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DISASTER RISK ASSESSMENT USING MIXED EXTREME VALUE DISTRIBUTIONS

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Abstract. *This paper presents mixed extreme value distributions to assess disaster risk. Various types of mixed distributions are employed for analyzing fatality risk induced by disasters that have affected the United States from 1837 to 2019. Stationary extreme value distribution may not fit the fatality distribution successfully, especially when multiple causes affect extreme outliers in the fatality data. To analyze the risk induced by the disaster, four types of parametric mixture models are investigated; the Weibull-generalized extreme value (GEV) model, the Weibull-generalized Pareto distribution (GPD) model, the GEV-GEV model, and the GEV-GPD model. We estimate parameters by the maximum likelihood (ML) method, except the mixing probability which is estimated by minimizing the integrated absolute errors (IAE), so as to assign more weight to the event in upper tail region. Exceedance frequency (FN) curves constructed from the parametric models are compared on a goodness-of-fit measure based on residuals. The stationary GPD model outperformed others in most criteria except the IAE, but the GPD model did not provide good estimates at high risk levels. The GEV-GEV model and the GEV-GPD model fitted better than other models in the IAE criteria. We constructed confidence bands of the FN-curve for the two models. The confidence interval of the return period for extremely high return level revealed that the GEV-GEV model provides more stable results. The proposed methodology can be applied to other disaster data affected by multiple causes.*

Keywords: Risk assessment, Mixed extreme value distribution, Exceedance frequency (FN) curve, Integrated absolute errors (IAE), Return level, Return period

1. Introduction.

The importance of risk analysis is getting higher as the consequences of disaster are getting more catastrophic. In this study, we analyze fatality risk induced by disasters in the United States, based on the data provided by Wikepedia [1]. Disasters inducing fifteen or more casualties are recorded, resulting in 395 in total. The disaster has many causes, but they can be classified into natural disaster and human-induced disaster.

We choose the exceedance frequency (FN) curve as a measure for societal risk, because it provides a convenient visual interpretation. Most risk analyses published so far have employed the empirical method for developing the FN-curve. However, parametric models are useful in estimating the confidence intervals and in predicting the return period of an extremely high return level. The parametric models based on a single distribution may not provide satisfactory results, especially when there are multiple causes. For these reasons, four types of parametric mixture models are investigated in this study.

We estimate parameters by maximum likelihood (ML) method, and derive exceedance frequency (FN) curve for each model. We also assess the goodness-of-fit of each model by integrating the absolute value of residuals. As for stationary distribution models, the GPD model outperforms the GEV or the Weibull model, but all the mixture models fit

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better than the GPD model. The GEV-GEV model and the GEV-GPD model fit better than other mixture models. Finally, we construct the confidence band of the FN-curve. Then the confidence intervals of the return periods for extremely high return levels are estimated.

This paper is organized as follows. The background of this research is described in Section 2. Statistical analyses of the fatality data employing extreme mixture models are explained in Section 3. Development of the FN-curves by fitting parametric models and the following risk analyses are described in Section 4. Finally, concluding remarks are presented in Section 5.

2. **Risk Assessment.**

2.1. **FN-curves.**
Societal risk is usually represented graphically in the FN-curve which displays the probability of exceedance as a function of the number of fatalities, on a double logarithmic scale. Let N be the random variable of fatalities, then the probability is

$$1 - F_N(x) = P(N > x) = \int_x^\infty f_N(y)dy \tag{1}$$

Then the expected loss of life can be obtained from the FN-curve:

$$E(N) = \int_0^\infty x f_N(x)\,dx = \int_0^\infty [1 - F_N(x)]dx \tag{2}$$

Another version of the FN-curve represents the frequency of exceedance rather than the probability. This type of FN-curve was introduced in Chapter 6 of the Reactor Safety Study (WASH-1400) [2], and similar types of curves have been utilized in many risk studies. The exceedance frequency per year can be easily obtained as the product of the exceedance probability and the expected number of events per year.

