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DISTRIBUTION SCHEDULING OF MULTI-TEMPERATURE COMPARTMENTS DELIVERY VEHICLES

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Abstract. *The rapid growth of perishable product supply chain requires efficient cold chain transportation of products to the end consumers that prevents temperature variation and quality degradation. This paper investigates effectiveness of the multi-compartment vehicles that are capable of joint delivery of mixed temperature products in order to replace multiple visits by traditional single temperature reefers within last mile delivery between hub and retailers. The proposed multi-compartment delivery problem is modeled as a generalization of inventory routing model. Also, a robust programming model for the multi-compartment delivery problem is developed that can protect inventory stockout caused by demand uncertainties. Proposed models are tested on the randomly generated instances that have comparable size to the previously reported computation of inventory routing problems.*

Keywords: Cold chain, Multi-compartment delivery problem, Robust programming

1. **Introduction.** As the global e-commerce becomes matured, on time delivery of perishable products such as fresh food, medical and pharmaceutical products to end-consumers becomes difficult tasks for logistics industries. The perishable products can be damaged by temperature variation and its quality degrades with time. The transportation of perishable products relies on cold chain technologies. Cold chain is defined as the transportation of temperature sensitive products along a supply chain through thermal and refrigerated packaging methods and the logistical planning to protect the integrity of the shipment [1]. Main elements of cold chain are cooling systems, cold storage, cold transport, and cold processing and distribution. The operations in the cold transport involve moving goods using refrigerated trucks and railcars, refrigerated cargo ships, reefers as well as by air cargo while maintaining required temperature and humidity conditions. Different products require different temperature standards to maintain product integrity. Required cold temperatures are commonly classified as banana (13°C), chill (2°C), frozen (−18°C), and deep frozen (−29°C). The medium to maintain cold temperatures are dry ice, gel packs, eutectic plates, liquid nitrogen, and reefers. Reefers are generic name for temperature controlled transport units including van, small truck, and forty foot ISO containers. Moving goods across the supply chain without any temperature degradation necessitates collaboration of logistical processes [2].

For the last mile delivery in cold supply chain, trucks and vans must meet the temperature requirements of the shipments. For orders containing multiple temperature requirements, several visits to the same retailer with different temperature reefers can happen. To minimize delivery cost, supplier needs to consolidate delivery visits while satisfying time and temperature requirements of orders. One effort to reduce delivery cost is the

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development of joint delivery of multi-temperature products within single reefers. MTJD system contains cold boxes with different levels of eutectic plates in order to provide multi-temperature compartments in a single vehicle. Other methods to provide multi-temperature in a single vehicle are to load several refrigerators, freezers, and isotherm containers [3]. Dividing vehicle cargo capacities to multiple compartments is also used in petroleum delivery trucks. For petroleum delivery, products in different compartments in a delivery truck cannot be mixed and each product is delivered to underground tanks in gas stations [4].

Previous researches on the cold chain delivery problems include determination of the multi-temperature delivery vehicles fleet size and vehicle loading and departure time adjustment problem for multi-temperature orders. A vehicle routing problem with modified objective function that models cooling and transport cost and a location-routing problem where the objective function includes refrigerating and carbon emission cost are introduced [5-8]. For vaccines and general pharmaceutical perishable products, product selection, production scheduling, vaccine allocation to developing countries, and vaccine distribution systems are reviewed [9,10]. Mathematical model to minimize vaccine delivery cost, stockout and obsolete penalty cost is formulated as a stochastic possibilistic optimization model [11].

If delivery vehicle contains one compartment, resulting problem becomes an inventory routing problem (IRP) [12]. IRP can be regarded as a combination of vehicle routing and inventory lot sizing problems. Branch-and-cut methods are developed where computational results for instances having up to 50 retailers and 3 to 7 time horizons are reported [13].

In this paper, we develop a mathematical model where multi-compartment vehicles replenish orders mixed with different temperature requirements while satisfying vehicle capacity and retailers' storage limit during planning horizon. Since the demands from retailers are uncertain for different temperature products, we also develop a robust programming model for the multi-temperature delivery problem. Previous robust formulation of the IRP assumes just nonnegativity of inventory level, while our robust formulation considers vehicle and storage capacities within multi-compartment vehicle environment.

