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DESIGN AND ANALYSIS OF A NOVEL FIVE-DIMENSIONAL HYPER-CHAOTIC SYSTEM

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Abstract. *This paper describes a novel five-dimensional hyper-chaotic system, where the novel system has twelve positive parameters and the basic features and dynamic behavior of the chaotic system are tested through the usage of equilibrium points, dissipativity, symmetry, Lyapunov exponents, waveform analysis, Kaplan-Yorke dimension, and sensitivity to initial conditions. It has found that the system has two unstable equilibrium points as well as it is a dissipative system, it has symmetric pairs of coexisting attractors, such as limit cycles or strange attractors, it has two non-negative Lyapunov exponents where the maximum non-negative Lyapunov exponent is (0.721821), Kaplan-Yorke dimension has been calculated as (2.66735), and it can be observed that the waveform of the time range has non-periodic properties. These results confirm that the nonlinear system is actually a hyperactivity system, random, and shows great complexity, as it is highly sensitive to initial conditions and therefore unpredictable for long periods.*
Keywords: Hyper-chaotic system, Five-dimensional, Lyapunov exponents, Waveform analysis, Equilibrium points

1. **Introduction.** Chaos is a prevalent phenome in nature that is being widely utilized in varied applications in numerous domains of study, like biology, information processing, mathematics, physics, secure communications, engineering, high-performance circuit design for telecommunications [1,2]. It has a noise-like behavior, and it exists in nonlinear dynamical systems. The engendered sequences from chaotic systems are very sensitive to their initial conditions; it has a long periodicity and spread spectrum [3].

These characteristics of chaotic system have a solid interconnection with features of cryptography. So, they have been a favorite selection for styling security primitives and cryptosystems in chaos-based cryptography for a long time [4]. In comparison with chaotic systems, hyper-chaotic systems have a tendency to have extra complex dynamics [5].

The hyper-chaotic system is often defined as a chaotic system with more than one positive Lyapunov exponent, i.e, the numeral of directions of pervasion is larger than one, which results the system to show the behavior of high disorder and randomness. Every day the number of articles that relates to introducing new hyper-chaotic system is increasing. Normally, generating a hyper-chaotic system with a more complex topological construction specially, making attractors with multi-wings or multi scrolls, is a motivating topic for analysis, and thus, becomes a fascinating mission and typically a key case for several engineering applications [6,7].

The first classical hyper-chaotic system is the known hyper-chaotic Rössler system [8]. Afterward, numerous hyper-chaotic systems have been advanced and therefore the applications of this model have been improved lately. Onto the bygone ages, numerous other hyper-chaotic systems have been presented, like hyper-chaotic Chen system [9],

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hyper-chaotic Lü system [10], hyper-chaotic Lorenz system [11], hyperchaos chua's circuit [12], hyper-chaotic Nikolov system [5]. The hyper-chaotic system has more complicated and intense construction than a chaotic system for the reason that numerous ingrained advantages, the hyper-chaotic system has been widely utilized in numerous domains such as information science, electronics, mathematics, physics, communication as given in [14].

The building of different hyper-chaotic systems and their study are beneficial to reconnoitering the nature of hyperchaos. They are extremely interesting to create different hyper-chaotic systems with the most hyper-chaotic nature and complicated dynamics [14]. The research has ripened at the trend of styling upper dimensional hyper-chaotic systems and a current concentration is on designing 5-D systems with best characteristics and dynamics [15,16].

In this paper, a novel five-dimensional chaotic system is presented and the dynamical behavior characteristics of the proposed system are studied and analyzed using the mathematica program. The new chaotic system is convenient to be utilized in many implementations and could be utilized in info encryption, because of the large group of keys that can be created from the system.

This paper is structured as follows. Section 2 introduces the construction of the novel five-dimensional system. Section 3 presents dynamics analysis of the system, equilibrium point, dissipativity, symmetry and invariability, Lyapunov exponents and Lyapunov dimensions, waveform analysis of the novel chaotic system, sensitivity to initial conditions. Section 4 provides the conclusion.