2.2. **Extreme value distributions.**
There are two famous distributions developed by the extreme value theory [3]. The first type is the generalized extreme value (GEV) distribution developed by block maxima approach. The GEV distribution with location (μ), scale (σ), and shape (ξ) parameters, is seen to be

$$G(x) = \begin{cases} \exp\left\{-\left[1 + \xi\left\{(x-\mu)/\sigma\right\}\right]_+^{-1/\varepsilon}\right\}, & \xi \neq 0 \\ \exp\left[-\exp\left\{-(x-\mu)/\sigma\right\}\right], & \xi = 0 \end{cases} \tag{3}$$

The GEV distribution comprises the Frechet ($\xi > 0$), the Gumbel ($\xi = 0$), and the Weibull ($\xi < 0$) distributions. The return level denotes the magnitude of consequence corresponding to a return period $T = 1/p$. Then the return level of the EVT is calculated to be

$$x_p = \begin{cases} \mu - \dfrac{\sigma}{\xi}\left[1 - \{-\log(1-p)\}^{-\xi}\right], & \xi \neq 0 \\ \mu - \sigma \log\{-\log(1-p)\}, & \xi = 0 \end{cases} \tag{4}$$

The second type is the generalized Pareto distribution (GPD) developed by peaks over threshold approach. It represents the limiting distribution of observations that exceed a selected threshold. The GPD with a threshold u is defined as

$$H(x) = \begin{cases} 1 - [1 + \xi(x-u)/\sigma]^{-1/\xi}, & \xi \neq 0 \\ 1 - \exp[-(x-u)/\sigma], & \xi = 0 \end{cases} \tag{5}$$

The GPD can be expressed by three extreme distributions: the Pareto ($\xi > 0$), the exponential ($\xi = 0$), and the Beta distributions ($\xi < 0$). Let ζ_u denote the probability that an observation exceeds the threshold u, and n_y denote the number of observations per year. Then the N-year return level for the GPD can be obtained to be

$$x_N = \begin{cases} u + \sigma/\xi \times \left[(N n_y \zeta_u)^{\xi} - 1\right], & \xi \neq 0 \\ u + \sigma \log(N n_y \zeta_u), & \xi = 0 \end{cases} \tag{6}$$

2.3. **Mixture distributions.** A finite mixture model is a convex combination of multiple distribution functions. Finite mixture models are appropriate to model heterogeneous populations. Since Hawkins [4] presented a normal mixture model, many other mixture models have been studied, including the log-normal mixture [5], the Weibull mixture [6], and mixture of extreme distributions [7].

A two-fold mixture cumulative distribution function (CDF) is obtained to be

$$F_{MIX}(x) = pF_1(x) + (1-p)F_2(x) \tag{7}$$

2.4. **Mixed extreme value distributions.** The Weibull-GEV model, the Weibull-GPD model, the GEV-GEV model, and the GEV-GPD model are employed. The Weibull CDF having scale parameter θ and shape parameter α is given by

$$W(x) = 1 - \exp\left\{-(x/\theta)^{\alpha}\right\}, \quad x > 0 \tag{8}$$

Then the mixed extreme distributions considered in this study are defined to be

$$F_{W-G}(x) = pW(x) + (1-p)G(x) \tag{9}$$

$$F_{W-H}(x) = pW(x) + (1-p)H(x) \tag{10}$$

$$F_{G-G}(x) = pG_1(x) + (1-p)G_2(x) \tag{11}$$

$$F_{G-H}(x) = pG(x) + (1-p)H(x) \tag{12}$$

3. Statistical Analysis.

3.1. **Descriptive statistical analysis.** Descriptive statistics are summarized in Table 1. The box plots in Figure 1 reveal highly extreme values in the fatalities. The magnitude caused by the natural disaster is much higher than that of human-induced disaster. For these reasons, mixed extreme models should be considered.

