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A MULTI-CRITERIA DECISION-MAKING APPROACH BASED ON DATA ENVELOPMENT ANALYSIS CONSIDERING INTERVAL WEIGHTS IN DEFENSE SECTOR

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Abstract. *The evaluation and selection of weapon systems is a multi-criteria decision-making problem that includes system performance, acquisition cost, and other factors. In this type of decision-making, most techniques, including AHP (analytic hierarchy process), are based on the crisp weights of evaluation criteria. However, if there is no large difference among the alternatives, the final priority may be changed, even if the criteria weights are slightly adjusted. In this study, we propose a DEA (data envelopment analysis) based model that is not sensitive to final priority variability by using an optimal weights combination for each alternative.*
Keywords: Alternatives selection, Multi-criteria decision-making, AHP, DEA

1. **Introduction.** Weapon system acquisition is a challenging task of selecting the best alternative among a set of alternatives at each decision-making stage by considering often mutually conflicting criteria. For example, the evaluation and selection of weapon systems traditionally includes weapon performance and cost factors, as well as interoperability, logistics support capabilities, and economic and technological effects. This makes it a multi-criteria decision-making (MCDM) problem.

MCDM is one of the fastest growing areas of operational research since the last two decades. It is also one of the most used decision methodologies in the sciences, business, government, and engineering fields for solving complex decision-making problems [1]. Although various MCDM techniques have been proposed, most of the public sector MCDM problems (including problems in the Korean defense sector) use the analytic hierarchy process (AHP) method. AHP is one of the most widely used MCDM tools; above all, it can provide a methodology for quantifying qualitative factors based on the intuitive judgment of experts.

In this method, the overall relative weights of the evaluation criteria are normally aggregated using the geometric (or arithmetic) mean of individual experts' judgments. However, the relative criteria weights determined through this method may vary depending on the composition and number of experts involved in the evaluation. If there is no large difference among the alternatives, the final priority may be changed, even if the criteria weights are slightly adjusted. It is sometimes unrealistic to make exact judgments in these complex and uncertain decision problems, while it is more realistic to provide interval judgments rather than exact judgments.

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Therefore, this study proposes a model based on data envelopment analysis (DEA) that maximizes the overall score of each alternative through an optimal combination within the given interval weights. Specifically, it is not sensitive to the final priority change.

The remaining paper is organized as follows. Section 2 presents the theoretical background of the research and reviews the extant literature. Section 3 describes the proposed mathematical model based on DEA. Section 4 offers a numerical example for selecting the best alternative to the weapon systems. Finally, Section 5 draws some conclusions and includes the final remarks.

2. Background and Preliminaries.

2.1. Analytic hierarchy process.

The AHP method was developed by Saaty [2] in the 1970s, and has been widely implemented as a useful technique for estimating relative weights. It is a powerful tool for solving MCDM problems because it can quantify qualitative factors based on the knowledge, experience, and experts' intuition.

A general AHP procedure is as follows. Step 1 defines a decision-making problem using a hierarchical structure with a goal, several layers of criteria, and a layer of alternatives. Step 2 carries out pairwise comparisons among the selected criteria and alternatives based on the opinions of experts. Step 3 synthesizes these judgments to yield a set of overall priorities for the hierarchy and to check the consistency of the judgments. The final step calculates the weighted sum of each alternative using Equation (1), and determines the final priorities of the alternatives.

$$\begin{aligned} &Score_j = \sum_{i=1}^{m} w_i x_{ij} \quad j = 1, 2, \ldots, n \\ &\text{with } \sum_{i=1}^{m} w_i = 1 \end{aligned} \tag{1}$$

where n is the number of alternatives; m is the number of criteria; w_i is the weight of the ith criterion; x_{ij} is the normalized value of the ith criterion for the jth alternative; and $Score_j$ is the overall score of the jth alternative as the weighted sum of the criterion values.

To increase objectivity and transparency in the decision-making process, a group (or panel) of experts is essential. When the importance weights of the criteria are derived by experts in the field, it is common to numerically integrate their individual judgments rather than to achieve their consensus through brainstorming or the Delphi method.

