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INITIAL ALIGNMENT FOR LARGE AZIMUTH MISALIGNMENT ANGLE WITH ITERATED EXTENDED KALMAN FILTER

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Abstract. *For the initial alignment of Inertial Navigation System (INS) with large azimuth misalignment angle, the accuracy of Extended Kalman Filter (EKF) will decrease due to the nonlinearity ignored by Taylor expansion, and this may cause the estimation result to diverge. An Iterated Extended Kalman Filter (IEKF) is proposed in the measurement update of EKF. The state estimation is iterated until estimation error is lower than the threshold and localization error aroused from Taylor expansion is reduced effectively. The convergent stability is improved by the method in this paper. The simulation results verify the effectiveness of the proposed algorithm.*
Keywords: Inertial navigation system, Iterated extended Kalman filter, Initial alignment

1. **Introduction.** With the continuous development of technology, Inertial Navigation System (INS) [1] has been widely used in the civilian field. INS plays an important role in aircraft, ships, automobile transportation, agricultural sowing and fertilization. The inertial navigation system adopts the inductive navigation method to obtain the position, velocity and attitude information of the carrier by the continuous integration operation of the acceleration and angular velocity signals measured by the Inertial Measurement Union (IMU) [2]. The inertial navigation system is required to provide the initial position, velocity and attitude information of the carrier before the inertial navigation system begins to work. The error in the initial information will increase the output error as the working time increases. In addition, due to the structural characteristics of the strapdown algorithm, these initial errors will accumulate quickly in a short period of time, which may eventually lead to large errors in the position, velocity and attitude [3]. Thus, a key factor in achieving accurate navigation is the initial alignment, providing accurate navigational initial position, velocity and attitude information. The accuracy of the initial alignment is directly related to the working precision of the inertial navigation system. Initial alignment is also one of the research hot topics in the current inertial navigation research field.

In initial alignment, filtering techniques are key technologies and have been widely used in practical engineering. Kalman Filter (KF) is a common traditional filter that estimates the state of the system. The original KF is applicable to linear systems and requires that the observation equation be linear. Bucy et al. proposed Extended Kalman Filter (EKF) in nonlinear systems [4]. The basic idea of EKF is to linearize nonlinear systems and then perform Kalman filtering. For inertial navigation, it is difficult to obtain accurate system model and noise parameters. The statistical characteristics of sensor noise are often uncertain due to changes in working environment [5]. This will cause the filter performance to drop. In order to overcome this shortcoming, some adaptive

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filtering methods, such as maximal a posteriori estimation, virtual noise compensation and dynamic deviation deconvolution estimation are generated. These methods enhance the robustness of the filtering to noise. This type of adaptive Kalman filtering method can automatically adjust the measurement noise covariance matrix.

In general, the initial alignment of the inertial navigation system is based on the inertial system error equation, and the filtering technique is used to estimate the misalignment angle of the system and correct it. The commonly used error equations default to small azimuth misalignment angles, but due to strong external interference factors, azimuth angles are large misalignment angles. Linear alignment models based on small misalignment angles cannot accurately describe the system [6]. The error transfer characteristic, the traditional Kalman filter algorithm cannot meet the requirements of alignment accuracy, so it is necessary to establish a nonlinear system model under large misalignment angle for nonlinear filtering.

Extended Kalman filtering and iterative filtering are commonly used estimation methods, but the computational complexity of the filtering gain matrix is large due to nonlinearity [7], and some approximation calculations have to be used, so the estimation accuracy is not perfect. The extended Kalman filter is a kind of filtering method based on nonlinear system model. However, because EKF omits high-order infinitesimals, the estimation will be inaccurate when nonlinear errors are serious. To deal with the error of EKF linearization, many scholars have done a lot of research work, such as iterative EKF algorithm, Uscent Kalman Filter (UKF) algorithm based on Uscent transform, central differential Kalman filter algorithm. For an INS subjected to multiple disturbances, a mixed H2/H1 filter was presented for stationary base self-alignment with enhanced disturbance rejection and attenuation performance in [8]. An anti-disturbance initial alignment approach for INS with the nonlinear dynamics and multiple disturbances was proposed in [9]. [10] proposed a new anti-disturbance fault tolerant alignment approach to improve the accuracy and reliability for an INS subjected to multiple disturbances and system faults. In this paper, IEKF is used in the initial alignment of the inertial navigation system under a large azimuth misalignment angle, and the nonlinear alignment system under large azimuth misalignment angle is simulated under static base. The simulation results verify the effectiveness of the method.