TABLE 1. Descriptive statistics of the fatality induced by disaster

Type	N	Unique N	Min	1st Q	Median	Mean	3rd Q	Max
Year	395	144	1837	1922	1962	1955	1989	2019
Total	395	198	15	33.5	66.0	233.2	166.5	9000
Natural	117	97	15	44.0	100.0	408.1	318.0	9000
Human	278	149	15	30.25	62.50	159.58	128.75	2996

3.2. **Estimation.** To estimate the parameters of each model, the maximum likelihood (ML) method is employed. We use the R-package [8] and add-on package 'MASS' [9] for the Weibull model and 'ismev' [10] for the GEV and GPD models. In risk analyses, it is very important to estimate the return level or return period at high risk levels. This motivates to employ a measure which is defined as the integrated absolute errors (IAE):

$$IAE = \int_{x_{\min}}^{x_{\max}} \left|\hat{F}_N(x) - F_N^*(x)\right| dx \tag{13}$$

The IAE assigns more weight to the event in upper tail region, because it takes account of the distance between events. The mixing probability is estimated by minimizing the IAE, while distribution parameters are estimated by the ML method. The fitted FN curves are displayed in Figure 2. The return frequencies for the stationary models show

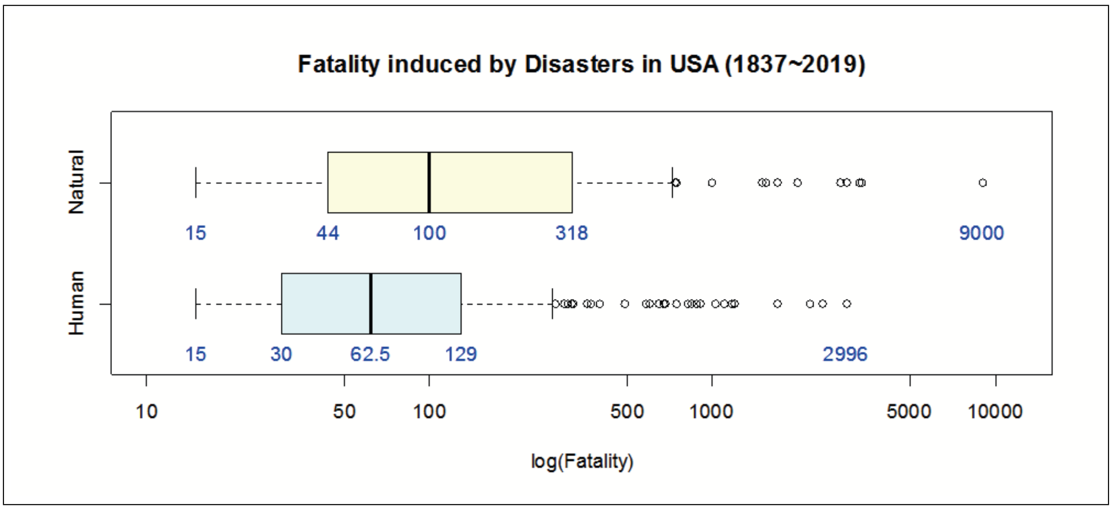

FIGURE 1. Box plot of the fatality induced by two types of disaster

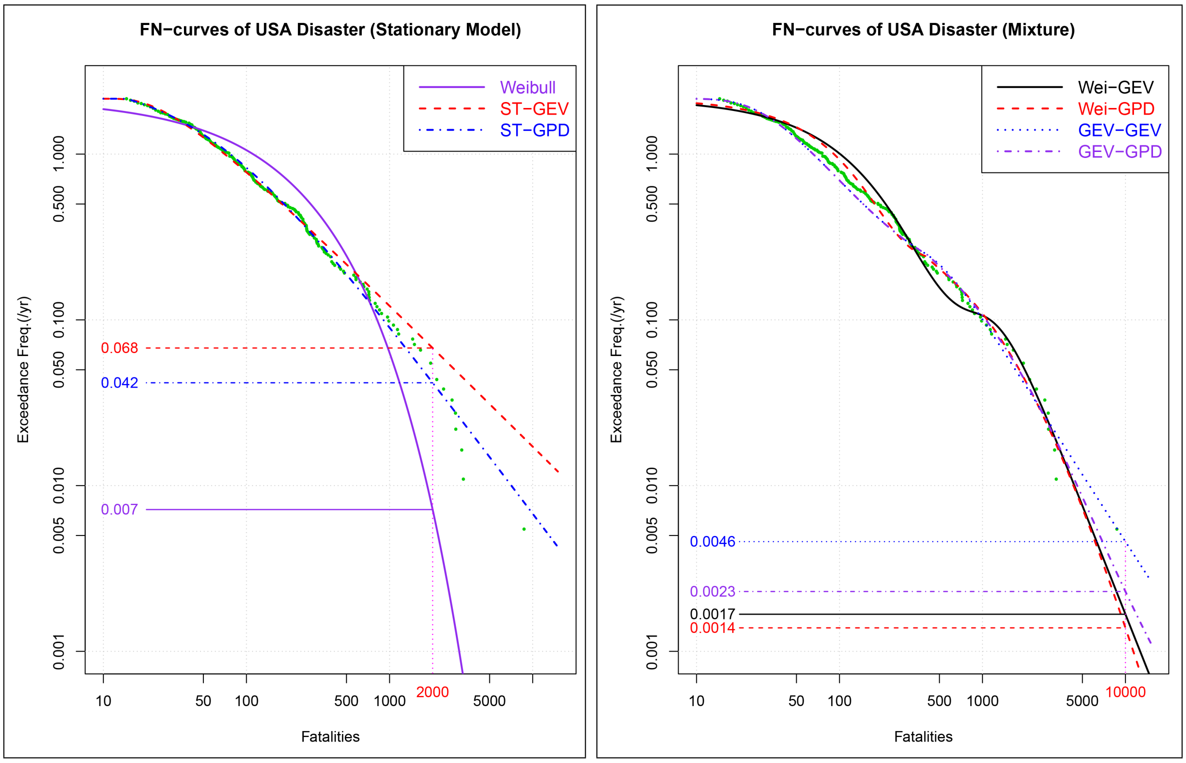

FIGURE 2. Exceedance frequency (FN) curves

