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ROBUST ADAPTIVE OUTPUT FEEDBACK CONTROLLER FOR A QUADRATIC BOOST CONVERTER

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Abstract. *Quadratic boost converters are nonlinear, high-order and nonminimum phase systems that present major challenges in the design of controllers to regulate their output voltages. In this express letter, it is shown that the simple adaptive control (SAC) that relies only on output voltage measurements can successfully regulate the output voltage of the quadratic boost converter. In order to apply successfully the SAC scheme and guarantee stability, and also to ensure that the output tracking error vanishes asymptotically, it requires that the controlled plant be almost strictly positive real (ASPR). A controlled plant that has a minimum phase transfer function of relative degree one is ASPR. Since the quadratic boost converter is not ASPR, a parallel feedforward compensator (PFC) is designed and is complemented with a proportional derivative (PD) control to ensure that the augmented system is ASPR and therefore, the SAC scheme can be successfully applied to the output regulation of the quadratic boost converter. Experimental results show that the SAC scheme possesses excellent features in terms of output tracking abilities, robustness to large input voltage and load step variations.*
Keywords: Quadratic boost converter, Simple adaptive control (SAC), DC-DC boost converter

1. **Introduction.** In recent years, major advances have been made in the development of energy sources such as solar arrays and fuel cells. Fuel cell systems have emerged as one of a very promising and environmentally friendly source of energy. However, their output voltage can be low and varying with load conditions that they require an interface with DC-DC boost converters with high static gain. High conversion rations are very hard to achieve with conventional DC-DC boost converters due to certain constraints imposed on them. An interesting boost topology that provides high duty ratios is the quadratic boost converter with a single active switch.

The design of controllers to regulate the output voltage of quadratic boost converters is a difficult task since these converters are nonlinear, high-order and nonminimum phase systems. Currently the basic control strategy for the quadratic boost converter consists of a cascade of an inner current loop and an outer voltage loop that uses a proportional-integral compensator to regulate the output voltage [1-4]. In [5], the authors considered a modified voltage-mode controller to regulate the output of the single-switch quadratic boost converter with the assumption that only the load resistances are unknown.

Adaptive controllers are widely used and well suited when DC-DC converters are subject to parameter uncertainties, large unknown variations associated with resistive loads and external input voltages. To the best of this author's knowledge, the application of adaptive control to the transformerless single switch quadratic boost converter considered in this work has appeared only in [6]. The authors in [6], used the property of passivity of the nonlinear incremental model to develop a stabilizing adaptive PI controller with

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three different estimators to estimate the unknown load. The controller developed is robust only to load variations and its implementation requires the exact knowledge of the inductances, capacitances and the input voltage. Moreover, its implementation requires four measurements, namely the currents in the two inductors and the voltages across the two capacitors. We should note here, that the SAC scheme developed in [8,9] has been successfully used in the work of [7] to regulate the ouput voltage of the conventional DC-DC boost converter. In that work, only simulations were performed and no experimental results were presented to validate the SAC scheme.

The main objective of this express letter is to show that the simple adaptive control scheme can be successfully implemented to regulate the output voltage of the quadratic boost converter. The implementation of the SAC requires only the measurement of the output voltage and its robustness is validated experimentally.

The structure of this express letter is as follows. In Section 2 the quadratic boost converter model is presented. In Section 3 a brief summary of the simple adaptive control algorithm is introduced. Section 4 presents the experimental results and finally Section 5 presents the conclusion.

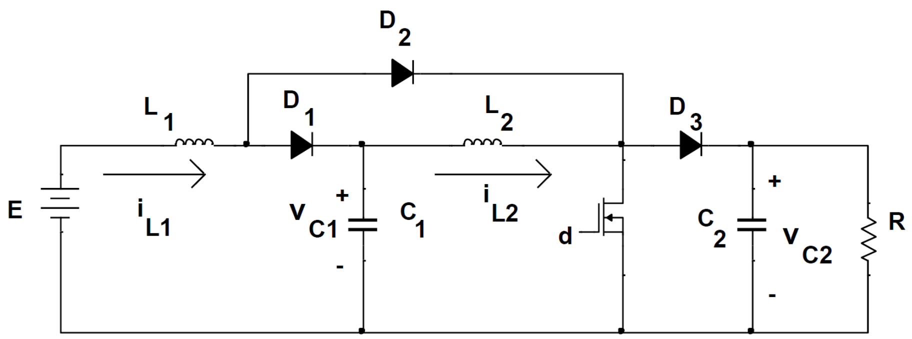

FIGURE 1. Quadratic boost converter

2. **Quadratic Boost Converter and Preliminaries.** A basic quadratic boost converter with a single active switch is shown in Figure 1. Under continuous conduction mode (CCM), the averaged model of the converter is