2. **Construction of the Novel Five-Dimensional System.** The novel five-dimensional autonomous system is gotten as follows:

$$\begin{aligned}\frac{dx}{dt} &= -ax + bv + cyzw\\ \frac{dy}{dt} &= dy - exzw - f\sinh(w)\\ \frac{dz}{dt} &= -gz + xy + hv\\ \frac{dw}{dt} &= iy - w - \tanh(v)x\\ \frac{dv}{dt} &= -jxz + kv + l\cosh(w)\end{aligned} \tag{1}$$

where x, y, z, w, v and $t \in \Re^+$ are called the states of the system and b, c, a, d, e, f, g, h, i, j, k and l are positive parameters of the system.

The 5-D system (1) displays a chaotic attractor, when the system parameter values are chosen as: $a = 8$, $b = 4.6$, $c = 1.3$, $d = 1.2$, $e = 0.3$, $f = 2$, $g = 2.5$, $h = 0.1$, $i = 5$, $j = 3$, $k = 1.8$, and $l = 7$ and the initial conditions as: $x(0) = 0.5$, $y(0) = 0.4$, $z(0) = 0.6$, $w(0) = 1.4$ and $v(0) = 1.8$. The strange attractors in three-dimension are shown in Figure 1, and the strange attractors in two-dimension are shown in Figure 2.

3. **Dynamics Analysis of the System.** In this department, fundamental features and complicated dynamics of the novel system (1) are inspected; the novel dynamic system has the next fundamental features.

3.1. **Equilibrium point.** We can acquire that system (1) has two equilibrium points:

$$\begin{aligned}0 &= -ax + bv + cyzw\\ 0 &= dy - exzw - f\sinh(w)\\ 0 &= -gz + xy + hv\\ 0 &= iy - w - \tanh(v)x\\ 0 &= -jxz + kv + l\cosh(w)\end{aligned} \tag{2}$$

