_{57} & \Phi_{58} \\ * & * & * & * & * & \Phi_{66} & \Phi_{67} & \Phi_{68} \\ * & * & * & * & * & * & \Phi_{77} & \Phi_{78} \\ * & * & * & * & * & * & * & \Phi_{88} \end{bmatrix}$$

$$\Phi_{11} = A^T P_1 A + \Gamma_1^T P_2 \Gamma_1 - P_1 + Q_1 + Q_2 + (A-I)^T \bar{d}^2 S_1 (A-I) + (A-I)^T \bar{\tau}^2 S_2 (A-I) \\ + \Gamma_1^T \bar{\tau}^2 T \Gamma_1 - S_1 - S_2$$

$$\Phi_{12} = \Gamma_1^T P_2 \Gamma_2 + \Gamma_1^T \bar{\tau}^2 T\left(\Gamma_2 - I\right),\ \Phi_{13} = -\bar{\alpha}\Gamma_1^T P_2 LC - \bar{\alpha}\Gamma_1^T \bar{\tau}^2 TLC + S_1$$

$$\Phi_{14} = \bar{\beta}A^T P_1 BK + \bar{\beta}\Gamma_1^T P_2 BK + \bar{\beta}(A-I)^T \bar{d}^2 S_1 BK + \bar{\beta}(A-I)^T \bar{\tau}^2 S_2 BK + \bar{\beta}\Gamma_1^T \bar{\tau}^2 TBK \\ + S_2$$

$$\Phi_{15} = -\bar{\beta}A^T P_1 BK - \bar{\beta}\Gamma_1^T P_2 BK - \bar{\beta}(A-I)^T \bar{d}^2 S_1 BK - \bar{\beta}(A-I)^T \bar{\tau}^2 S_2 BK \\ - \bar{\beta}\Gamma_1^T \bar{\tau}^2 TBK$$

$$\Phi_{16} = \Phi_{17} = \Phi_{18} = 0,\ \Phi_{22} = \Gamma_2^T P_2 \Gamma_2 - P_2 + R - T + \left(\Gamma_2 - I\right)^T \bar{\tau}^2 T\left(\Gamma_2 - I\right)$$

$$\Phi_{23} = -\bar{\alpha}\Gamma_2^T P_2 LC - \bar{\alpha}\Gamma_2^T \bar{\tau}^2 TLC,\ \Phi_{24} = \bar{\beta}\Gamma_2^T P_2 BK + \bar{\beta}\left(\Gamma_2 - I\right)^T \bar{\tau}^2 TBK$$

$$\Phi_{25} = -\bar{\beta}\Gamma_2^T P_2 BK - \bar{\beta}\left(\Gamma_2 - I\right)^T \bar{\tau}^2 TBK,\ \Phi_{26} = \Phi_{27} = \Phi_{28} = 0$$

$$\Phi_{33} = \left(\bar{\alpha}^2 + \alpha_1^2\right)(LC)^T P_2 LC + \left(\bar{\alpha}^2 + \alpha_1^2\right)(LC)^T \bar{\tau}^2 TLC - 2S_1$$

$$\Phi_{34} = -\bar{\alpha}\bar{\beta}(LC)^T P_2 BK - \bar{\alpha}\bar{\beta}(LC)^T \bar{\tau}^2 TBK$$

$$\Phi_{35} = \bar{\alpha}\bar{\beta}(LC)^T P_2 BK + \bar{\alpha}\bar{\beta}(LC)^T \bar{\tau}^2 TBK,\ \Phi_{36} = S_1,\ \Phi_{37} = \Phi_{38} = 0$$

$$\Phi_{44} = \left(\bar{\beta}^2 + \beta_1^2\right)(BK)^T P_1 BK + \left(\bar{\beta}^2 + \beta_1^2\right)(BK)^T P_2 BK + \left(\bar{\beta}^2 + \beta_1^2\right)(BK)^T \left(\bar{d}^2 S_1 \right. \\ \left. + \bar{\tau}^2 S_2\right) BK + \left(\bar{\beta}^2 + \beta_1^2\right)(BK)^T \bar{\tau}^2 TBK - 2S_2$$