This paper is organized as follows. Section 2 introduces proposed multi-compartment delivery problem and its robust programming formulation. Section 3 discusses computational results for both deterministic and robust formulations. Finally, Section 4 summarizes the result and concludes the paper.

2. **The Multi-Temperature Compartment Delivery Problem.** While traditional IRP models consider single type of delivery vehicles, this study investigates the benefits of consolidating deliveries considering time, space, and temperature dimensions. For planning periods $t \in T$, it is assumed that a single supplier would supply perishable products to set I of retailers. Possible set R of route r with length c_r is available. At the supplier's depot, there are two kinds of vehicles. The ordinary delivery vehicle (vehicle type Y_q) has capacity O_q with fixed temperature q and multi-compartment vehicle (vehicle type X) has multiple compartments such that each compartment has different temperatures with capacity P_q. We assume that type X vehicle can serve all the temperature requirements of retailers. Retailer i's q-temperature product's initial inventory level is I_{iq}, daily consumption rate is u_{iq}, maximum inventory capacity is C_{iq}, and unit holding cost is h_{iq}. Decision variable x_r^t has value one if retailer i is visited using type X vehicle in period t and has value zero otherwise. Similarly y_{rq}^t has value one if q-temperature type Y vehicle delivers to retailer i in period t, has value zero, otherwise. Variables d_{irq}^t (z_{irq}^t) denote the amount of delivery to retailer i using route r in period t by type X vehicle (type Y vehicle). Notice that inventory level at retailer i in period t is $I_{iq} + \sum_r \sum_{s \le t} d_{irq}^s - tu_{iq}$.

Objective function of the proposed model is to minimize delivery and inventory holding cost and constraints are vehicle capacity, retailers' storage limit and nonnegativity of inventory levels. We assume that in each period and for each temperature q, single X and Y_q type vehicles are deployed. Based on the above discussion, we could formulate a multi-compartment delivery problem (MCIRP) as follows.

$$
\begin{aligned}
\text{(MCIRP)} \quad \min \quad & \sum_t \sum_r c_r \left(x_r^t + \sum_q y_{rq}^t \right) + \sum_t \sum_i \sum_q h_{iq} I_{iqt} && (1)\\
\text{s.t.} \quad & \sum_{i \in r} d_{irq}^t \le P_q x_r^t \quad \forall t, r, q && (2)\\
& \sum_{i \in r} z_{irq}^t \le O_q y_{rq}^t \quad \forall r, q, t && (3)\\
& I_{iqt} = I_{iqt-1} + \sum_{i \in r} \left(d_{irq}^t + z_{irq}^t \right) - u_{iqt} \quad \forall i, q, t && (4)\\
& \sum_r x_r^t \le 1 \quad \forall t && (5)\\
& \sum_r y_{rq}^t \le 1 \quad \forall t, q && (6)\\
& d_{irq}^t + z_{irq}^t \le C_{iq} - I_{iqt} \quad \forall i, r, q, t && (7)\\
x_r^t,\ y_{rq}^t \text{ binary} \quad & \forall r, t, q, \quad I_{iqt}, d_{irq}^t, z_{irq}^t \ge 0 \quad \forall i, r, q, t && (8)
\end{aligned}
$$

The objective function (1) is to minimize the sum of the transportation cost and the customer's inventory holding cost. Constraints (2), and (3) denote vehicle capacity limits on route r delivery, and (4) represents inventory level of retailer i during period t for temperature q products. Constraints (5), and (6) require that single vehicle for each type is deployed in any period for any temperature q product. (7) corresponds to retailer's storage capacity limit, and (8) denotes nonnegativity and binary constraints.

This model requires the route length c_r as input data for faster computation, but one can add arc routing variables and subtour elimination constraints and solve the expanded model using branch-and-cut algorithm [13]. Our formulation could be regarded as a restricted master problem with pre-generated set of columns to obtain a good heuristic solution. Because of data c_r, the optimal value of the proposed model is an upper bound for the expanded model with arc routing variables.