significant differences among the models, even with a return level of 2000 much lower than 10,000.

The estimates and standard errors (in parenthesis) are summarized in Table 2. The GPD model looks best among the stationary models, but it overestimates the exceedance frequency in upper tail area. Diagnostic plots for the GEV model and the GPD model are illustrated in Figure 3. Significant discrepancy in the quantile plot reveals that both the stationary GEV and GPD models may not provide satisfactory results.

3.3. **Goodness-of-fit measures.** To compare the goodness-of-fit for each model, the following measures are employed. Let $\hat{F}_N(x)$ and $F^*_N(x)$ be the fitted FN-curve and the empirical FN-curve, respectively, and let $x_{(i)}$, $i = 1, \ldots, N$ be the order statistics. Then

TABLE 2. Estimation results for the parametric models

Model	shape1	scale1	location1	shape2	scale2	location2	mix-p1
Weibull	0.693 (0.023)	162.3 (12.5)	–	–	–	–	–
GEV	1.168 (0.081)	41.55 (3.55)	44.15 (2.48)	–	–	–	–
GPD	0.874 (0.095)	57.03 (5.57)	15 (–)	–	–	–	–
Wei-GEV	0.939 (0.035)	123.0 (7.2)	–	0.466 (0.300)	656.9 (176.1)	1630 (190)	0.950
Wei-GPD	1.160 (0.046)	98.5 (4.79)	–	0.335 (0.222)	707.3 (188.3)	400.0 (–)	0.893
GEV-GEV	0.761 (0.081)	31.63 (2.32)	41.48 (2.10)	0.751 (0.216)	397.7 (85.15)	773.6 (76.4)	0.930
GEV-GPD	0.761 (0.081)	31.63 (2.32)	41.48 (2.10)	0.294 (0.213)	769.6 (201.9)	416.2 (–)	0.930