The individual judgments of experts are typically aggregated using a geometric or arithmetic mean method [3] to obtain common judgments. However, even experts in the field can have reasonably large differences in their preferences among criteria, so it is difficult to state that the weights calculated by the mean method are absolute. If there is no large difference among the alternatives, the final priority may be changed, even if the criteria weights are slightly adjusted. Some studies [4-6] have suggested that it is more appropriate to generate interval weight estimates rather than one exact value to reflect the uncertainty of judgments in real-world decision problems. This is our motivation to propose an MCDM approach that considers the interval weights among the evaluation criteria instead of the exact (or crisp) values by AHP method.

2.2. Data envelopment analysis.

The DEA, which was developed by Charnes et al. in 1978 [7], is an MCDM tool based on linear programming for measuring the relative efficiency of alternatives with multiple inputs and multiple outputs. The efficiency is typically defined as the ratio of outputs to inputs; an alternative whose efficiency score equals 1 is called efficient in DEA.

Unlike other MCDM methods, DEA can calculate the most favorable weight combination to maximize the efficiency for each alternative without using common weights. In other words, the weights of input and output criteria that maximize its efficiency score are decision variables in the DEA model.

We suppose that n is the number of alternatives, m is the number of inputs, and s is the number of outputs. Then, the input-oriented DEA model proposed by Charnes et al. [7] is formulated as a linear programming problem:

$$\begin{aligned} \text{Max} \quad & E_k = \sum_{r=1}^{s} u_r y_{rk} \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{ik} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \qquad j = 1, 2, \ldots, n \\ & u_r, v_i \geq \varepsilon \qquad \forall r, i \end{aligned} \tag{2}$$

where k is the index for the alternative under evaluation (k ranges over $1, 2, \ldots, n$); j is the alternative index ($j = 1, 2, \ldots, n$); i is the input index ($i = 1, 2, \ldots, m$); r is the output index ($r = 1, 2, \ldots, s$); E_k is the efficiency score of the kth alternative; v_i is the weight given to the ith input criterion; u_r is the weight given to the rth output criterion; x_{ik} and x_{ij} are the values of the ith input criterion for the kth and jth alternatives, respectively; y_{rk} and y_{rj} are the values of the rth output criterion for the kth and jth alternatives, respectively; and ε is a small non-Archimedean value.

In the traditional DEA model above, many alternatives may be evaluated as efficient – even if they are inefficient – by assigning extreme and unrealistic weights (i.e., some weights are too close to zero or too large). A way to overcome the low discrimination power due to unrealistic weight distribution of the DEA model is through the restriction of weights. One weight restriction method is the assurance region (AR) model by Thompson et al. [8]. The AR refers to the lower and upper limits that are imposed on the ratios of input and output weights for each criterion. The DEA-AR model can be formulated by adding the following constraints to the traditional DEA model:

$$L_{1,i} \leq \frac{v_i}{v_1} \leq U_{1,i}, \quad L_{1,r} \leq \frac{u_r}{u_1} \leq U_{1,r} \quad (i = 2, 3, \ldots, m;\ r = 2, 3, \ldots, s) \tag{3}$$

where v_1 and v_i are the weights of the 1st and ith input criteria, respectively; u_1 and u_r are the weights of the 1st and rth output criteria, respectively; $L_{1,i}$ and $U_{1,i}$ are the lower and upper bound limits of the input weight ratio (v_i/v_1); and $L_{1,r}$ and $U_{1,r}$ are the lower and upper bound limits of the output weight ratio (u_r/u_1).

The proposed model based on DEA with interval weights will be restricted by the lower and upper limits generated from the AHP results of a group of experts.

2.3. **Previous studies in defense sector.** Due to the sensitive nature of defense work, the review of previous studies on MCDM problems was limited. While studies using the DEA method have been conducted, the proportion of the literature based on the AHP method was expectedly high. The AHP method has been applied to analyzing the combat effectiveness of weapon systems [9], to develop the evaluation index for weapon systems or their core components selection [10,11], and to evaluate R&D projects [12].

Lee et al. [13] presented a goal programming model to select the best alternative among the weapon systems considering both the subjective weights by AHP and the objective weights by principal component analysis. Karaburun and Alaykiran [14] used AHP to obtain the criteria weight values and technique for order preference by similarity to ideal solution to determine preferences among alternatives in the weapon selection problem.

