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EVOLUTIONARY GAME ANALYSIS ON THE CHOICE OF VEHICLE SHARING TRAVEL MODES

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Abstract. *In order to effectively solve the problem of traffic congestion, we set up an evolutionary game model for the choice of traffic control departments and traveler from the two modes of car sharing and private cars travel, we also analyze the relationship between the traveler's choice of private cars travel game benefit and the game benefit of car sharing trip (the relationship between $B - A$ and $C + D$), the relationship between the sum of the rewards for due diligence and negligence of the traffic control department and the cost of management (the relationship between $E + F$ and G). The results of the model analysis show that the incentive effect of the traffic management department on car sharing and the control effect on private cars play a crucial role in the evolution of traveler choice behavior. In addition, we have analyzed the stable equilibrium points under six kinds of evolution conditions and the results. The analytical conclusions obtained in this paper can provide reference for the traffic management department to formulate management strategies.*

Keywords: Traffic congestion, Car sharing, Private cars, Stability, Evolutionary game

1. **Introduction.** The car sharing travel mode is a kind of repetitive electric car rental service, which is between the private car and the car sharing, to meet the needs of residents' short-term car use. With the implementation of the national "Internet +" development strategy, on the basis of limited electric vehicles, through the rational allocation of travel, the reduction of private car travel volume and retention, effectively alleviated urban traffic congestion.

Overseas research on the concept of car sharing was earlier. Shaheen and Martin [1] studied the feasibility of introducing car sharing projects. Shaheen [2] analyzed the impact of introducing car sharing on the current urban transport system. Firnkorn and Müller [3] explored the effect of cartogo (a kind of car sharing model) on the reduction of private cars. Few scholars have studied the concept of car sharing in China. Wang [4] studied the feasibility of the promotion of pure electric vehicles in the city. Zhou et al. [5] analyzed the main influencing factors for the choice of private car and car sharing by travelers.

Evolutionary game theory is a science derived from game theory and biology. The object of research is the effect of mutual protection between groups and the influence of individuals within a group. In recent years, many researchers have applied game theory to the study of traffic choice behavior. For example, Gong [6] used the traffic information to induce the traveler's path selection behavior, and analyzed the mutual influence and game relationship of traveler's path selection behavior under the condition of induced information. Chen et al. [7] analyzed the game and evolution process of the mode of traffic travel, and used evolutionary game theory to deduce the evolutionary stability strategy of travel mode choice.

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Based on evolutionary game theory, we consider the influence of car sharing on traveler selection behavior, and establish an evolutionary game model of traveler selection under the management of traffic management department, and analyze the dynamic model of driver selection behavior.

2. The Evolutionary Game Model of Traffic Control Departments and Travelers.

Each time the user travels to generate expenditure, the user's income can be regarded as a negative expenditure. The user's travel expenses and the management of the management department have a great influence on the choice of travel mode. For example, in the selection of travel methods, due to personal identities of passengers, the parking problem, emergencies on the roads of vehicles, and the management of the traffic management department and other influencing factors, the travel cost of the private car will not be less than that of the car sharing mode. Therefore, under the management of transportation departments, there is a game relationship between travelers who choose to travel.

2.1. The establishment of a revenue matrix.

There are two options for travelers to travel: car sharing and private cars; there are two strategies for traffic management: management and non-management. The following symbols are introduced:

A: The game benefits of the traveler when choosing a car to share a trip;
B: The benefits of choosing private cars for travelers;
C: Award for car sharing for the traffic control department;
D: Penalty for private cars (levying parking fees and congestion charges);
E: The reward for the due diligence of the management department;
F: Penalty for dereliction of duty on the traffic control department;
G: The cost of management for the traffic control department.

Without loss of generality: $B > A$. The game benefit matrix, with the participation of the management department, is shown in Table 1.

TABLE 1. Game benefit matrix

	Traffic management department management	Traffic management department does not manage
Car sharing	$A + C, -G$	$A, 0$
Private cars	$B - D, -G + E$	$B, -F$

2.2. Solution of the game benefit matrix.

Set the proportion of travelers who choose to share travel with the car as "x", then choose the proportion of private cars to travel as "$1 - x$". Assuming that the possibility of management of the traffic control department is "y", the probability that the traffic control department will not manage is "$1 - y$".