$$\Phi_{45} = -\left(\bar{\beta}^2+\beta_1^2\right)(BK)^TP_1BK - \left(\bar{\beta}^2+\beta_1^2\right)(BK)^TP_2BK - \left(\bar{\beta}^2+\beta_1^2\right)(BK)^T\Big(\bar{d}^2S_1 + \bar{\tau}^2S_2\Big)BK - \left(\bar{\beta}^2+\beta_1^2\right)(BK)^T\bar{\tau}^2TBK$$

$$\Phi_{46}=0,\ \Phi_{47}=S_2,\ \Phi_{48}=0$$

$$\Phi_{55} = \left(\bar{\beta}^2+\beta_1^2\right)(BK)^TP_1BK + \left(\bar{\beta}^2+\beta_1^2\right)(BK)^TP_2BK + \left(\bar{\beta}^2+\beta_1^2\right)(BK)^T\left(\bar{d}^2S_1 + \bar{\tau}^2S_2\right)BK + \left(\bar{\beta}^2+\beta_1^2\right)(BK)^T\bar{\tau}^2TBK - 2T$$

$$\Phi_{56}=\Phi_{57}=0,\ \Phi_{58}=T,\ \Phi_{66}=-Q_1-S_1,\ \Phi_{67}=\Phi_{68}=0,\ \Phi_{77}=-Q_2-S_2,\ \Phi_{78}=0$$

$$\Phi_{88}=-R-T$$

If $\Phi<0$, we obtain that $E\{\Delta V(k)\}<0$. Following a similar proof procedure in [10], it yields:

$$E\{\Delta V(k)\} \le \xi^T(k)\Phi\xi(k) < \eta^T(k)\Phi\eta(k) < -\lambda_{\min}(-\Phi)\eta^T(k)\eta(k) < -\mu\eta^T(k)\eta(k) \quad (25)$$

where

$$0<\mu<\min\left(\lambda_{\min}(-\Phi),\sigma\right), \sigma : \max\left\{\lambda_{\max}\left(P_i\right), \lambda_{\max}\left(Q_i\right), \lambda_{\max}(R), \lambda_{\max}\left(S_i\right), \lambda_{\max}(T)\right\}$$
$$i=1,2$$

From $E\{\Delta V(k)\}<0$ it yields

$$E\{\Delta V(k)\} < -\mu\eta^T(k)\eta(k) < -\frac{\mu}{\sigma}V(k) = -\psi_2V(k) \quad (26)$$

Hence, it can be obtained from Lemma 2.1 and Definition 2.1 that the closed-loop system is stochastically stable.

$\Phi<0$ is a stability condition of the closed-loop system and obviously it is not LMI. Then in order to solve the observer-based controller design problem, we need a condition in the form of LMI.

Notice that $\Phi<0$ is equal to

$$\begin{bmatrix} \bar{\Phi}_{11} & \bar{\Phi}_{12} & \bar{\Phi}_{13} & \bar{\Phi}_{14} & \bar{\Phi}_{15} & \bar{\Phi}_{16} & \bar{\Phi}_{17} & \bar{\Phi}_{18} \\ * & \bar{\Phi}_{22} & \bar{\Phi}_{23} & \bar{\Phi}_{24} & \bar{\Phi}_{25} & \bar{\Phi}_{26} & \bar{\Phi}_{27} & \bar{\Phi}_{28} \\ * & * & \bar{\Phi}_{33} & \bar{\Phi}_{34} & \bar{\Phi}_{35} & \bar{\Phi}_{36} & \bar{\Phi}_{37} & \bar{\Phi}_{38} \\ * & * & * & \bar{\Phi}_{44} & \bar{\Phi}_{45} & \bar{\Phi}_{46} & \bar{\Phi}_{47} & \bar{\Phi}_{48} \\ * & * & * & * & \bar{\Phi}_{55} & \bar{\Phi}_{56} & \bar{\Phi}_{57} & \bar{\Phi}_{58} \\ * & * & * & * & * & \bar{\Phi}_{66} & \bar{\Phi}_{67} & \bar{\Phi}_{68} \\ * & * & * & * & * & * & \bar{\Phi}_{77} & \bar{\Phi}_{78} \\ * & * & * & * & * & * & * & \bar{\Phi}_{88} \end{bmatrix} + \Omega_1^TP_1\Omega_1 + \Omega_2^TP_2\Omega_2 + \Omega_2^TP_2\Omega_2$$
$$+\Omega_3^T\left(\bar{d}^2S_1+\bar{\tau}^2S_2\right)\Omega_3 + \Omega_4^T\bar{\tau}^2T\Omega_4 + \Omega_5^T\left(P_2+\bar{\tau}^2T\right)\Omega_5 + \Omega_6^T\left(P_2+\bar{\tau}^2T\right)\Omega_6$$
$$+\Omega_7^T\Big(P_1+P_2+\bar{d}^2S_1+\bar{\tau}^2S_2+\bar{\tau}^2T\Big)\Omega_7$$
$$+\Omega_8^T\left(P_1+P_2+\bar{d}^2S_1+\bar{\tau}^2S_2+\bar{\tau}^2T\right)\Omega_8 < 0 \quad (27)$$