2. Nonlinear Equation of Large Azimuth Misalignment with Initial Alignment.

Under the condition of static base, the horizontal error angle φ_E and φ_N are considered to be small, while the azimuth error angle φ_U is large. Taking the geographic coordinate system as the navigation coordinate system n, the coordinate system established by the position solved by the inertial navigation system is the calculation coordinate system g, and the actual position of the carrier is the carrier coordinate system b. At this time, the coordinate transformation matrix from n to b can be written as:

$$C_n^c \approx \begin{bmatrix} \cos\varphi_U & \sin\varphi_U & -\varphi_N \\ -\sin\varphi_U & \cos\varphi_U & \varphi_E \\ \varphi_N\cos\varphi_U + \varphi_E\sin\varphi_U & \varphi_N\sin\varphi_U - \varphi_E\cos\varphi_U & 1 \end{bmatrix} \tag{1}$$

Ignoring the influence of position error, the error angle of calculation coordinate system g and navigation coordinate system n caused by position error is 0. Thus, the projection error of the earth's rotation angular velocity ω_{ie} and gravitational acceleration g in the n is also zero, that is $\delta\omega_{ie}^n = 0$, $\delta g^n = 0$. Since the discussion in this paper is a static base, the velocity V^n and rotation angular velocity ω_{en}^n of the carrier are both 0. According to error model of Bar-Itzhachk and Bermant [11,12] error equations of speed and error equations of attitude are as follows:

$$\begin{cases} \delta\dot{V} = \left(C_n^b - I\right) f^n - 2\omega_{ie}^n \times \delta V^n + C_b^c \nabla^b \\ \dot{\varphi} = \left(I - C_n^c\right) \omega_{ie}^n + \delta\omega_{en}^n + C_b^c \varepsilon^b \end{cases} \tag{2}$$

where C_b^c is the coordinate transformation matrix from the load system b to the calculated coordinate system c, which can be calculated by the roll angle, pitch angle and yaw angle of the carrier:

$$C_b^c = \{C_{ij}\}_{i,j=1,2,3} \tag{3}$$

φ is the attitude error vector; δV is the velocity error vector; f^n is the output of accelerometer; ∇^b is the measurement error of the accelerometer; ε^b is the measurement error of the gyro; $\delta\omega_{en}^n$ is the calculation error of ω_{en}, and it depends on the velocity error:

$$\delta\omega_{en}^n = \begin{bmatrix} 0 & -1/(R_m + h) & 0 \\ 1/(R_n + h) & 0 & 0 \\ \tan L/(R_n + h) & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta V_E \\ \delta V_N \\ \delta V_U \end{bmatrix} \tag{4}$$

where L is geographical latitude; R_m is the radius of curvature along the meridian; R_n is the radius of curvature along the prime ring; h is the altitude of the aircraft in the sky; Ω is the angular velocity of earth's rotation. The projection of earth rotation angular velocity and gravity acceleration g on n system is respectively: $\omega_{ie}^n = \begin{bmatrix} 0 & \Omega\cos L & \Omega\sin L \end{bmatrix}^T$, $f^n = \begin{bmatrix} 0 & 0 & g \end{bmatrix}^T$.

The output relations of gyro and accelerometer under the navigation system and load system satisfy the following relations:

$$C_b^c \begin{bmatrix} \nabla_x^b & \nabla_y^b & \nabla_z^b \end{bmatrix}^T = \begin{bmatrix} \nabla_E & \nabla_N & \nabla_U \end{bmatrix}^T \tag{5}$$

$$C_b^c \begin{bmatrix} \varepsilon_x^b & \varepsilon_y^b & \varepsilon_z^b \end{bmatrix}^T = \begin{bmatrix} \varepsilon_E & \varepsilon_N & \varepsilon_U \end{bmatrix}^T \tag{6}$$