$$
\begin{aligned}
\frac{di_{L_1}}{dt} &= -(1-d)\frac{v_{C_1}}{L_1} + \frac{E}{L_1} \\
\frac{di_{L_2}}{dt} &= -(1-d)\frac{v_{C_2}}{L_2} + \frac{v_{C_1}}{L_2} \\
\frac{dv_{C_1}}{dt} &= (1-d)\frac{i_{L_1}}{C_1} - \frac{i_{L_2}}{C_1} \\
\frac{dv_{C_2}}{dt} &= (1-d)\frac{i_{L_2}}{C_2} - \frac{v_{C_2}}{RC_2}
\end{aligned}
\tag{1}
$$

where i_{L_1} and i_{L_2} represent the currents through the inductors L_1 and L_2 respectively, and v_{C_1} and v_{C_2} voltages across the capacitors C_1 and C_2 respectively. The external input voltage is represented by E and the load by the resistor R. The control input d to the converter is the duty ratio function. Solving for the equilibrium point of (1) results in the following steady-state operating conditions given by

$$
\begin{aligned}
&I_{L_1} = \frac{E}{R(1-D)^4}; \quad I_{L_2} = \frac{E}{R(1-D)^3} \\
&V_{C_1} = \frac{E}{(1-D)}; \quad V_{C_2} = \frac{E}{(1-D)^2}; \quad D = 1 - \sqrt{\frac{E}{V_{C_2}}}
\end{aligned}
\tag{2}
$$

where I_{L_1}, I_{L_2}, V_{C_1}, V_{C_2} and D are the equilibrium values of the average state variables i_{L_1}, i_{L_2}, v_{C_1}, v_{C_2} and d, respectively. The ideal static gain of the converter is

$$M(D) = \frac{V_{C_2}}{E} = \frac{1}{(1-D)^2} \tag{3}$$

The system is linearized with respect to the equilibrium point to yield the following plant equation

$$\begin{aligned} \dot{x}_p &= A_p x_p + B_p u_p \\ y_p &= C_p x_p \end{aligned} \tag{4}$$

where

$$A_p = \begin{bmatrix} 0 & 0 & -\frac{(1-D)}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{(1-D)}{L_2} \\ \frac{(1-D)}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{(1-D)}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \tag{5}$$

$$B_p = \begin{bmatrix} \frac{V_{C_1}}{L_1} \\ \frac{V_{C_2}}{L_2} \\ -\frac{I_{L_1}}{C_1} \\ -\frac{I_{L_2}}{C_2} \end{bmatrix}, \quad C_p = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

with $x_p = \begin{bmatrix} x_{p1} & x_{p2} & x_{p3} & x_{p4} \end{bmatrix}^T$ and $x_{p1} = i_{L_1} - I_{L_1}$, $x_{p2} = i_{L_2} - I_{L_2}$, $x_{p3} = v_{C_1} - V_{C_1}$, $x_{p4} = v_{C_2} - V_{C_2}$, $u_p = d - D$ and y_p is the measurable output tracking error.

3. **Simple Adaptive Controller.** In the SAC algorithm presented in [8,9], the plant is described by the linear time-invariant system

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p u_p(t) \\ y_p(t) &= C_p x_p(t) \end{aligned} \tag{6}$$

The objective here is the design of an adaptive controller $u_p(t)$, without the explicit knowledge of A_p and B_p, such that the plant output $y_p(t)$ is required to track asymptotically the output $y_m(t)$ of the reference model

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m u_m(t) \\ y_m(t) &= C_m x_m(t) \end{aligned} \tag{7}$$

Defining the measurable output tracking error $e_y(t)$ and the vector $r(t)$ by

$$e_y(t) = y_m(t) - y_p(t); \; r(t) = \begin{bmatrix} e_y \\ x_m \\ u_m \end{bmatrix} \tag{8}$$

The simple adaptive controller $u_p(t)$ is defined as

$$u_p(t) = K(t) r(t) \tag{9}$$

where the total adaptive controller $K(t)$ is given by

$$K(t) = K_I(t) + K_p(t) \tag{10}$$

with

$$\dot{K}_I(t) = \left[\begin{array}{ccc} \dot{K}_{Ie}(t) & \dot{K}_{Ix}(t) & \dot{K}_{Iu}(t) \end{array} \right] = e_y r^T \Gamma_I \tag{11}$$

and the proportional adaptive gain K_p given by

$$K_p(t) = e_y r^T \Gamma_p \tag{12}$$

where Γ_I and Γ_p are coefficient matrices that control the rate of adaptation.