Then: W_{d1} is the expected return of the traveler who chooses the way of car sharing, W_{d2} is the expected return of the traveler who chooses the travel mode of the private car, and W_d is the average expected return of the traveler.

$$W_{d1} = (A + C)y + A(1 - y) = A + Cy \tag{1}$$

$$W_{d2} = (B - D)y + B(1 - y) = B - Dy \tag{2}$$

$$W_d = xW_{d1} + (1 - x)W_{d2} = x(A + Cy) + (1 - x)(B - Dy) \tag{3}$$

W_{t1} is the expected revenue for the traffic management department; W_{t2} is the expected revenue that the traffic management does not manage; W_t is the average expected return of the traffic control department.

$$W_{t1} = -Gx + (-G + E)(1 - x) = -G + E - Ex \tag{4}$$

$$W_{t2} = 0 \cdot x + (-F)(1 - x) = -F + Fx \tag{5}$$

$$W_t = yW_{t1} + (1 - y)W_{t2} = y(-G + E - Ex) + (1 - y)(-F + Fx) \tag{6}$$

According to the theory of evolutionary game, the speed of dynamic change in the game party's game selection ratio is the core of the game of limited rationality game. This speed depends on the learning ability and speed of the game player. In a large group of repeated games, copying dynamic evolutionary games is the most common one. Replication dynamics is a dynamic differential equation describing the proportion or frequency with which a particular strategy is used in a population. Its dynamic rate of change can be expressed by a dynamic differential equation $\frac{dx}{dt} = x(W_s - W)$, where x is the proportion of the gamers strategy s in a population, W_s is the expected return of the game player's choice strategy, and W is the total expected return of the game player in the whole game [8].

The dynamic change rate of x and y in this game is:

$$\frac{dx}{dt} = x(W_{d1} - W_d) = x(1 - x)(W_{d1} - W_{d2}) \tag{7}$$

$$\frac{dy}{dt} = y(W_{t1} - W_t) = y(1 - y)(W_{t1} - W_{t2}) \tag{8}$$

Bring (1), (2), (4), (5) into (7), (8) to obtain the replication dynamic equations of the pedestrian group and traffic control department:

$$\frac{dx}{dt} = x(1 - x)[A - B + (C + D)y] \tag{9}$$

$$\frac{dy}{dt} = y(1 - y)[E + F - G - (E + F)x] \tag{10}$$

In this equation, $A-B$, $C+D$, $E+F-G$ and $E+F$ are the expressions of the double population balance parameters. It is easy to know that $B - A$ and $C + D$ reflect the relative income of the traveler who chooses to share the trip with the car and the car to share the trip. This paper is called the relationship between the choice of the car sharing mode and the income of the private car travel mode; Then $E+F-G$ and $E+F$ reflect the relationship between the sum of the reward of the due diligence and the negligence of the traffic control department and the management cost. The combination of the two styles is an evolutionary game model of the traveler's travel mode selection behavior, which shows the dynamic change and adjustment process of traveler's choice of travel mode.

3. Analysis of the Model.

3.1. The stability of the system.

Proposition 3.1. *If $y = \frac{B-A}{C+D}$, then $\frac{dx}{dt} = 0$, this time is stable state for all x.*

Proposition 3.2. *If $0 < y < \frac{B-A}{C+D}$, there are two equilibrium points $x_1 = 0$ and $x_2 = 1$ in Equation (9) in the dynamic system, and $x_1 = 0$ is the local asymptotic stability point of Equation (9), $x_2 = 1$ is not that.*

Proof: Let $f(x) = x(1-x)[A-B+(C+D)y] = 0$, when $\frac{B-A}{C+D} < 1$ and $0 \leq x \leq 1$, the equilibrium points are $x_1 = 0$ and $x_2 = 1$, calculating $f'(x) = (1-2x)[A-B+(C+D)y]$.

(a) Consider the position $x_1 = 0$, $f'(0) = A-B+(C+D)y < 0$, so $x_1 = 0$ is the locally asymptotically stable point of Equation (9) in the dynamic system;

(b) Consider the position $x_2 = 1$, $f'(1) = A - B + (C + D)y > 0$, so $x_2 = 1$ is not a locally asymptotically stable point for Equation (9) in a dynamic system.

The same reasoning can be for Propositions 3.3, 3.5 and 3.6.