Next, we describe the robust programming model for MCIRP. Our presentation uses similar notations from Bertsimas and Thiele which describe robust formulation of inventory lot sizing problem [14]. Among model parameters, retailer demand u_{iqt} is generally unknown and is estimated using historic data. Suppose that demand parameter u_{iqt} for retailer i for product q is a random variable with distribution given as $u_{iqt} \in [\bar{u}_{iqt} - \hat{u}_{iqt}, \bar{u}_{iqt} + \hat{u}_{iqt}]$ where $\bar{u}_{iqt}$ and $\hat{u}_{iqt}$ are average and half length parameters of demand. Let $z_{iqt} = (u_{iqt} - \bar{u}_{iqt})/\hat{u}_{iqt}$, then $z_{iqt} \in [-1, 1]$. Assume that $\sum_{k=0}^t z_{iqk} \le \Gamma_{iqt}$. In the robust linear programming literature, Γ_{iqt} is called budget of robustness parameter. The only assumption on Γ_{iqt} is that it is increasing in t. Because of space limitation, we just state robust formulation of models (1)-(8) as the following mixed integer programming model.

$$
\begin{aligned}
\text{(RMCIRP)} \quad \min \quad & \sum_t \sum_r c_r \left(x_r^t + \sum_q y_{rq}^t \right) + \sum_t \sum_i \sum_q h_{iq} w_{iqt} && (9)\\
& \sum_{i \in r} d_{irq}^t \le P_q x_r^t \quad \forall r, q, t && (10)
\end{aligned}
$$

$$\sum_{i\in r} z^t_{irq} \le O_q y^t_{rq} \quad \forall r,q,t \tag{11}$$

$$\bar{I}_{iqt} = I_{iq0} + \sum_{k=0}^{t-1}\left(\sum_{i\in r}\left(d^k_{irq} + z^k_{irq}\right) - \bar{u}_{iqk}\right) \quad \forall i,q,t \tag{12}$$

$$\Gamma_{iqt-1} q_{iqt-1} + \sum_{k=0}^{t-1} r_{iqk} \le \bar{I}_{iqt} \quad \forall i,q,t \tag{13}$$

$$w_{iqt} - \bar{I}_{iqt} + \Gamma_{iqt-1} q_{iqt-1} + \sum_{k=0}^{t-1} r_{iqk} \ge 0 \quad \forall i,q,t \tag{14}$$

$$q_{iqt-1} + r_{iqk} \ge \hat{u}_{iqk} \quad \forall i,q,t,k \le t-1 \tag{15}$$

$$\bar{I}_{iqt} + \Gamma_{iqt-1} q_{iqt-1} + \sum_{k=0}^{t-1} r_{iqk} + d^t_{irq} + z^t_{irq} \le C_{iq} \quad \forall i,r,q,t \tag{16}$$

$$\sum_{r} x^t_r \le 1 \quad \forall t \tag{17}$$

$$\sum_{r} y^t_{rq} \le 1 \quad \forall t,q \tag{18}$$

$$d^t_{irq} \ge 0,\ q_{iqt} \ge 0,\ r_{iqk} \ge 0 \quad \forall i,r,q,t,k \le t-1 \tag{19}$$

$$x^t_r, y^t_{rq} \text{ binary} \qquad \forall r,t,q, \quad I_{iqt}, d^t_{irq}, z^t_{irq} \ge 0 \quad \forall i,r,q,t \tag{20}$$

3. **Computational Results.** We have performed computational experiments for MCIRP and its robust counterpart RMCIRP using randomly generated instances. All formulations are coded using AMPL employing CPLEX 12.8 as solver and a time limit of 10 minutes is set for all computations. In the following, notations I, R, T also denote cardinality of set I, R, T. For route cost c_r, we first generated R random subsets of retailers from total I retailers and computed optimal routing cost for each subset of retailers by solving symmetric traveling salesman problem.