Expanding (2), the specific form is as follows:

$$\delta\dot{V}_E = \varphi_N g + 2\Omega\sin L\delta V_N + \nabla_E \tag{7}$$

$$\delta\dot{V}_N = -\varphi_E g - 2\Omega\sin L\delta V_E + \nabla_N \tag{8}$$

$$\dot{\varphi}_E = -\sin\varphi_U\Omega\cos L + \varphi_N\Omega\sin L - \delta V_U/(R_m + h) + \varepsilon_E \tag{9}$$

$$\dot{\varphi}_N = (1 - \cos\varphi_U)\Omega\cos L - \varphi_E\Omega\sin L + \delta V_E/(R_n + h) + \varepsilon_N \tag{10}$$

$$\dot{\varphi}_U = (\varphi_E\cos\varphi_U - \varphi_N\sin\varphi_U)\Omega\cos L + \delta V_E\tan L/(R_n + h) + \varepsilon_U \tag{11}$$

In the formula, the subscripts E, N and U are corresponding to east, north, up of the navigation frame n; φ is the misalignment angle; δV is speed error; the model error vector $W = \begin{bmatrix} \nabla_E & \nabla_N & \varepsilon_E & \varepsilon_N & \varepsilon_U \end{bmatrix}^T$. Take two horizontal velocity errors δV_E and δV_N for the observation. Therefore, the nonlinear state equation of initial alignment for constructing an inertial navigation system [11-13] is as follows:

$$\begin{cases} \dot{X} = f(X) + GW \\ Z = h(X) + v \end{cases} \tag{12}$$

In the formula, model error perturbation matrix $G = I_{5\times5}$, define five dimension system state vector $X = \begin{bmatrix} \delta V_E & \delta V_N & \varphi_E & \varphi_N & \varphi_U \end{bmatrix}^T$, and $f(X)$ is as follows:

$$f(X) = \begin{bmatrix} \varphi_N g + 2\Omega\sin L\delta V_N \\ -\varphi_E g - 2\Omega\sin L\delta V_E \\ -\sin\varphi_U\Omega\cos L + \varphi_N\Omega\sin L - \delta V_U/(R_m + h) \\ (1 - \cos\varphi_U)\Omega\cos L - \varphi_E\Omega\sin L + \delta V_E/(R_n + h) \\ (\varphi_E\cos\varphi_U - \varphi_N\sin\varphi_U)\Omega\cos L + \delta V_E\tan L/(R_n + h) \end{bmatrix} \tag{13}$$

$Z = \begin{bmatrix} Z_1 & Z_2 \end{bmatrix}^T = \begin{bmatrix} \delta V_E & \delta V_N \end{bmatrix}^T$ is the observation vector, $h(X) = HX$, $H = \begin{bmatrix} I_{2\times 2} & O_{2\times 3} \end{bmatrix}$ is the observation matrix, and v is the measurement noise of the observation equation.

After discretization of the nonlinear continuous system model by the Runge-Kutta methods [14], discrete system equations of state and measurement can be described as the formula:

$$\begin{cases} x_{k+1} = f(x_k) + w_k \\ z_k = h(x_k) + v_k \end{cases} \tag{14}$$

In the formula, system noise w_k and measurement noise v_k are mutually independent with time varying mean and covariance of the normal white noise sequences.

$$\begin{cases} E\left[w_k\right] = 0, \quad Cov[w_k, w_j] = E\left[w_k w_j^T\right] = Q_k \delta_{kj} \\ E[v_k] = 0, \quad Cov[v_k, v_j] = E\left[v_k v_j^T\right] = R_k \delta_{kj} \\ Cov[w_k, v_j] = E\left[w_k v_j^T\right] = 0 \end{cases} \tag{15}$$

Q_k is the variance intensity matrix of the discrete system noise w_k; R_k is the variance intensity matrix of the discrete measurement noise v_k; δ_{kj} is the Kronecker-function.

3. Iterated Extended Kalman Filter.

3.1. Extended Kalman filter. For the estimation of nonlinear systems and measurement equations, EKF is the simplest and widely used Gaussian nonlinear filtering method [15]. As the simplest and most direct form of KF [16], EKF performs a one-order Taylor expansion [17] on the nonlinear function, which linearizes the nonlinear function and then performs linear KF processing [18]. The calculation process of the EKF algorithm is as follows.