Proposition 3.3. *If $\frac{B-A}{C+D} < y < 1$, there are two equilibrium points $x_1 = 0$ and $x_2 = 1$ in Equation (9) in the dynamic system, and $x_2 = 1$ is the local asymptotic stability point of Equation (9), $x_1 = 0$ is not that.*

Proposition 3.4. *If $x = \frac{E+F-G}{E+F}$, then $\frac{dy}{dt} = 0$, this is a stable state or all y.*

Proposition 3.5. *If* $0 < x < \frac{E+F-G}{E+F}$, *there are two equilibrium points* $y_1 = 0$ *and* $y_2 = 1$ *in Equation (10) in the dynamic system, and* $y_2 = 1$ *is the local asymptotic stability point of Equation (10),* $y_1 = 0$ *is not that.*

Proposition 3.6. *If* $\frac{E+F-G}{E+F} < x < 1$, *there are two equilibrium points* $y_1 = 0$ *and* $y_2 = 1$ *in Equation (10) in the dynamic system, and* $y_1 = 0$ *is the local asymptotic stability point of Equation (10),* $y_2 = 1$ *is not that.*

To sum up, the possible stability points of the dynamic system are:

$$(0,0)\ (0,1)\ (1,0)\ (1,1)\ \left(\frac{E+F-G}{E+F}, \frac{B-A}{C+D}\right)$$

① When the conditions $\frac{E+F-G}{E+F} < x < 1$ and $0 < y < \frac{B-A}{C+D}$ are satisfied, the stability point may be $(0,0)$, that is, if the traveler who chooses to share the car is larger than a certain percentage, the traffic control department will relax management, and the traveler will choose to travel by private car.

② When the conditions $0 < x < \frac{E+F-G}{E+F}$ and $0 < y < \frac{B-A}{C+D}$ are satisfied, the stability point may be $(0,1)$, that is, the traveler who chooses to share the car is less than a certain percentage, while the traffic management department does not strengthen the management, the traveler will choose the private car to travel.

③ When the conditions $\frac{E+F-G}{E+F} < x < 1$ and $\frac{B-A}{C+D} < y < 1$ are satisfied, the stability point may be $(1,0)$. That is, when the probability of management by the traffic control department is greater than a given value, the traveler will select the car to share the trip.

④ When the conditions $0 < x < \frac{E+F-G}{E+F}$ and $\frac{B-A}{C+D} < y < 1$ are satisfied, the stability point may be $(1,1)$. That is, when the number of people who choose to share the car is less than a certain percentage, the traffic control department will strengthen management, and the traveler will select the car to share the trip.

⑤ When the conditions $y = \frac{B-A}{C+D}$ and $x = \frac{E+F-G}{E+F}$ are satisfied, $\frac{dx}{dt} = 0$, $\frac{dy}{dt} = 0$, all x and y are stable states, and the stability point may be $\left(\frac{E+F-G}{E+F}, \frac{B-A}{C+D}\right)$.

3.2. **Analysis of evolution stability by using Jacobian matrix.** Jacobian matrix of the evolutionary game: Jacobian matrix is similar to the derivative of the single variable function. It is assumed that the copy dynamic equations in evolutionary game are $f(x)$ and $g(y)$. The partial derivatives (if any) of the equations can form a matrix, which is called Jacobian matrix of the evolutionary game (the matrix is denoted by the symbol: J):

$$J = \begin{bmatrix} \frac{\partial f(x)}{\partial x} & \frac{\partial f(x)}{\partial y} \\ \frac{\partial g(y)}{\partial x} & \frac{\partial g(y)}{\partial y} \end{bmatrix}$$

The conditions of equilibrium stability are: in the equilibrium state, the first derivative of the system dynamic equation is zero, and the second derivative is greater than zero.