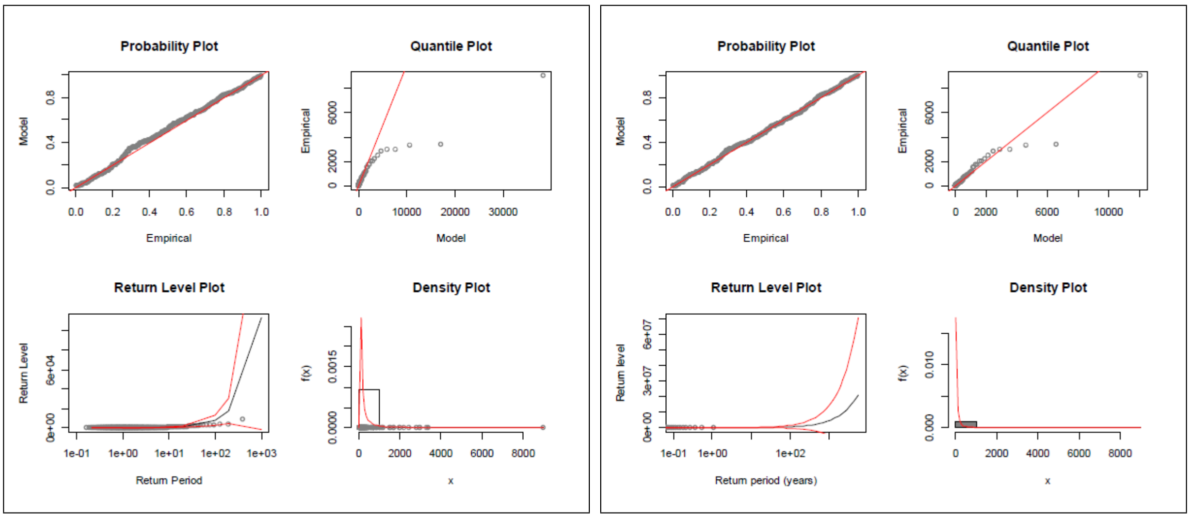

FIGURE 3. GEV and GPD diagnostic plots

the Kolmogorov-Smirnov type statistic is

$$KS = \max_{1\le i\le N} \left| \hat{F}_N\left(x_{(i)}\right) - F_N^*\left(x_{(i)}\right) \right| \tag{14}$$

The mean absolute error (MAE) and the mean squared error (MSE) are

$$MAE = \frac{1}{N}\sum_{i=1}^{N} \left| \hat{F}_N\left(x_{(i)}\right) - F_N^*\left(x_{(i)}\right) \right| \tag{15}$$

$$MSE = \frac{1}{N}\sum_{i=1}^{N} \left[\hat{F}_N\left(x_{(i)}\right) - F_N^*\left(x_{(i)}\right) \right]^2 \tag{16}$$

Let m be the number of parameters to be estimated. Then the Akaike's information criterion (AIC) [11] and the Bayesian information criterion (BIC) [12] are

$$AIC = -2\ln\left[L\left(x_1,\ldots,x_N;\hat{\theta}\right)\right] + 2m \tag{17}$$

$$BIC = -2\ln\left[L\left(x_1,\ldots,x_N;\hat{\theta}\right)\right] + m\ln(N) \tag{18}$$

The goodness-of-fit measures evaluated for each model are summarized in Table 3. The stationary GPD model outperformed others in most criteria except the IAE. The mixture models fitted better than stationary models in the IAE criteria.