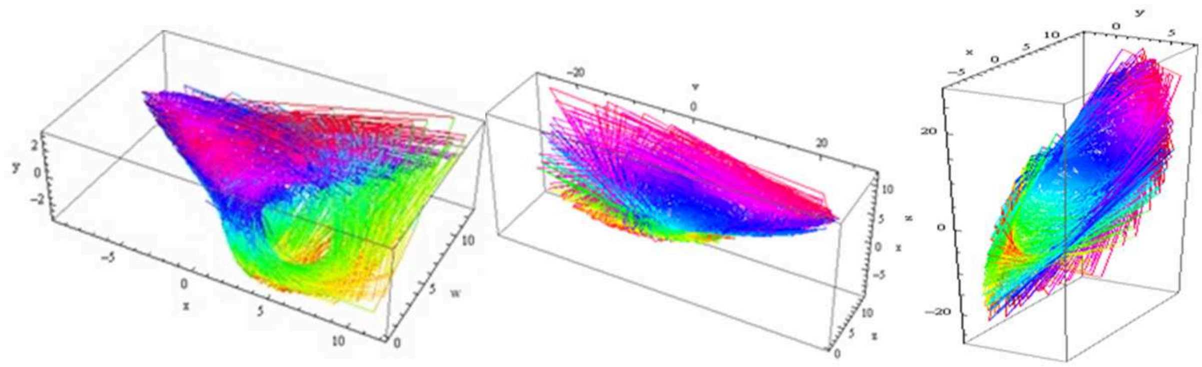

Figure 1. Chaotic attractors three-dimensional view

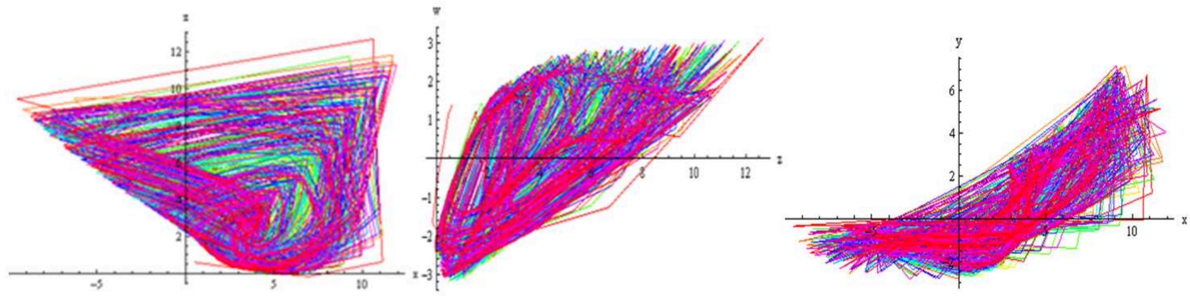

Figure 2. Chaotic attractors two-dimensional view

When $a = 8$, $b = 4.6$, $c = 1.3$, $d = 1.2$, $e = 0.3$, $f = 2$, $g = 2.5$, $h = 0.1$, $i = 5$, $j = 3$, $k = 1.8$, and $l = 7$, the two equilibrium points become:

$E_0\{x = 0, y = 0, z = 0, w = 0, v = 0\}$,

$E_1\{x = -7.16838, y = -6.25877, z = 16.1932, w = -43.8217, v = -56.8716\}$.

The Jacobian matrix uses a partial differential equation to obtain the equilibrium point, the Jacobian is found at each equilibrium point to confirm its unstability. The Jacobian matrix of the system (1), let:

$$f = \begin{cases} f_1 = \frac{dx}{dt} = -ax + bv + cyzw \\ f_2 = \frac{dy}{dt} = dy - exzw - f\sinh(w) \\ f_3 = \frac{dz}{dt} = -gz + xy + hv \\ f_4 = \frac{dw}{dt} = iy - w - \tanh(v)x \\ f_5 = \frac{dv}{dt} = -jxz + kv + l\cosh(w) \end{cases} \tag{3}$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} & \frac{\partial f_1}{\partial w} & \frac{\partial f_1}{\partial v} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} & \frac{\partial f_2}{\partial w} & \frac{\partial f_2}{\partial v} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} & \frac{\partial f_3}{\partial w} & \frac{\partial f_3}{\partial v} \\ \frac{\partial f_4}{\partial x} & \frac{\partial f_4}{\partial y} & \frac{\partial f_4}{\partial z} & \frac{\partial f_4}{\partial w} & \frac{\partial f_4}{\partial v} \\ \frac{\partial f_5}{\partial x} & \frac{\partial f_5}{\partial y} & \frac{\partial f_5}{\partial z} & \frac{\partial f_5}{\partial w} & \frac{\partial f_5}{\partial v} \end{bmatrix} \tag{4}$$

$$J = \begin{bmatrix} -a & czw & cyw & cyz & b \\ -ezw & d & -exw & -exz & -f\cosh(w) \\ y & x & -g & 0 & h \\ \tanh(v) & i & 0 & -1 & x\,\text{sech}\tanh(v) \\ -jz & 0 & -jx & l\sinh(w) & k \end{bmatrix} \tag{5}$$

For equilibrium point $E_0\{x = 0, y = 0, z = 0, w = 0, v = 0\}$, and $a = 8$, $b = 4.6$, $c = 1.3$, $d = 1.2$, $e = 0.3$, $f = 2$, $g = 2.5$, $h = 0.1$, $i = 5$, $j = 3$, $k = 1.8$, and $l = 7$, the Jacobian matrix has the following result:

$$J = \begin{bmatrix} -8 & 0 & 0 & 0 & 4.6 \\ 0 & 1.2 & 0 & 0 & -2 \\ 0 & 0 & -2.5 & 0 & 0.1 \\ 0 & 5 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1.8 \end{bmatrix} \tag{6}$$

To gain its eigenvalues, let $|\lambda I - J| = 0$. Then the eigenvalues that corresponding to the equilibrium $E_0(0, 0, 0, 0, 0)$ are respectively obtained as follows:

$$\lambda_1 = -8, \quad \lambda_2 = -2.5, \quad \lambda_3 = 1.8, \quad \lambda_4 = 1.2, \quad \lambda_5 = -1$$