To simplify Equations (9) and (10) in dynamic systems, let $H_1 = B - A$ ($H_1 > 0$); $H_2 = C + D$; $I_1 = E + F - G$ and $I_2 = E + F$. The stability of the system which is composed of Equations (9) and (10) can be obtained by the stability of the Jacobian matrix, the Jacobian matrix of Equation (9) and Equation (10) in the dynamic system is as follows:

$$J = \begin{bmatrix} (1-2x)[-H_1 + H_2 y] & H_2 x(1-x) \\ -I_2 y(1-y) & (1-2y)[I_1 - I_2 x] \end{bmatrix}$$

The determinant of the Jacobian matrix is:

$$\det(J) = (1-2x)[-H_1 + H_2 y](1-2y)[I_1 - I_2 x] + H_2 x(1-x) I_2 y(1-y)$$

The trace of the Jacobian matrix is:

$$tr(J) = (1 - 2x)[-H_1 + H_2 y] + (1 - 2y)[I_1 - I_2 x]$$

$H_1 < H_2$, $I_1 > 0$

The system has 5 equilibrium points:

$$(0, 0)\ (0, 1)\ (1, 0)\ (1, 1)\ \left(\frac{I_1}{I_2}, \frac{H_1}{H_2}\right)$$

① At the equilibrium point $(0, 0)$, the Jacobian matrix, determinant and trace of the system are:

$$J = \begin{bmatrix} -H_1 & 0 \\ 0 & I_1 \end{bmatrix}, \quad \det(J) = -H_1 I_1, \quad tr(J) = -H_1 + I_1$$

Because of $H_1 > 0$, $I_1 > 0$ and $\det(J) < 0$, the balance point $(0, 0)$ is the saddle point of the system.

② At the equilibrium point $(0, 1)$, the Jacobian matrix, determinant and trace of the system are:

$$J = \begin{bmatrix} -H_1 + H_2 & 0 \\ 0 & -I_1 \end{bmatrix}, \quad \det(J) = (H_1 - H_2) I_1, \quad tr(J) = -H_1 + H_2 - I_1$$

Because of $\det(J) < 0$, the balance point $(0, 1)$ is the saddle point of the system.

③ At the equilibrium point $(1, 0)$, the Jacobian matrix, determinant and trace of the system are:

$$J = \begin{bmatrix} H_1 & 0 \\ 0 & I_1 - I_2 \end{bmatrix}, \quad \det(J) = H_1(I_1 - I_2), \quad tr(J) = H_1 + I_1 - I_2$$

Because $H_1 > 0$, $I_1 - I_2 < 0$, $\det(J) < 0$, the balance point $(1, 0)$ is the saddle point of the system. This is the most ideal situation, that is, the traffic control department does not manage and the travelers choose to share the trip with the car, this is impossible to achieve in real life.

④ At the equilibrium point $(1, 1)$, the Jacobian matrix, determinant and trace of the system are:

$$J = \begin{bmatrix} H_1 - H_2 & 0 \\ 0 & I_2 - I_1 \end{bmatrix}, \quad \det(J) = (H_1 - H_2)(I_2 - I_1), \quad tr(J) = H_1 - H_2 + I_2 - I_1$$

Because of $H_1 - H_2 < 0$, $I_2 - I_1 > 0$, $\det(J) < 0$ the balance point $(1, 1)$ is the saddle point of the system.

⑤ At the equilibrium point $\left(\frac{I_1}{I_2}, \frac{H_1}{H_2}\right)$, the Jacobian matrix, determinant and trace of the system are:

$$J = \begin{bmatrix} 0 & \dfrac{H_2 I_1 (I_2 - I_1)}{I_2^2} \\ \dfrac{-I_2 H_1 (H_2 - H_1)}{H_2^2} & 0 \end{bmatrix}, \quad \det(J) = \frac{H_2 I_1 (I_2 - I_1) I_2 H_1 (H_2 - H_1)}{I_2^2 H_2^2},$$

$tr(J) = 0$

The following analysis of the symbols of $H_2 I_1 (I_2 - I_1)$ and $I_2 H_1 (H_2 - H_1)$, we know $H_1 > 0$, $H_2 > 0$, $I_2 > 0$ and $I_2 - I_1 > 0$, $I_1 > 0$, $H_2 - H_1 > 0$, $\det(J) > 0$.

In discussing the value range of $\frac{I_1}{I_2}$, $\frac{H_1}{H_2}$, it is easy to know $H_1 > 0$, $H_2 > 0$, $I_2 > 0$, $I_1 > 0$, $\frac{I_1}{I_2} < 1$ and $\frac{H_1}{H_2} < 1$ $(B - A < C + D)$, the equilibrium point $\left(\frac{I_1}{I_2}, \frac{H_1}{H_2}\right)$ is the unstable point.

In summary, the equilibrium points of the Jacobian matrix are shown in Table 2.