Set the nonlinear error equation and nonlinear measurement equation of extended Kalman filter as

$$\begin{cases} x_{k+1} = f(x_k) + w_k \\ z_{k+1} = h(x_{k+1}) + v_{k+1} \end{cases} \tag{16}$$

In the formula, $f()$ is the state transition function; $h()$ is the measurement transfer function; x_k is system state; w_k is system noise; v_{k+1} is measuring noise.

First, in each calculation time step of EKF, the Jacobi matrix [19] is solved for the nonlinear function:

$$F_k = \left.\frac{\partial f}{\partial x}\right|_{x=\hat{x}_k} H_{k+1} = \left.\frac{\partial h(x)}{\partial x}\right|_{x=\hat{x}_{k+1|k}} \tag{17}$$

The prior state estimation and covariance are predicted by nonlinear equation of state when the system noise is ignored:

$$\hat{x}_{k+1|k} = f\left(\hat{x}_{k|k}\right) \tag{18}$$

$$P_{k+1|k} = F_k P_{k|k} F_k^T + Q_k \tag{19}$$

The innovation matrix and the innovation variance matrix are calculated by measuring equation when the measurement noise is ignored:

$$e_{z,k+1|k} = z_{k+1} - \hat{z}_{k+1|k} = z_{k+1} - h\left(\hat{x}_{k+1|k}\right) \tag{20}$$

$$S_{k+1} = H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1} \tag{21}$$

Finally, the gain of EKF is calculated K_{k+1}, state estimation update and state error variance matrix:

$$K_{k+1} = P_{k+1|k} H_{k+1}^T {S_{k+1}}^{-1} \tag{22}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} e_{z,k+1|k} \tag{23}$$

$$P_{k+1|k+1} = (I - K_{k+1}H_{k+1})\, P_{k+1|k} \tag{24}$$

3.2. **The combining of iterative filter and extended Kalman filter.** If the error between the estimated value $\hat{x}_{k+1|k}$ and the true value is large in the EKF, the first-order approximation will not accurately approximate the true value. It will affect the accuracy of the EKF. Therefore, the following IEKF will be applied to correcting these errors effectively.

Based on the above ideas, the IEKF algorithm used in this paper involves the following three steps.

The first step:

$$\hat{x}_{k+1|k} = f\left(\hat{x}_{k|k}\right) \tag{25}$$

$$P_{k+1|k} = F_k P_{k|k} F_k^T + Q_k \tag{26}$$

$$K_{k+1} = P_{k+1|k} H_{k+1}^T \left(H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1}\right)^{-1} \tag{27}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\left[z_{k+1} - h\left(\hat{x}_{k+1|k}\right)\right] \tag{28}$$

The second step:

$$\hat{X}_{k|k+1} = \hat{x}_{k|k} + P_{k|k} F_K^T P_{k+1|k}^{-1}\left[\hat{X}_{k+1|k+1} - \hat{X}_{k+1|k}\right] \tag{29}$$

The third step:

$$P_{k+1|k} = F_k P_{k|k} F_k^T + Q_k \tag{30}$$

$$K_{k+1} = P_{k+1|k} H_{k+1}^T \left(H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1}\right)^{-1} \tag{31}$$

$$P_{k+1|k+1} = (I - K_{k+1}H_{k+1})\, P_{k+1|k} \tag{32}$$

$$\hat{x}_{k+1|k} = f\left(\hat{x}_{k|k+1}\right) \tag{33}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\left[z_{k+1} - h\left(\hat{x}_{k+1|k}\right)\right] \tag{34}$$

The algorithm of Equations (25)-(34) can be represented by Figure 1, it can be clearly seen from the figure that IEKF has two calculation circuits, the one on the left is the