Therefore, the equilibrium $E_0(0, 0, 0, 0, 0)$ is a saddle point. So, the hyper-chaotic system is unstable at the point E_0. At the same time, it is easy to prove that the equilibrium point E_1 is also an unstable saddle point. For equilibrium point $E_1\{x = 3.99397, y = 0.850088, z = 1.63191, w = 0.25648, v = 6.84548\}$ and $a = 8$, $b = 4.6$, $c = 1.3$, $d = 1.2$, $e = 0.3$, $f = 2$, $g = 2.5$, $h = 0.1$, $i = 5$, $j = 3$, $k = 1.8$, and $l = 7$, the Jacobian matrix has the following result:

$$J = \begin{bmatrix} -8 & 1.3*1.631*0.526 & 1.3*0.850*0.256 & 1.3*0.850*1.631 & 4.6 \\ -0.3*1.631*0.256 & 1.2 & -0.3*3.993*0.256 & -0.3*3.993*1.631 & -2*\cosh(0.256) \\ 0.850 & 3.993 & -2.5 & 0 & 0.1 \\ \tanh(6.845) & 5 & 0 & -1 & 3.993*\text{sech}\tanh(6.845) \\ -3*1.631 & 0 & -3*3.993 & 7*\sinh(0.256) & 1.8 \end{bmatrix} \tag{7}$$

In the same way, the eigenvalues corresponding to an equilibrium point E_1 are obtained as:

$$\lambda_1 = -7.03841, \quad \lambda_2 = -1.9232 + 5.31805i, \quad \lambda_3 = -1.9232 - 5.31805i, \quad \lambda_4 = 3.08341$$
$$\lambda_5 = -0.698601$$

where i indicates the unity of fanciful number. With regard to the equilibrium point E_1, the results display that λ_1, λ_4 and λ_5 are positive and negative real numbers, λ_2 and λ_3 become a pair of complex conjugate eigenvalues with negative real parts. So, equilibrium point E_1 is saddle-focus point; thus, these equilibrium points are all unstable.

3.2. **Dissipativity.** System (1) can be expressed in vector notation as:

$$f = \begin{cases} f_1 = \frac{dx}{dt} = -ax + bv + cyzw \\ f_2 = \frac{dy}{dt} = dy - exzw - f\sinh(w) \\ f_3 = \frac{dz}{dt} = -gz + xy + hv \\ f_4 = \frac{dw}{dt} = iy - w - \tanh(v)x \\ f_5 = \frac{dv}{dt} = -jxz + kv + l\cosh(w) \end{cases} \tag{8}$$

The divergence of the vector field f on R^5 is given by

$$\nabla \cdot f = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z} + \frac{\partial f_4}{\partial w} + \frac{\partial f_5}{\partial v} \tag{9}$$

We note that $\nabla \cdot f$ measures the rate at which volumes change under the flow Φ_t of f.

Let D be a region in R^5 with a smooth boundary and let $D(t) = \Phi_t(t)$, the image of D under Φ_t, the time t of the flow of f. Let $V(t)$ be the volume of $D(t)$. By Liouville's theorem, we get:

$$\frac{dV}{dt} = \int_{D(t)} (\nabla \cdot f)\, dxdydzdwdv \tag{10}$$

For system (1), we find that
$\nabla \cdot f = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z} + \frac{\partial f_4}{\partial w} + \frac{\partial f_5}{\partial v} = -a + d - g - 1 + k < 0$ because a, d and k are positive constants. Substituting (9) into (10) and simplifying, we get

$$\begin{aligned} \frac{dV}{dt} &= (-a + d - g - 1 + k) \int_{D(t)} dxdydzdwdv \\ &= (-a + d - g - 1 + k)V(t) \\ &= e^{-8.5}V(t) \end{aligned} \tag{11}$$

Solving the first order linear differential equation (11), we obtain the unique solution

$$\begin{aligned} V(t) &= V(0)e^{(-a+d-g-1+k)t} \\ &= V(0)e^{-8.5t} \end{aligned} \tag{12}$$

Equation (12) shows that any volume $V(t)$ must shrink exponentially fast to zero with time. Thus, the dynamical system described by (1) is a dissipative system. As (1) is a dissipative system, all orbits of the system (1) are eventually confined to a specific of R^5 that has zero volume. Hence, the asymptotic motion of the system (1) settles onto an attractor of system (1).