In Table 1, we evaluated the effect of multi-compartment delivery vehicles in terms of delivery cost, vehicle capacity utilization, retailers' average inventory level and holding cost. For this particular instance, $(I,R,T) = (35,68,12)$, $O_q = 900$, $P_q = 300$ and annual holding cost is set as 20%. Average daily demand u_{iqt} for product q is randomly generated from uniform $(20,50)$ and retailer's storage capacity C_{iq} is set at between 100 and 150. For inventory routing problems and its robust model, the parameters I between 5 and 50, T up to 7 are reported in the previous research [13,15]. Also, for the multi-compartment delivery problem similar to MCIRP, instances with 50 retailers and 5 time horizons are considered [16]. We denote $f_X = \sum_{r,t} x^t_r$, $f_Y = \sum_{r,q,t} y^t_{rq}$ as the total number of visits by the multi-compartment and single-compartment vehicles and $vol_X = \sum_{i,r,q,t} d^t_{irq}$ and $vol_Y = \sum_{i,r,q,t} z^t_{irq}$ as the total delivery volume by type X and type Y_q vehicles. Table 1 compares the results when (1) only single-temperature vehicles are used, (2) only multi-compartments vehicles are used, and (3) one multi-compartment vehicle and many single-temperature vehicles are used. The second and third columns in Table 1 denote the optimal value z_{MCIRP} and transportation cost in z_{MCIRP}, (f_X, f_Y) column denotes total delivery visit by vehicle types X and Y, 'average load factor' column denotes average delivery volume divided by vehicle capacity, $avg(I_{iqt})$ average inventory level of retailers, and the last column denotes average holding cost of retailers. Note that case 2 has the minimum objective value since deploying multi-compartment vehicles reduces both the number of visit and delivery volumes to retailers. Also, vehicle load factor shows multi-compartment vehicles can utilize its capacity better than single-temperature vehicles

Table 1. Comparison of single and multi-compartment vehicle delivery schedule

Scenarios	z_{MCIRP}	Transportation cost	$(f_X,\ f_Y)$	Average load factor	$avg(I_{iqt})$	$avg\left(\sum_{t,q} h_{iq} I_{iqt}\right)$
(1)	7289.26	6329.26	$(0, 32)$	56%	856.82	171.36
(2)	4194.18	3534.18	$(22, 0)$	82%	519.83	103.97
(3)	5913.09	5013.09	$(16, 14)$	60%	711.70	142.34

cases. Retailer's average inventory level and holding cost also indicate benefits of multi-compartment vehicle deliveries.

To evaluate the performance of the robust model RMCIRP, we generated three more instances of MCIRP and corresponding RMCIRP instances by varying budget of robustness Γ_{iqt} and demand half length $\hat{u}_{iqt}/\bar{u}_{iqt}$. Note that the budget of robustness Γ_{iqt} has property that $\Gamma_{iqt-1} \leq \Gamma_{iqt}$ for each $t \in T$. For time horizon T, we let the maximum robustness budget $\Gamma_{iqt} = \{1, 2, \ldots, T\}$, and for a fraction $u \in (0, 1)$, $u\Gamma_{iqt}$ denotes sequences $\{\lfloor u1 \rfloor, \lfloor u2 \rfloor, \ldots, \lfloor uT \rfloor\}$. For simplicity, we assume that the budget of robustness Γ_{iqt} is same for all (i, q). In Table 2, $vol_X\%$ denotes the ratio of delivery volume by vehicle type X in percentage and 'gap' column denotes $z_{incumbentIP}/z_{bestLP} - 1.0$ from CPLEX branch-and-bound tree. Thus 'gap' column denotes the distance from optimality. Column z_{MCIRP} is the best integer programming objective value at the termination. For three instances of MCIRP in Table 2, we can see that as the model parameters (I, R, T) increase, optimality gap increases. And the ratio of delivery volume between type X and type Y vehicles does not show clear correlation with the model size. We can speculate that optimal delivery schedule depends on all parameters including vehicle capacities, average demand, and storage capacity as well as route parameters.