Eo and Park [15] proposed an integrated AHP-DEA method to evaluate the alternatives of defense acquisition programs. After they constructed the weighted sum composite scores using AHP from the lowest level of the hierarchy to the next highest one, DEA was applied to identifying efficient alternatives at the highest level. Further, Moon and Kang [16] presented a two-stage model combining DEA and AHP for the acquisition of a weapon system. In the first stage, DEA was used to select the efficient group, and then the second stage applied AHP to determining the best alternative.

3. **Proposed Model.** The proposed model formulation is based on the traditional DEA model mentioned above, but employs only output variables. Since an MCDM problem can be considered a problem with a DEA approach that has no inputs or has the same amount of inputs, DEA can be applied to identifying non-dominated alternatives [17]. A DEA model without input variables can be derived by simply using a dummy input [18] with a value equal to 1 for all alternatives in Equation (2). The modified DEA model without inputs for calculating the efficiency score (E_k) of the kth alternative can be found in [17] and [19].

Next, to avoid the extreme and unrealistic weight distribution of the DEA model, we consider restriction conditions with interval weights for each criterion. This provides realistic and practical situations; it also increases the discrimination power of the DEA analysis. The lower and upper bounds of the interval weights are gathered from the AHP results of a group of experts. Then, we add constraints such that the weight (u_r) given to the rth criterion is placed within the lower and upper bounds represented by the intervals $[L_r, U_r]$. As shown in Equation (3), the DEA-AR model [8] sets the lower and upper bounds as the ratio between the weights so as to generate a feasible solution.

In the case of the proposed model, the weights of the upper and lower bounds obtained from the AHP results (normalized to $[0, 1]$ range) can be used without the ratio conversion, by performing a proper normalization of the characteristic data. However, when such absolute weight restrictions are imposed on DEA models, the models may be infeasible [20]. Our proposed model is formulated with the big-M method for removing infeasibility of the models; if the model becomes infeasible, then a penalty is imposed in order to have minimum deviation from the weight interval considered, as shown in [21].

Another consideration is the sum of the criteria weights, which is usually normalized to 1 in MCDM methods. While this consideration is intuitive and reasonable, it is rarely used in DEA methods because the DEA's data normalization and weight allocation scheme are realized by the model itself. Unlike the existing DEA models, we can set a reasonable constraint such that the sum of all criteria weights is equal to 1 ($\sum u_r = 1$) because each criterion weight is restricted and bounded between 0 and 1 from the AHP results. The proposed model based on the DEA approach can be summarized in the following linear programming problem:

$$
\begin{aligned}
\text{Max} \quad & Score_k = \sum_{r=1}^{s} u_r \hat{y}_{rk} - M\left(\sum_{r=1}^{s} \underline{d}_r^- + \sum_{r=1}^{s} \overline{d}_r^+\right) & & \\
\text{s.t.} \quad & \sum_{r=1}^{s} u_r \hat{y}_{rj} \le 1 & & j = 1, 2, \ldots, n \\
& u_r + \underline{d}_r^- - \underline{d}_r^+ = L_r & & \forall r \\
& u_r + \overline{d}_r^- - \overline{d}_r^+ = U_r & & \forall r \\
& \sum_{r=1}^{s} u_r = 1 & & \\
& \underline{d}_r^-, \underline{d}_r^+, \overline{d}_r^-, \overline{d}_r^+ \ge 0 & & \forall r
\end{aligned}
\tag{4}
$$

where $Score_k$ is the overall score of the kth alternative as the weighted sum of the criterion values; u_r is the weight given to the rth criterion; $\hat{y}_{rk}$ and $\hat{y}_{rj}$ are the normalized values of the rth criterion for the kth and jth alternatives, respectively; L_r and U_r are the lower bound and upper bound of the rth criterion weight, respectively, with the AHP method; $\underline{d}_r^-$ and $\underline{d}_r^+$ are the negative and positive deviation variables from the lower bound of the rth criterion weight; $\overline{d}_r^-$ and $\overline{d}_r^+$ are the negative and positive deviation variables from the upper bound of the rth criterion weight; and M is a very big positive number.