The residual plots for the stationary models and for the mixture models are shown in Figure 4, where the sum of area above and below the horizontal axis corresponds to the IAE measure. The residuals of the stationary GPD model reveal stable patterns across the whole range, which explains its outperformance in most measures. However, focusing on the range above 1000 fatalities, all the mixture models are more stable than the GPD

TABLE 3. Goodness-of-fit measures for the parametric models

Model	KS	MAE	MSE	AIC	BIC	IAE
Weibull	0.377	0.151	0.030	4947.5	4955.5	220.4
GEV	0.111	0.031	0.001	4703.2	4715.1	192.8
GPD	0.074	0.017	0.001	4678.1	4686.1	77.0
Wei-GEV	0.265	0.090	0.014	4857.6	4877.3	31.9
Wei-GPD	0.206	0.067	0.008	4822.5	4842.2	36.5
GEV-GEV	0.149	0.065	0.006	4731.5	4754.5	34.7
GEV-GPD	0.150	0.065	0.006	4731.8	4754.9	28.9

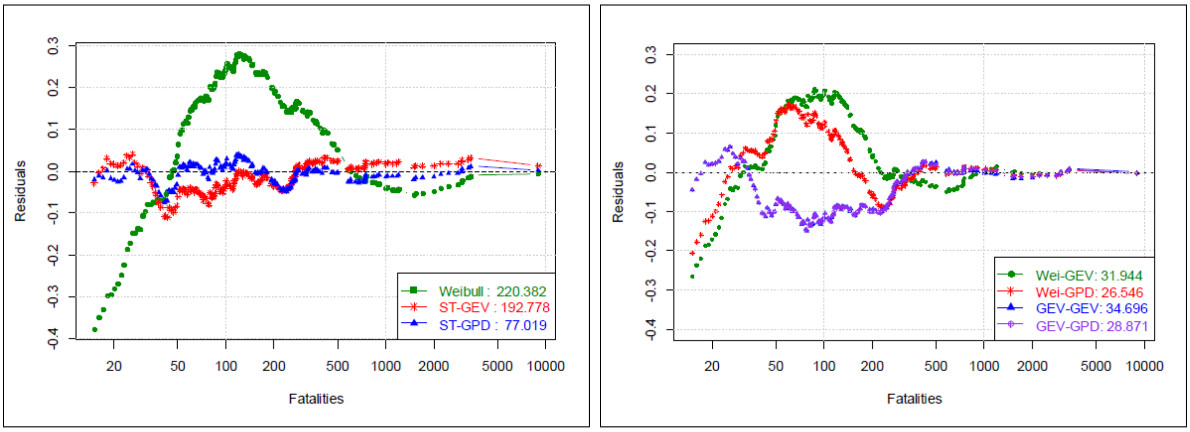

FIGURE 4. Residual plots of the FN curves: fatalities in log-scale

model. This is why their IAE measures are significantly lower than those of the stationary models.

4. **Risk Assessment of the Disaster Induced Fatality.** The GEV-GEV model and the GEV-GPD model are selected for further risk assessment, because they show preferable IAE measures. The other two mixture models are excluded, since they show worse performance in almost all measures. The 90% and 95% confidence bands of the FN-curves are estimated for the GEV-GEV model and the GEV-GPD model, respectively, by Monte-Carlo simulation, as depicted in Figure 5. They look similar, but the height of the confidence band for the GEV-GPD model is wider than that of the GEV-GEV model, at upper tail area. The confidence intervals of the return periods for selected return levels are summarized in Table 4. The confidence intervals are comparable up to the return level of 2000, but the GEV-GEV model has much tighter intervals for levels 5000 and 10000. Therefore, the GEV-GEV model is more preferable. As an illustration, we may say that the return period corresponding to return level 10000 is at least 92.8 (yr) and at most 780.7 (yr), at a significance level of 5% from the GEV-GEV model.

5. **Concluding Remarks.** To analyze the risk induced by disaster in the United States, we applied three stationary models and four mixture models including extreme value distributions. A new goodness-of-fit criterion IAE was introduced to estimate the mixture probability in the model. As for the stationary model, the GPD model fitted best in most criteria. However, in risk analyses, the IAE criterion is more important than other criteria, because the estimation in upper tail area is frequently required. Considering six goodness-of-fit criteria, two mixture models were selected for the risk assessment. We constructed the confidence bands of the FN-curves for the GEV-GEV model and the GEV-GPD model, respectively. The GEV-GEV model provided more tightened confidence intervals of return periods corresponding to high return levels.