It is known from Table 2 that there is no stable point, when $C + D$ is large enough, it can be known from $B - A < C + D$ that $A + C > B - D$, that is, the interest of the traveler who chooses to share the car is large; from $E + F > G$, it can be known that

TABLE 2. Balance point conditions

Equilibrium point	det(J)	$tr(J)$	Stability
$(0,0)$	$-$		saddle point
$(0,1)$	$-$		saddle point
$(1,0)$	$-$		saddle point
$(1,1)$	$-$		saddle point
$\left(\frac{E+F-G}{E+F}, \frac{B-A}{C+D}\right)$	$+$		unstable point

TABLE 3. Balance point conditions

Equilibrium point	det(J)	$tr(J)$	Stability
$(0,0)$	$-$		saddle point
$(0,1)$	$+$	$-$	stable point
$(1,0)$	$-$		saddle point
$(1,1)$	$+$	$+$	unstable point

TABLE 4. Balance point conditions

Equilibrium point	det(J)	$tr(J)$	Stability
$(0,0)$	$+$	$-$	stable point
$(0,1)$	$-$		saddle point
$(1,0)$	$-$		saddle point
$(1,1)$	$+$	$+$	unstable point

TABLE 5. Balance point conditions

Equilibrium point	det(J)	$tr(J)$	Stability
$(0,0)$	$+$	$-$	stable point
$(0,1)$	$+$	$+$	unstable point
$(1,0)$	$-$		saddle point
$(1,1)$	$-$		saddle point

$E-G > -F$, that is, the proceeds from the due diligence of the traffic control department are greater than the proceeds from negligence; then the traffic control department will perform due management and charge the private cars. Most people will choose to share the travel by car to reduce the pressure of congestion.

Similarly, it can be seen that the Jacobian matrix of these three cases judges the equilibrium point, as shown in Table 3, Table 4 and Table 5.

It is known from Table 3 that $(0,1)$ is a stable point. In this case, the probability that the traveler chooses to share the car is "0", and that of the traffic control department is "1". The reason for this situation might be that although incentives for car sharing have been provided, the degree of incentives is not enough, or private cars have been managed but the management is not strong enough, making it more profitable to choose a private car to travel than to choose a car to share a trip, so people tend to choose private cars to travel.

It is known from Table 4 that $(0,0)$ is a stable point. In this case, the probability that the traveler chooses to share the car is "0", and the probability that the traffic control department does not manage is "1". Because the due diligence of the traffic control department is less than the loss of the dereliction of duty, the choice of not managing this situation is what we do not want to see. This suggests that we must increase the management of private car travel, and increase the reward and punishment of due diligence to the traffic control department, so that the traffic control department plays a supervisory role.

It is known from Table 5 that $(0, 0)$ is a stable point. In this case, the probability of the traveler choosing the private car is "1" and the probability of the traffic management department is "0". Although the income of choosing a car-sharing trip is greater than that of a private car, because the traffic control department chooses not to manage because the due diligence is less than the negligence gain, it evolves to $(0, 0)$ state.

Based on the above situation and the current situation of car sharing, the following suggestions are made.

1) Increase the incentives for car sharing, for example, adopting economic subsidies; government-led operation of public transport; corporate operations, government subsidies; government shares, business operations.

2) Control the travel of private cars, for example, limit number, limit license, congestion fee and parking fee.

3) Carry out vigorous publicity for car sharing through the Internet, media and other means.

4. **Conclusion.** According to the stability theory, this paper obtains five balance points $(0, 0)$, $(0, 1)$, $(1, 0)$, $(1, 1)$ and $\left(\frac{E+F-G}{E+F}, \frac{B-A}{C+D}\right)$ of the profit matrix by analyzing six propositions. The relationship between the income from car trips and the income from car sharing trips, and the possible relationship between the sum of rewards for due diligence and negligence of the traffic control department and the costs of management may be explained in detail using the replication dynamics system of the Jacobian matrix analysis model. Discussions and conclusions on the stability of various stable points have led to the conclusion that the strength of the traffic management department's management, namely the penalty for private cars and the reward for car sharing, will influence the choice of travelers.

This model only considers the two game players of the two strategies. It is more idealistic. The future research should analyze the stable state of multiple strategies of multiple game players and analyze the parameters to make it better reflect the traffic sector. Under management, travelers choose to travel and are more in line with reality.

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