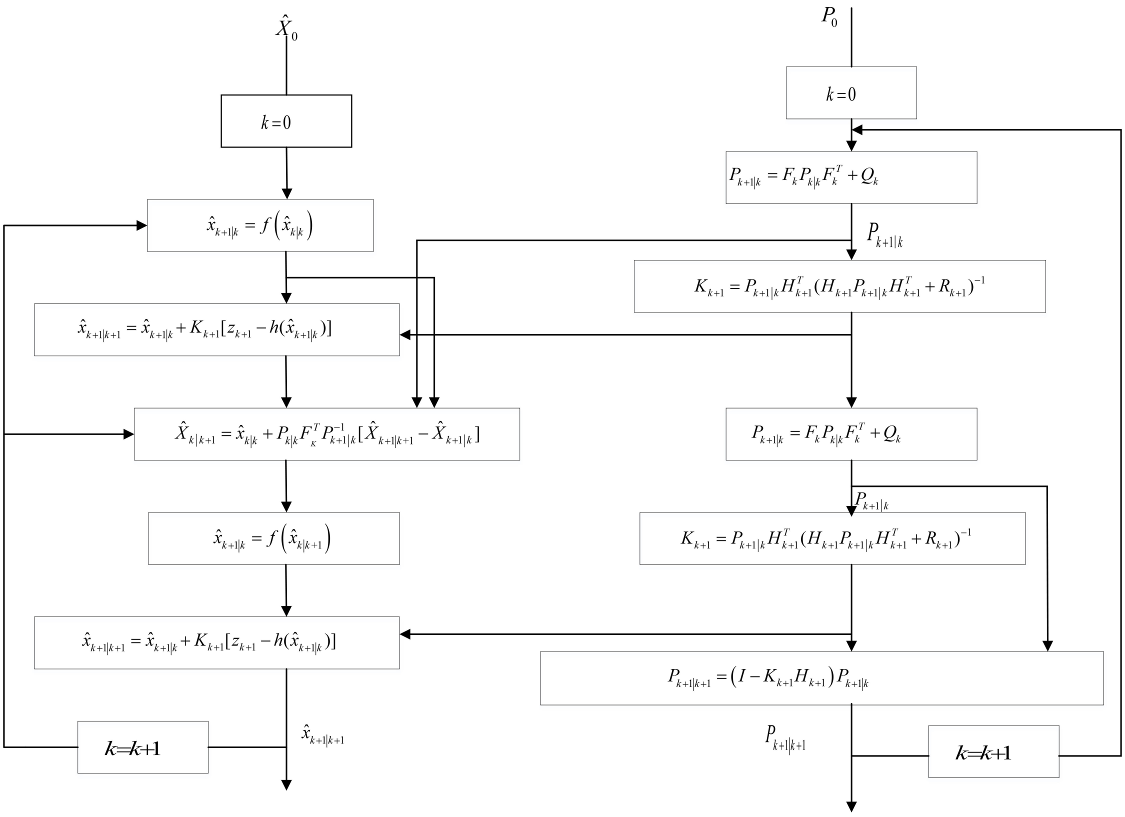

FIGURE 1. The flowchart of IEKF

filtering calculation circuit, and the one on the right is the gain calculation circuit. The gain calculation circuit is independent of the gain calculation circuit, while the filter calculation circuit depends on the gain calculation circuit. In a filtering cycle, IEKF has two obvious information updating processes: time updating process and measurement updating process.

4. **Simulation Examples.** In this section, the initial alignment of a Strapdown Inertial Navigation System (SINS) with large azimuth misalignment angle is considered. Simulation conditions are as follows.

- Local latitude is 44.9deg north latitude; local longitude is 116.4deg east longitude.
- Initial misalignment angles φ_E, φ_N, φ_U are chosen as 1, 1 and 10deg, respectively.
- Gyroscopes: the constant drift is 0.1deg/hr, and the random drift is 0.05deg/hr/Hz.
- Accelerometers: the constant bias is 100μg, and the random bias is 50μg/Hz.

Both system noise and measurement noise are Gaussian white noise; $P(0)$, Q, R are as follows:

$$P(0) = dig\left\{(1\text{deg})^2, (1\text{deg})^2, (10°)^2, (0.05\text{m/s})^2, (0.05\text{m/s})^2,\right.$$
$$\left.(0.1°/\text{hr})^2, (0.1°/\text{hr})^2, (0.1°/\text{hr})^2, (100\mu\text{g})^2, (100\mu\text{g})^2\right\}$$
$$Q = dig\left\{(0.05°/\text{hr})^2, (0.05°/\text{hr})^2, (0.05°/\text{hr})^2, (50\mu\text{g})^2, (50\mu\text{g})^2, 0, 0, 0, 0, 0\right\}$$
$$R = dig\left\{(0.05\text{m/s})^2, (0.05\text{m/s})^2\right\}$$