4. **Numerical Example.**

4.1. **Data and modeling.** In this section, we apply the proposed model for selecting the best alternative among a set of weapon systems. The data set of this example has been taken from Lee et al. [13], as shown in Table 1. In this example, there are six missile systems, three criteria, and 19 sub-criteria.

TABLE 1. Characteristic data for six alternatives, and weights derived from AHP

Criteria	Sub-criteria	Alternative missile systems						Criteria weights		
		no. 1	no. 2	no. 3	no. 4	no. 5	no. 6	Crisp value	Lower bound	Upper bound
Basic capabilities	Range	150	160	135	140	155	170	0.049	0.021	0.075
	Altitude	24	28	22	24	28	30	0.037	0.023	0.053
	Hit probability	0.75	0.80	0.75	0.75	0.80	0.80	0.107	0.073	0.139
	Reaction time	12	9	13	12	10	9	0.059	0.028	0.093
	Setup time	5.5	5.0	6.0	5.5	5.0	5.0	0.046	0.034	0.052
	Detection targets	95	110	85	95	100	100	0.044	0.021	0.096
	Engagement targets	6	9	6	6	8	8	0.069	0.021	0.180
Operational capabilities	Interoperability	0.75	0.80	0.65	0.65	0.70	0.70	0.081	0.037	0.125
	ECM	0.65	0.75	0.65	0.65	0.75	0.75	0.123	0.072	0.232
	Anti-ARM	0.75	0.80	0.65	0.70	0.70	0.80	0.114	0.079	0.148
	Mobility	0.65	0.65	0.75	0.75	0.75	0.70	0.048	0.028	0.083
	Trainability	0.75	0.75	0.70	0.65	0.65	0.65	0.030	0.017	0.042
	ILS availability	0.80	0.80	0.75	0.75	0.75	0.75	0.053	0.016	0.093
Cost & technical effects	Acquisition cost	1,100	1,250	950	1,050	1,050	1,100	0.029	0.013	0.050
	Maintenance cost	12	14	8	8	9	12	0.018	0.009	0.027
	Offset trade	0.50	0.45	0.75	0.60	0.60	0.45	0.014	0.006	0.020
	Technological effect	0.90	0.90	1.10	1.00	0.90	0.90	0.037	0.024	0.052
	Industrial effect	0.80	0.80	1.20	1.10	1.00	0.80	0.021	0.012	0.044
	Corporation growth	0.90	0.90	1.10	1.00	1.00	0.80	0.024	0.014	0.038

First, the characteristic data on six missile systems should be normalized to eliminate the scale differences. Among several types of normalization methods in MCDM techniques, we apply the most commonly used ratio normalization: The beneficial criterion (the higher the better) is normalized as $\hat{y}_{rj} = y_{rj}/\max(y_{rj})$, while the non-beneficial criterion (the lower the better) is normalized as $\hat{y}_{rj} = \min(y_{rj})/y_{rj}$. Reaction time, setup time, acquisition cost, and maintenance cost are the non-benefit criteria, while the others are benefit criteria in this case. When this ratio normalization is used in the proposed model, the 1st constraint of Equation (4) may be omitted; the 1st constraint always holds due to the presence of the 4th constraint.

Meanwhile, the relative weights of the 19 sub-criteria shown on the right side of Table 1 were obtained from a group of experts by using AHP. The crisp value, lower bound, and upper bound of each criterion respectively represent the mean, minimum, and maximum weights based on the judgment of individual experts. Both the lower and upper bound values of each criterion are used as absolute weight restrictions in the proposed model.

The numerical modeling for six missile systems is formulated by the proposed model in Equation (4) and the data reported in Table 1.

4.2. **Results.** LINGO (ver. 17.0), an optimization software, was used to solve the above linear programming problem. Table 2 reports the results for this example using the proposed model. Between the six alternatives, missile 2, with an overall score of 0.973, was selected as the best. The order of the alternatives' priorities is as follows: missile 2 > 6 > 5 > 3 > 1 > 4. Table 2 also reports the results of the optimal weights for the 19 criteria to maximize the overall score of each missile system. Notably, these weight combinations for each alternative meet both lower and upper bound values of Table 1. This means that even if the big-M method is not applied in Equation (4), all problems become feasible in this example.