In order for the simple adaptive control to guarantee stability and ensure that the output tracking error vanishes asymptotically with bounded adaptive gains, it is required that the controlled plant be almost strictly positive real. A controlled plant that has a minimum phase transfer function of relative degree one is ASPR. In the case that the original plant is not ASPR, then a parallel feedforward compensator (PFC) can be used such that the augmented system is ASPR. If the original plant is stabilizable by a controller $G_{c1}(s)$, then adding the inverse $G_{c1}^{-1}(s)$ as a PFC in parallel with the original plant $G_p(s) = \frac{y_p(s)}{u_p(s)}$ will make the augmented system $G_a(s) = G_p(s) + G_{c1}^{-1}(s)$ minimum phase.

In order to ensure that the augmented plant is minimum phase with a relative degree of one and facilitate the design of the PFC, the controller $G_{c1}(s)$ is complemented with the classical PD controller

$$G_{c2}(s) = K\left(\frac{s}{s_o} + 1\right) \tag{13}$$

that maintains plant stability such that the closed loop transfer function

$$T(s) = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)} \tag{14}$$

with $G_c(s) = G_{c1}(s)G_{c2}(s)$ is asymptotically stable. In this case, the inverse $G_{c2}^{-1}(s)$ given by

$$G_{c2}^{-1}(s) = \frac{K^{-1}}{\frac{s}{s_o} + 1} \tag{15}$$

can be used as a PFC and the augmented system $G_a(s) = G_{c1}(s)G_p(s) + G_{c2}^{-1}(s)$ is minimum phase with relative degree of one. Consequently, the simple adaptive control can be applied directly to the quadratic boost converter. Please see Figure 2 for the summary of the SAC algorithm as presented in [9].

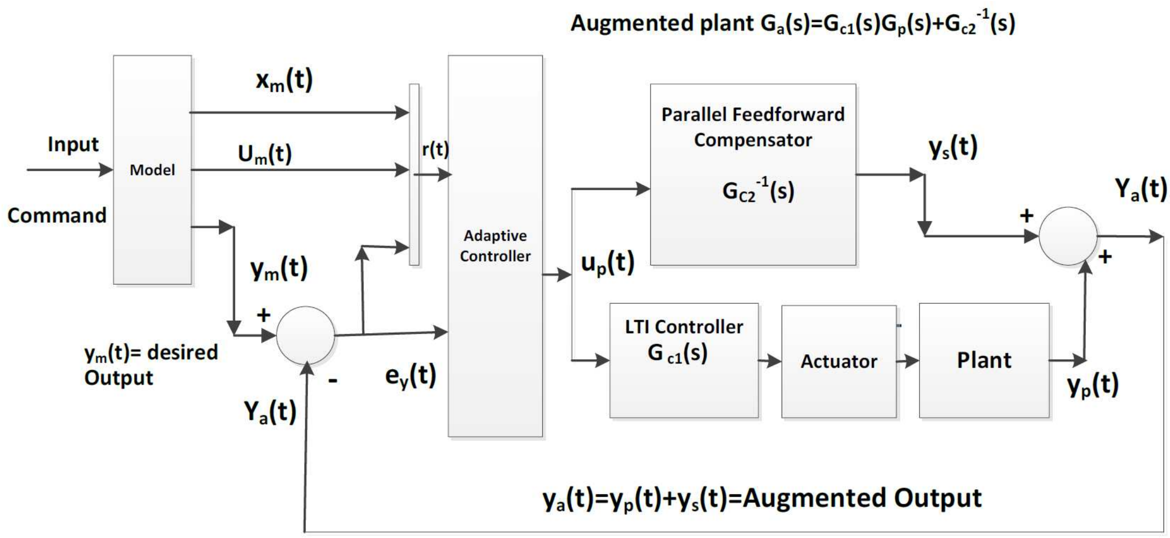

FIGURE 2. Simple adaptive control

Please note that the successful application of the simple adaptive controller will ensure that the augmented output $y_a(t)$ and not the plant output $y_p(t)$ tracks the model output $y_m(t)$ asymptotically. To minimize the steady-state error that may result, the gain K in (13) should be chosen as the highest finite gain that still maintains stability in order to ensure that $G_{c2}^{-1}(s)$ given in (15) has a small gain. As seen in Figure 2, this choice ensures that $y_s(t)$ remains small compared to $y_p(t)$ and therefore maintains $y_a(t) \approx y_p(t)$.