The problem of initial alignment is considered based on the IEKF. The fowchart of the proposed algorithm is described in Figure 1. The estimation errors of three misalignment angles are described in Figures 2-4, where the red thin lines represent the estimation errors by the proposed iterated extended Kalman flter in this paper and blue thick lines denote the estimation errors based on extended Kalman flter. The Root Mean Square (RMS) for the estimation errors of three misalignment angles between the proposed approach and EKF are shown in Table 1. From Figures 2-4 and Table 1, it can be seen that the proposed predictive iterated extended Kalman flter has a good performance for initial alignment of an SINS. It is shown that the precision of the proposed flter is better than EKF in the estimation accuracy of azimuth error angle.

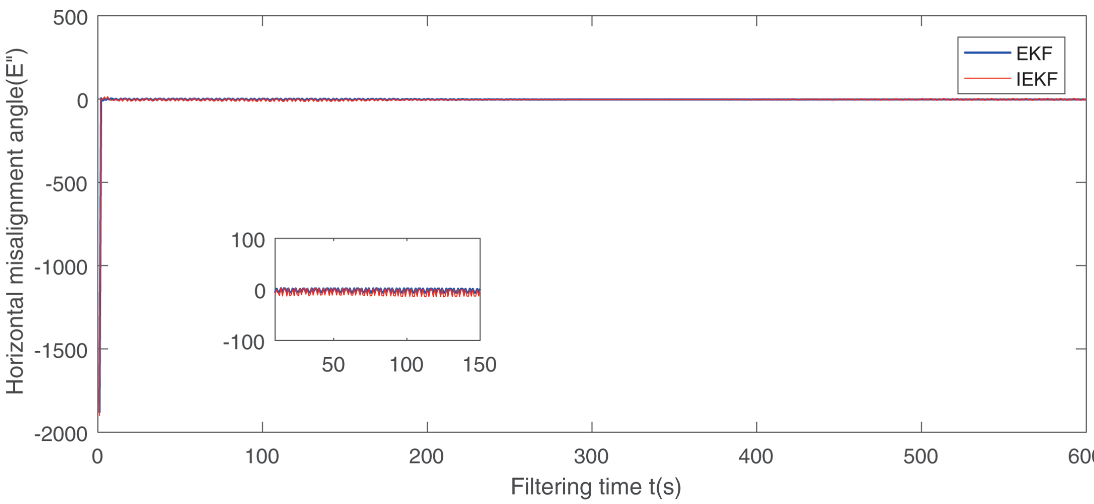

FIGURE 2. Estimation errors of φ_E

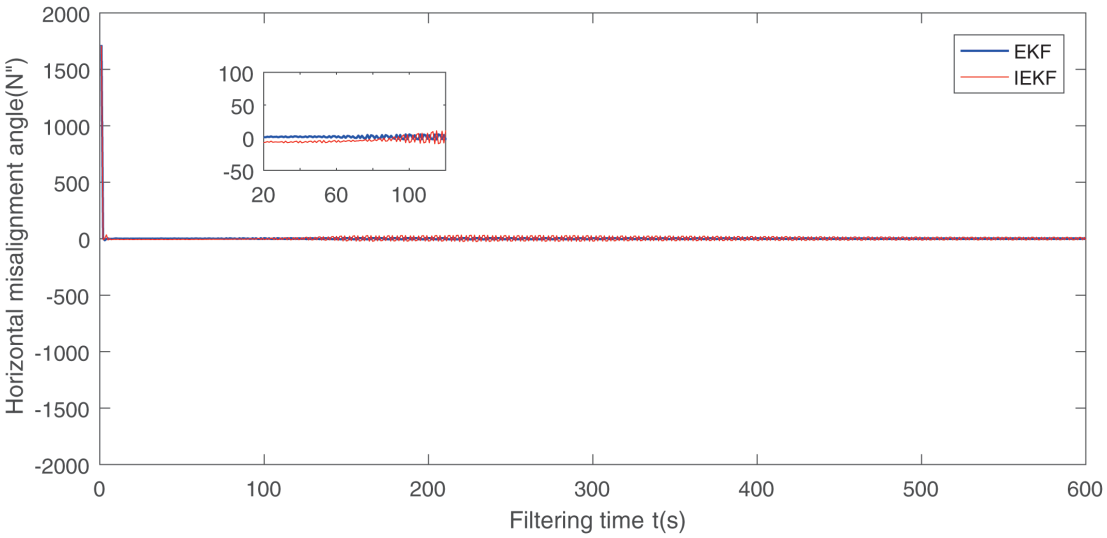

FIGURE 3. Estimation errors of φ_N

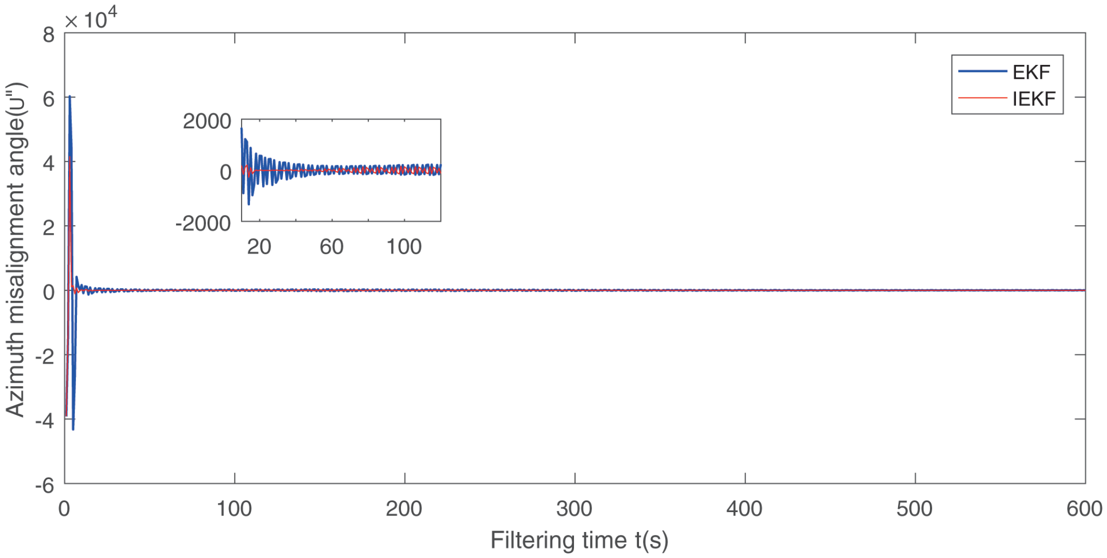

FIGURE 4. Estimation errors of φ_U

TABLE 1. Comparison with two kinds of filtering alignment results

Method	φ_E ($''$)	φ_N ($''$)	φ_U ($''$)
EKF	-21.28	28.67	-489.25
IEKF	-20.59	31.28	-252.12