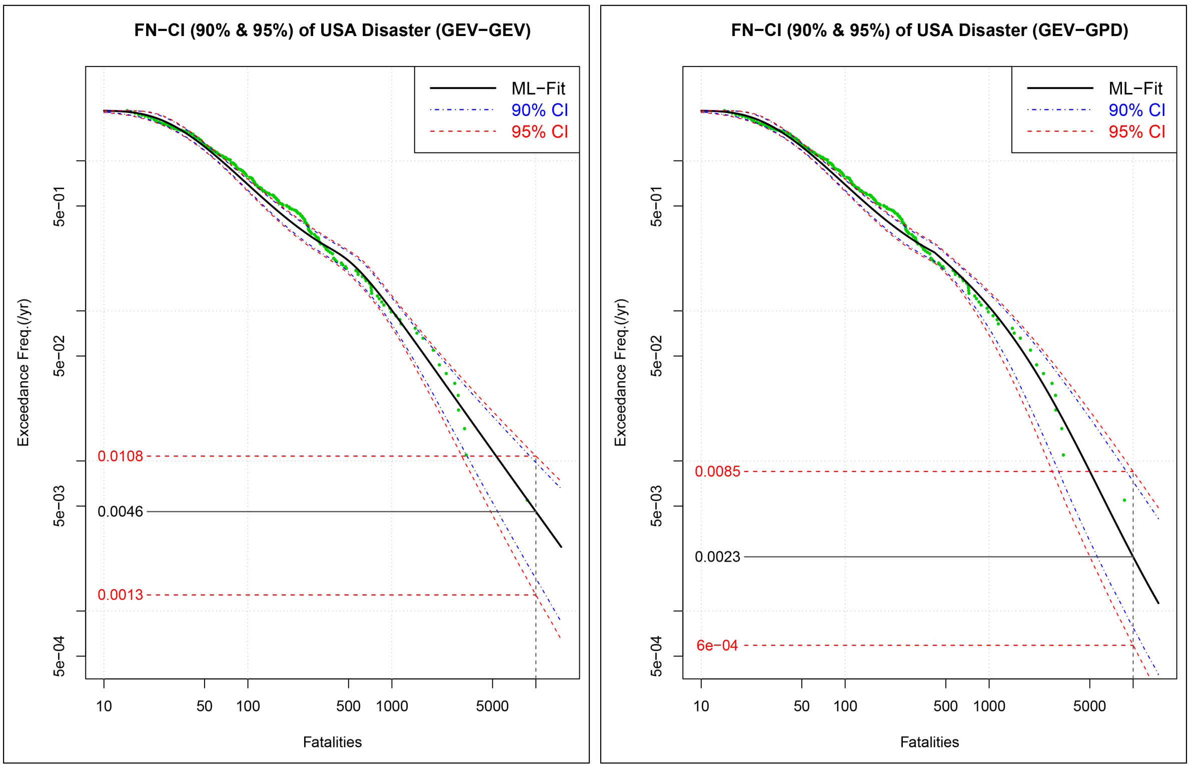

FIGURE 5. Confidence band of FN-curves

TABLE 4. Estimated confidence intervals of return periods

Conf. Level	Return Level	1000	2000	5000	10000
90%	GEV-GEV	[8.2, 12.3]	[18.5, 38.0]	[50.3, 189.0]	[102.4, 606.9]
	GEV-GPD	[7.7, 13.1]	[16.0, 47.4]	[51.7, 341.2]	[137.1, 1291.1]
95%	GEV-GEV	[7.9, 12.9]	[17.7, 42.2]	[46.6, 229.7]	[92.8, 780.7]
	GEV-GPD	[7.4, 14.5]	[15.1, 57.2]	[46.5, 433.0]	[117.6, 1685.9]

The mixed extreme value model will be useful especially when two or more causes affect the extreme consequences. Further study can be done in two directions. First, the data classified into natural disaster and human-induced disaster can be analyzed separately, so that the characteristic of each disaster type may be investigated. Second, piecewise type models can be developed, and compared to the mixture models.

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