Table 2. Results for MCIRP instances

	(I, R, T)	P_q	O_q	$vol_X\%$	$vol_Y\%$	gap	z_{MCIRP}
1	$(6, 25, 7)$	50	150	78.7	21.3	5.7%	1332
2	$(10, 40, 7)$	150	300	100.0	0	8.2%	1436
3	$(10, 40, 12)$	150	300	86.5	13.5	19.2%	1665

In Table 3, the first column denotes the MCIRP problem number in Table 2 and the second column budget of robustness, third column demand half width, Inc_{cost} and Inc_{vol} show increase percentage of the objective value and total delivery volume relative to MCIRP solution. Last column in Table 3 shows holding cost percentage in the objective value z_{RMCIRP}. We can see that as the budget of robustness and demand half width increase, Inc_{cost} and Inc_{vol} increase. Also, as the demand half width increases, $vol_Y\%$ increases. This could be caused by the capacity limits of type X vehicles. For second group of RMCIRP instances from row 6 to 14, increase of P_q results in delivery by type X vehicles only. We still can see that as the budget of robustness and demand half width increase, the delivery volume increases. For the third group of instances from row 15 to 17, as the planning horizon increases, delivery by type Y vehicles increases.

4. **Conclusions.** Traditional reefers used in the last mile delivery of cold chain have single temperature zone set in its compartment. If an order contains mixed temperature requirements across several products, multiple visits to the same retailer can happen. In order to provide cost-effective last mile delivery of perishable products, we propose a multi-compartment delivery model where each compartment has different temperature zones that allow joint delivery of mixed order. For protecting stockout caused by demand forecast error or lack of vehicle capacity, we also formulated a robust programming model

Table 3. Results for RMCIRP for instances in Table 2

	Γ_{iqt}	$\hat{u}_{iqt}/\bar{u}_{iqt}$	$vol_X\%$	$vol_Y\%$	$Inc_{cost}\%$	$Inc_{vol}\%$	$c_H\%$
1	$0.3\Gamma_{iqt}$	0.05	78.5	21.5	1.5	4.8	3.1
	$0.5\Gamma_{iqt}$	0.025	78.4	21.6	3.6	4.0	2.7
	$0.5\Gamma_{iqt}$	0.05	78.9	21.1	6.2	6.4	2.5
	$0.5\Gamma_{iqt}$	0.1	63.5	36.5	14.2	12.0	2.1
	Γ_{iqt}	0.05	77.7	22.3	12.3	9.9	2.0
2	$0.3\Gamma_{iqt}$	0.025	100.0	0	2.9	12	7.0
	$0.3\Gamma_{iqt}$	0.05	100.0	0	1.2	15	7.1
	$0.3\Gamma_{iqt}$	0.1	100.0	0	3.3	15	7.4
	$0.5\Gamma_{iqt}$	0.025	100.0	0	1.2	14	7.0
	$0.5\Gamma_{iqt}$	0.05	100.0	0	2.9	16	7.0
	$0.5\Gamma_{iqt}$	0.1	100.0	0	3.3	19	7.1
	Γ_{iqt}	0.025	100.0	0	3.3	13	7.4
	Γ_{iqt}	0.05	100.0	0	2.9	19	7.4
	Γ_{iqt}	0.1	100.0	0	13.2	33	6.1
3	$0.3\Gamma_{iqt}$	0.025	87.5	12.5	0.29	8.85	6.4
	$0.3\Gamma_{iqt}$	0.05	89.3	10.7	0.41	11.95	6.0
	$0.3\Gamma_{iqt}$	0.1	100.0	0	3.86	13.55	5.6

for the multi-compartment delivery problem. Our model assumes pre-generated routes data, so that the optimal solution from the proposed model is a good heuristic solution. Our approach can be regarded as a column generation heuristic algorithm with a limited set of columns.

From the randomly generated instances, we observed that multi-compartment vehicles can reduce number of visits to retailers that reduces total transportation cost and increase utilization of cargo capacities compared to traditional single temperature reefers. From the robust programming instances, we observed that the capacities of multi-compartment, budget of robustness, and routing structures affect delivery patterns, but as the mixed order increases, we observed that multi-compartment vehicle's loading increases.

Based on randomly generated case study, we found that by deploying mixed fleet of single- and multi-temperature vehicles, one can save transportation cost and retailers' average inventory holding cost and can achieve efficient frequent small quantity delivery to all retailers. In the follow-up study, we will compare the proposed model with the expanded model without route data by adopting branch-and-cut method.

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