5. **Conclusions.** In this article, the problem of inertial navigation initial alignment of static base under large azimuth misalignment angle is studied. For the shortcoming of the extended Kalman filter algorithm, an iterative extended Kalman filter method is designed to measure the algorithm update. The state estimate of the phase is updated with a multi-step iteration so that it iteratively ends when a certain threshold is met [20]. The iterative EKF algorithm uses multi-step iteration for the state estimation of the measurement update based on the original EKF, thereby reducing the truncation error caused by Taylor expansion [21], further improving the accuracy of the inertial alignment

under the large azimuth misalignment angle. Finally, simulations for stationary initial alignment of an SINS are given to show the efficiency of the proposed approach.

This paper mainly focuses on SINS initial alignment under static base. In practical engineering, there are also many SINS initial alignment under dynamic base. Next work can be done to study SINS initial alignment under dynamic base.

REFERENCES

[1] Y. Huang, Y. Zhang and X. Wang, Kalman-filtering-based in-motion coarse alignment for odometer-aided SINS, *IEEE Trans. Instrumentation and Measurement*, vol.66, no.12, pp.3364-3377, 2017.

[2] G. H. Cheng, S. Y. Cao, L. Guo and W. Chen, Initial alignment of inertial navigation system based on a predictive iterated Kalman filter, *The 37th Chinese Control Conference (CCC)*, Wuhan, pp.4655-4660, 2018.