3.3. **Symmetry and invariability.** When the coordinate (x, y, z, w, v) is transformed into $(-x, -y, z, -w, -v)$, the novel system is invariant and has the symmetry about the z-axis. In order to demonstrate the conclusion, let:

$$x = -x,\ y = -y,\ z = z,\ w = -w \text{ and } v = -v \tag{13}$$

And then we have:

$$-\frac{dx}{dt} = \frac{dx}{dt},\quad -\frac{dy}{dt} = \frac{dy}{dt},\quad \frac{dz}{dt} = \frac{dz}{dt},\quad -\frac{dw}{dt} = \frac{dw}{dt}, \text{ and } -\frac{dv}{dt} = \frac{dv}{dt} \tag{14}$$

$$\begin{aligned} -\frac{dx}{dt} &= ax - bv - cyzw \\ -\frac{dy}{dt} &= -dy + exzw + f\sinh(w) \\ \frac{dz}{dt} &= -gz + xy + hv \\ -\frac{dw}{dt} &= -iy + w + \tanh(v)x \\ -\frac{dv}{dt} &= jxz - kv - l\cosh(w) \end{aligned} \tag{15}$$

According to Equation (14) and Equation (15), the result is obtained as follows:

$$\begin{aligned} \frac{dx}{dt} &= -ax + bv + cyzw \\ \frac{dy}{dt} &= dy - exzw - f\sinh(w) \\ \frac{dz}{dt} &= -gz + xy + hv \\ \frac{dw}{dt} &= iy - w - \tanh(v)x \end{aligned} \tag{16}$$

$$\frac{dv}{dt} = -jxz + kv + l\cosh(w)$$

It is easy to see that system (1) is invariant under the coordinates transformation $(x, y, z, w, v) \to (-x, -y, z, -w, -v)$ which persists for all values of the system parameters. Thus, system (1) has rotation symmetry about the z-axis. It is also easy to see that the z-axis is invariant under the flow of system (1). It means this five-dimensional system could also have symmetric pairs of coexisting attractors, such as limit cycles or strange attractors.

3.4. **Lyapunov exponents and Lyapunov dimensions.** According to the nonlinear dynamical theory, a quantitative mensuration method of the sensitive dependence on the initial conditions is calculating the Lyapunov exponent. It is the average rate of divergence (or convergence) of two neighboring trajectories. Furthermore, the five Lyapunov exponents of the nonlinear dynamical system (1) with parameters $a = 8$, $b = 4.6$, $c = 1.3$, $d = 1.2$, $e = 0.3$, $f = 2$, $g = 2.5$, $h = 0.1$, $i = 5$, $j = 3$, $k = 1.8$, and $l = 7$, are obtained as follows: $L_1 = 0.721821$, $L_2 = 0.133932$, $L_3 = -1.28232$, $L_4 = -2.36733$ and $L_5 = -5.70623$.