Table 2. Overall score and optimal weight of six alternatives using the proposed model

No.	Score	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
		u_{11}	u_{12}	u_{13}	u_{14}	u_{15}	u_{16}	u_{17}	u_{18}	u_{19}	$\sum u_r$
Missile 1	0.901	0.075	0.023	0.139	0.028	0.052	0.021	0.021	0.125	0.126	0.148
		0.028	0.042	0.093	0.013	0.009	0.006	0.024	0.012	0.014	1.000
Missile 2	0.973	0.021	0.023	0.139	0.028	0.034	0.096	0.083	0.125	0.232	0.079
		0.028	0.017	0.016	0.013	0.009	0.006	0.024	0.012	0.014	1.000
Missile 3	0.905	0.021	0.023	0.139	0.028	0.034	0.021	0.021	0.037	0.148	0.079
		0.083	0.042	0.093	0.050	0.027	0.020	0.052	0.044	0.038	1.000
Missile 4	0.897	0.021	0.023	0.139	0.028	0.052	0.021	0.021	0.037	0.099	0.148
		0.083	0.017	0.093	0.050	0.027	0.006	0.052	0.044	0.038	1.000
Missile 5	0.950	0.065	0.053	0.139	0.028	0.052	0.021	0.021	0.037	0.232	0.079
		0.083	0.017	0.093	0.013	0.009	0.006	0.024	0.012	0.014	1.000
Missile 6	0.966	0.075	0.041	0.139	0.093	0.052	0.021	0.021	0.037	0.232	0.148
		0.028	0.017	0.016	0.013	0.009	0.006	0.024	0.012	0.014	1.000

We also compared the results between the proposed model and other existing methods, as shown in Table 3. First, the DEA-AR model proposed by Thompson et al. [8] cannot identify the best alternative because the overall scores of missile systems 2, 5, and 6 are equally 1. Although the AR was restricted to the ratios of the criteria weights from the AHP results, it did not provide a full ranking of alternatives. However, we can obtain a full ranking of alternatives by using the proposed model, which has a realistic and stringent constraint such that the sum of all criteria weights is equal to 1.

Next, we compare the results of the proposed model with those of the AHP-weighted sum (WS) method. Both results are the same up to the 1st to 3rd ranks, but the 4th

Table 3. Comparison of rankings between the proposed model and other existing methods

No.	AHP-WS method		DEA-AR model		Proposed model	
	Score	Rank	Score	Rank	Score	Rank
Missile 1	0.865	4	0.905	6	0.901	5
Missile 2	0.951	1	1.000	1	0.973	1
Missile 3	0.856	6	0.945	4	0.905	4
Missile 4	0.864	5	0.928	5	0.897	6
Missile 5	0.924	3	1.000	1	0.950	3
Missile 6	0.932	2	1.000	1	0.966	2

to 6th ranks are different from each other. The score and rank of the AHP-WS method in Table 3 are the results dealing with the criteria weights as crisp values of AHP. If the criteria weights are changed within a given interval of Table 1, the priority may be reversed when the AHP-WS method is applied. In the proposed model, each alternative was able to obtain a maximum overall score through the optimal combination for the criteria weights within the AHP interval. That is, the proposed model is not sensitive to final priority variability when considering the interval weights rather than the exact and crisp values.

5. **Conclusion.** Weapon system acquisition in the defense sector is an important issue of national security. The task to select the best alternative among a set of alternatives is difficult, especially considering the complex and diverse evaluation factors that must be prioritized. However, decision makers prefer conceptually simple and clear analytical methods. No matter how complex the analytical environment may be, there is always a methodology that can effectively use simple tools.

In this study, we proposed a DEA-based model to maximize the overall score of each alternative through an optimal combination within the given interval weights. The proposed model is not sensitive to final priority variability within the interval weights, unlike many MCDM techniques that include AHP. Although the selection problem of weapon systems in the final stage was analyzed as a numerical example, the proposed model can be used throughout the acquisition program. For example, it can be used to determine the means of acquiring weapon systems (i.e., R&D, production by technical transfer, or overseas purchase) and the selection of R&D contractors or core components.

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