[3] T. Z. Kang, J. C. Fang and W. Wang, In-flight calibration approach based on quaternion optimization for POS used in airborne remote sensing, *IEEE Trans. Instrumentation and Measurement*, vol.62, no.11, pp.2882-2889, 2013.

[4] R. S. Bucy, C. Hecht and K. D. Senne, New methods for nonlinear filtering, *Revue Francaise D Automatique Informatique Recherche Operationnelle*, vol.7, no.7, pp.3-54, 1973.

[5] M. Attari, Z. Luo and S. Habibi, An SVSF-based generalized robust strategy for target tracking in clutter, *IEEE Trans. Intelligent Transportation Systems*, vol.17, no.5, pp.1381-1392, 2016.

[6] H. Shao, L. Miao and W. Gao, Ensemble particle filter based on KLD and its application to initial alignment of SINS in large misalignment angles, *IEEE Trans. Industrial Electronics*, vol.65, no.11, pp.8946-8955, 2018.

[7] Y. G. Zhang, L. Luo, T. Fang and G. Q. Wang, An improved coarse alignment algorithm for odometer-aided SINS based on the optimization design method, *Sensors*, vol.18, no.1, p.195, 2018.

[8] S. Y. Cao and L. Guo, Multi-objective robust initial alignment algorithm for Inertial Navigation System with multiple disturbances, *Aerosp. Sci. Technol.*, vol.21, no.1, pp.1-6, 2012.

[9] L. Guo and S. Y. Cao, Initial alignment for nonlinear inertial navigation systems with multiple disturbances based on enhanced anti-disturbance filtering, *Int. J. Contr.*, vol.85, no.5, pp.491-501, 2012.

[10] S. Y. Cao, L. Guo and W. H. Chen, Anti-disturbance fault tolerant initial alignment for inertial navigation system subjected to multiple disturbances, *Aerosp. Sci. Technol.*, vol.72, pp.95-103, 2018.

[11] Y. L. Mao, J. B. Chen, C. L. Song, W. S. Wu et al., Scaled UKF with reduced sigma points for initial alignment of SINS, *The 30th Chinese Control Conference*, Yantai, China, pp.106-110, 2011.

[12] S. A. Gadsden and A. S. Lee, Advances of the smooth variable structure filter: Square-root and two-pass formulations, *Journal of Applied Remote Sensing*, vol.11, no.1, pp.15-18, 2017.

[13] J. C. Fang and S. Yang, Study on innovation adaptive EKF for in-flight alignment of airborne POS, *IEEE Trans. Instrumentation and Measurement*, vol.60, no.4, pp.1378-1388, 2011.

[14] X. L. Wang, X. J. Guan, J. C. Fang and H. N. Li, A high accuracy multiplex two-position alignment method based on SINS with the aid of star sensor, *Aerosp. Sci. Technol.*, vol.42, pp.66-73, 2015.

[15] J. Li, J. Fang and M. Du, Error analysis and gyro-bias calibration of analytic coarse alignment for airborne POS, *IEEE Trans. Instrumentation and Measurement*, vol.61, no.11, pp.3058-3064, 2012.

[16] B. M. Scherzinger, Inertial navigator error models for large heading uncertainty, *Proc. of Position, Location and Navigation Symposium (PLANS'96)*, USA, pp.477-484, 1996.

[17] I. Y. Bar-Itzhack and N. Berman, Control theroretic approach to inertial navigation systems, *Journal of Guidance Control and Dynamics*, vol.11, no.3, pp.237-245, 1988.

[18] S. Garg, L. Morrow and R. Mamen, Strapdown navigation technology: A literature survey, *Journal of Guidance Control and Dynamics*, vol.1, no.3, pp.287-291, 1978.

[19] J. C. Fang and X. N. Gong, Predictive iterated Kalman filter for INS/GPS integration and its application to SAR motion compensation, *IEEE Trans. Instrumentation and Measurement*, vol.59, no.4, pp.909-915, 2010.

[20] S. L. Lu, L. Xie and J. B. Chen, New techniques for initial alignment of strapdown inertial navigation system, *Journal of the Franklin Institute*, vol.346, no.10, pp.1021-1037, 2009.

[21] L. Wang, W. Wu and X. Pan, RLG SINS dynamic error compensation under vibration environments, *Gyroscopy and Navigation*, vol.9, no.1, pp.35-44, 2018.