It can be seen that the biggest Lyapunov exponent is positive, showing that the system has chaotic properties. L_1 and L_2 are positive Lyapunov exponents, and the remaining three Lyapunov exponents are negative. Hence, the system is hyper-chaotic. The fractal dimension is also a typical characteristic of chaos calculated Kaplan-Yorke dimension by Lyapunov exponents, and D_{KY} can be expressed as [17]:

$$D_{KY} = j + \frac{1}{|L_{j+1}|}\sum_{i=1}^{j} L_i \tag{17}$$

where j says the first j Lyapunov exponent is nonnegative, namely, j is the maximum value of i value which meets both $\sum_{i=1}^{j} L_i > 0$ and $\sum_{i=1}^{j+1} L_i < 0$ at the same time. L_i is in descending order of the sequence according to the sequence of Lyapunov exponents. D_{KY} is the upper bound of the dimension of the system information. For the system in this work, by observing the values of five Lyapunov exponents in the above, we determine that the value of j is two, and then the Kaplan-Yorke dimension can be expressed from the above due to $L_1 + L_2 > 0$ and $L_1 + L_2 + L_3 + L_4 + L_5 < 0$, the Lyapunov dimension of the novel chaotic system is:

$$D_{KY} = j + \frac{1}{|L_{j+1}|}\sum_{i=1}^{j} L_i \longrightarrow$$

$$D_{KY} = 2 + \frac{1}{|L_{j+1}|}\sum_{i=1}^{2} L_i = 2 + \frac{L_1 + L_2}{L_3} = 2 + \frac{0.72182 + 0.13393}{1.28232} = 2.66735$$

It means that the Lyapunov dimension of system (1) is fractional. Because of the fractal nature, the new system has non-periodic orbits; what is more, its nearby trajectories diverge. Therefore, there is really chaos in this nonlinear system.

3.5. **Waveform analysis of the novel chaotic system.** The waveform of a chaotic system should be aperiodic to demonstrate that the proposed system is a chaotic system. The waveforms of ($x(t)$, $y(t)$, $z(t)$, $w(t)$, $v(t)$) in the time domain are shown in Figure 3. The waveforms of ($x(t)$, $y(t)$, $z(t)$, $w(t)$, $v(t)$) are aperiodic. It can be observed that the time domain waveform has non-cyclical properties.

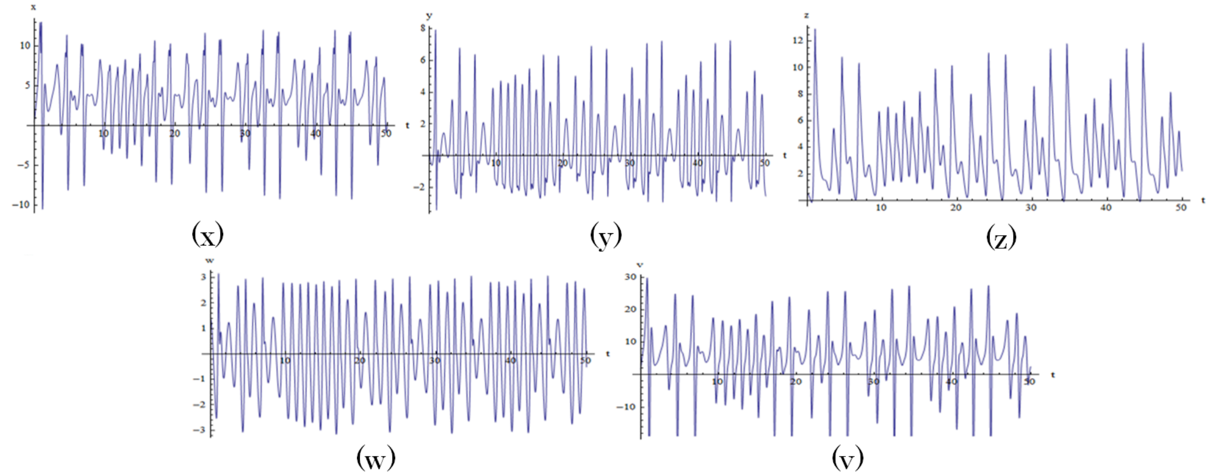

Figure 3. Time versus (x, y, z, w, v) of a new chaotic scheme

3.6. **Sensitivity to initial conditions.** Long-term unpredictability is the most characteristic feature of a chaotic system. This comes about because of sensitive dependence of solutions on initial conditions. Two different initial conditions, no matter how close, will eventually become widely separated. Thus, for any finite number of digits of accuracy in an initial condition, there will be a future time at which no accurate predictions can be made about the state of the system.