By using Lemma 2.3, (27) can be transformed into the following inequality:

$$\begin{bmatrix} \bar{\Phi} & \Omega_1 & \Omega_2 & \Omega_3 & \Omega_4 & \Omega_5 & \Omega_6 & \Omega_7 & \Omega_8 \\ * & -P_1^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -P_2^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\left(\bar{d}^2S_1+\bar{\tau}^2S_2\right)^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \left(-\bar{\tau}^2T\right)^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\left(P_2+\bar{\tau}^2T\right)^{-1} & 0 & 0 & 0 \\ * & * & * & * & * & * & -\left(P_2+\bar{\tau}^2T\right)^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & -\Pi^{-1} & 0 \\ * & * & * & * & * & * & * & * & -\Pi^{-1} \end{bmatrix} < 0 \quad (28)$$

where $P_1+P_2+\bar{d}^2S_1+\bar{\tau}^2S_2+\bar{\tau}^2T=\Pi$. It is obvious that (28) is not strict LMI so it cannot be solved directly. According to [17], for any positive scalars ε_i, $i=1,2,3,4$, there exist the following inequalities:

$$-P_1^{-1} \leq \Theta_1,\ -P_2^{-1} \leq \Theta_2,\ -\left(\bar{d}^2 S_1 + \bar{\tau}^2 S_2\right)^{-1} \leq \Theta_3,\ -\left(\bar{\tau}^2 T\right)^{-1} \leq \Theta_4$$
$$-\left(P_2 + \bar{\tau}^2 T\right)^{-1} \leq \Theta_5,\ -\left(P_1 + P_2 + \bar{d}^2 S_1 + \bar{\tau}^2 S_2 + \bar{\tau}^2 T\right) \leq \Theta_6 \quad (29)$$

According to (29), it can be concluded that if (13) holds, (28) holds. The proof is completed.

Remark 3.1. *By using the inequalities (29), the non-strict LMI (28) can be transformed into the form of LMI. Differently, some other existing papers apply the idea of the cone complementarity algorithm (CCL) developed in [18] to transform the original non-convex feasibility problem to a nonlinear optimization problem. Compared with CCL method, the basic matrix inequality method has lower computational cost and is easy to understand.*

4. **Numerical Example.** In this section, a numerical simulation result is presented to show the validity of the designed observer-based controller. The system parameters are shown as follows:

$$A = \begin{bmatrix} -1.06 & 0.5 \\ 0.2 & 0.3 \end{bmatrix},\ B = \begin{bmatrix} 2 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}$$

The state response of open-loop system ($u(k) = 0$) is shown in Figure 2 and it is clear that the open-loop system is unstable. Some parameters are given in the following:

$$\bar{d} = \bar{\tau} = 3,\ \bar{\alpha} = 0.2,\ \bar{\beta} = 0.1,\ \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = \varepsilon_6 = 1$$

By solving the LMI (13) in Theorem 3.1 it yields $t_{\min} = -0.0010$ which implies that the LMI is feasible and we obtain that

$$K = \begin{bmatrix} 0.1222 & -0.1356 \end{bmatrix},\ L = \begin{bmatrix} -9.8039 \\ -0.0189 \end{bmatrix}$$

By the observer-based controller (6) and (7) with gains above, the states trajectories are shown in Figure 3. Obviously, the closed-loop system is stabilized by the observer-based controller we designed which demonstrate that the proposed method is effective.