Figure 4 shows that the evolution of the chaos trajectories is very sensitive to initial conditions, where the initial values of the system are set to: $x(0) = 0.5$, $y(0) = 0.4$, $z(0) = 0.6$, $w(0) = 1.4$ and $v(0) = 1.8$, which represent in solid (blue) line and the initial values $x(0) = 0.5$, $y(0) = 0.4$, $z(0) = 0.00000006$, $w(0) = 1.4$ and $v(0) = 1.8$ represent in dashed line.

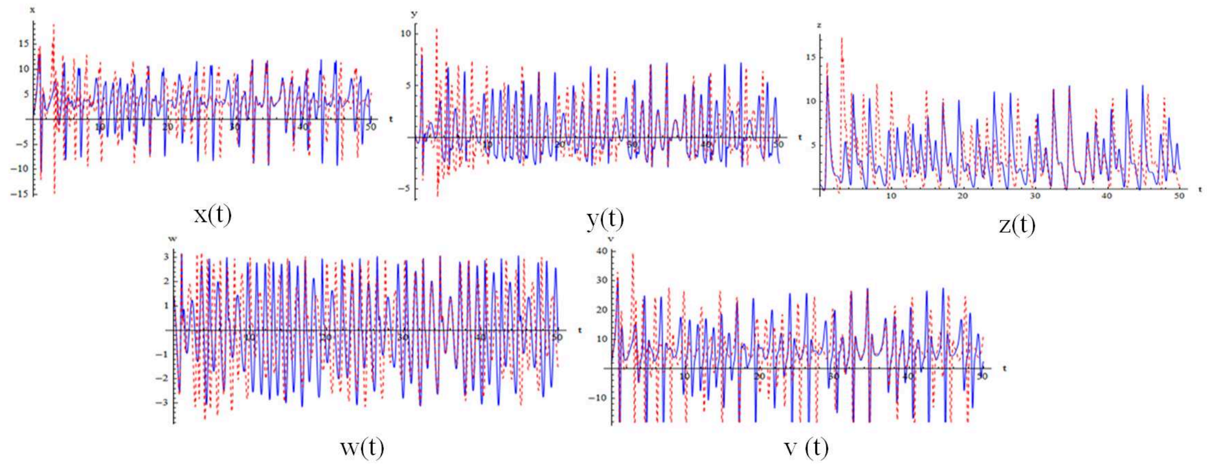

Figure 4. Sensitivity tests of the novel system

4. **Conclusion.** In this paper, we have introduced a new 5-D hyper-chaotic system, which has been successfully validated by the system analysis by means of equilibrium points, dissipativity, symmetry, Lyapunov exponents, waveform and sensitivity to initial conditions. The system generates the hyper chaos behavior when parameter values are: $a = 8$, $b = 4.6$, $c = 1.3$, $d = 1.2$, $e = 0.3$, $f = 2$, $g = 2.5$, $h = 0.1$, $i = 5$, $j = 3$, $k = 1.8$, and $l = 7$ and the initial conditions as: $x(0) = 0.5$, $y(0) = 0.4$, $z(0) = 0.6$, $w(0) = 1.4$ and $v(0) = 1.8$, the Lyapunov exponents for the system $L_1 = 0.721821$, $L_2 = 0.133932$, $L_3 = -1.28232$, $L_4 = 2.36733$ and $L_5 = -5.70623$. It means the system is hyper chaotic

because it has two positive Lyapunov exponents, the fractal dimension is 2.66735, the new system has two unstable equilibrium points, and the new system characterizes with high sensitivity to initial condition and generates complex chaotic attractor. The novel hyper-chaotic system is suitable to be used in numerous applications and could be employed in information encryption, because of the big group of keys that can be generated from the system. In the future, the novel hyper-chaotic system can be adopted in the image processing, artificial intelligence and other computer science fields, also can merge the proposed system with other techniques like watermark or steganography techniques.

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