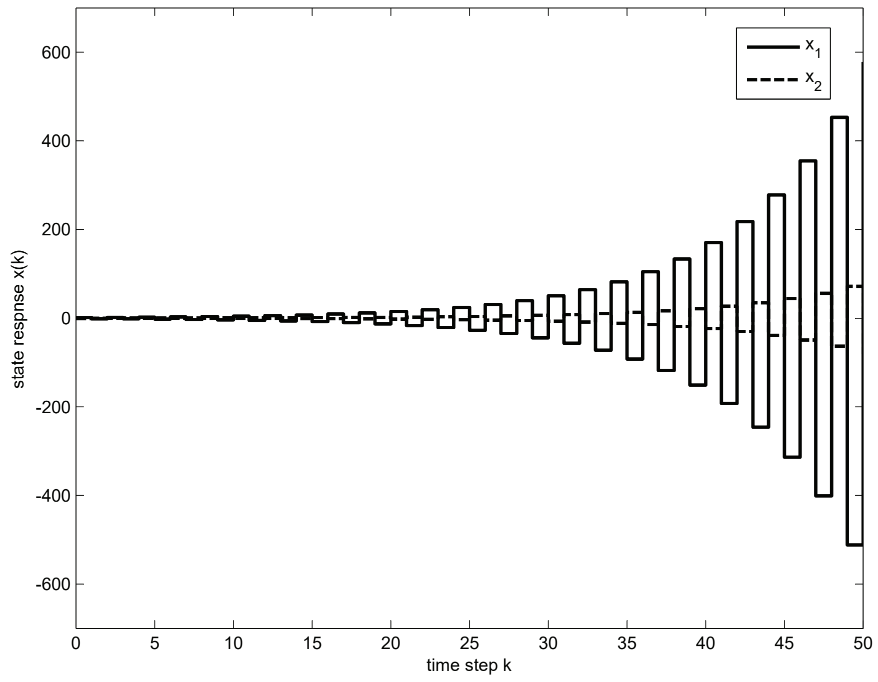

FIGURE 2. State responses of the open-loop system

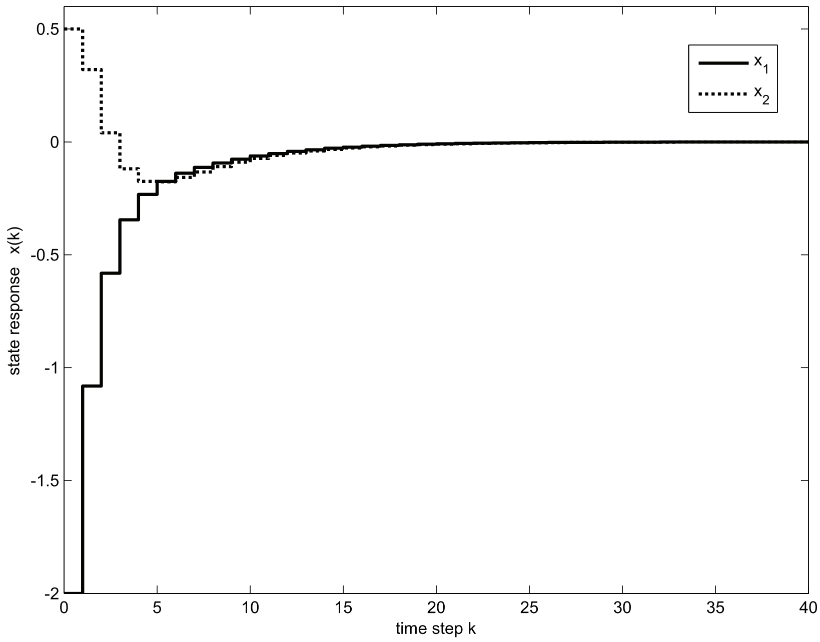

FIGURE 3. State responses of the closed-loop system

5. **Conclusions.** In this paper, networked-induced delays and data packet dropouts are considered simultaneously in both channels. The data packet dropout process is modeled as random Bernoulli process. By employing a Lyapunov-Krasovskii functional candidate, the observer gain and controller gain can be obtained through a sufficient condition based on LMI technique. At last, a numerical example is given to show the effectiveness of our proposed method.

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