4. **Experimental Results.** The nominal parameters of the quadratic boost converter are: $E = 6$ V, $L_1 = 180$ µH, $L_2 = 180$ µH, $C_1 = 20$ µF, $C_2 = 20$ µF, $R = 1000$ Ω and $V_d = 20$ V. Using (4) and (5), the transfer function of output voltage-to-duty ration is derived and given by

$$G_p(s) = \frac{y_p(s)}{u_p(s)} = \frac{-1825.7419\,(s - 1.66\times 10^6)\,(s^2 - 283s + 1.667\times 10^8)}{(s^2 + 41.86s + 1.622\times 10^7)\,(s^2 + 8.145s + 4.282\times 10^8)} \tag{16}$$

This transfer function exhibits a fourth-order system and is nonminimum phase with three zeros in the right-half plane. The LTI controller $G_{c1}(s)$ is designed using the root locus techniques to stabilize the plant $G_p(s)$ and is given by

$$G_{c1}(s) = 0.233\,\frac{\left(\frac{s}{100}+1\right)^2}{(s+1)^2} \tag{17}$$

In order to facilitate the design of the PFC, the controller $G_{c1}(s)$ is complemented with the classical PD controller $G_{c2}(s)$ given by (13) that maintains plant stability and such that the closed loop transfer function (14) is asymptotically stable. The design of the PD controller using root locus techniques yields

$$G_{c2}(s) = 250\left(\frac{s}{250}+1\right) \tag{18}$$

The PFC is implemented as $G_{c2}^{-1}(s)$ and is given by

$$G_{c2}^{-1}(s) = \frac{0.004}{\frac{s}{250}+1} \tag{19}$$

In this case, the augmented system $G_a(s) = G_{c1}G_p(s) + G_{c2}^{-1}$ is minimum phase with all the zeros in the left-hand plane located at

$$-69862,\ -1575 \pm 12972j,\ -1011,\ -30 \pm 41j \tag{20}$$

Moreover, $G_a(s)$ is of relative degree of one and therefore is ASPR. As a result, the simple adaptive control can be used successfully.

To reduce the number of tuning parameters, the adaptation weighting matrices are chosen as $\Gamma_I = \alpha I_3$ and $\Gamma_p = \beta I_3$ where I_3 denotes a 3×3 identity matrix. Matlab/Simulink-based simulations were performed to determine the parameters α, β and the reference model given by (7) in order to achieve a satisfactory response. The parameter values selected are $\alpha = 10$, $\beta = 10$ with the following reference model

$$\begin{aligned} \dot{x}_m(t) &= -100x_m(t) + 100u_m(t) \\ y_m(t) &= x_m(t) \end{aligned} \tag{21}$$

with $u_m = V_d = 20$ V being the input command. The initial conditions for the adaptation gains are $K_{Ieo} = 10.0$, $K_{Ixo} = 10.0$ and $K_{Iuo} = 10.0$.

A prototype of the quadratic boost converter was constructed. The simple adaptive control given by (9) is implemented using the dSPACE 1104 real-time controller board with the switching frequency of the PWM modulator set at 200 KHz. The experimental results are depicted in Figures 3, 4 and 5. Figure 3 shows the response due to a step change of the reference voltage V_d from 20 V to 30 V. Figure 4 depicts the robustness of the adaptive controller to step load variations. In this case, the load resistor R varies

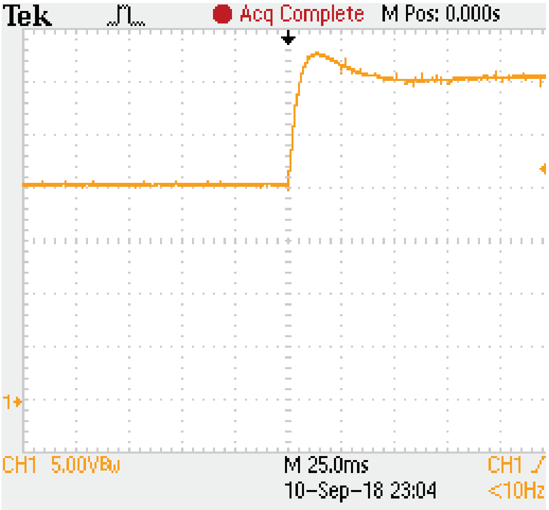

FIGURE 3. Output response (5 V/div, 25 ms/div) with step reference voltage change from 20 V to 30 V

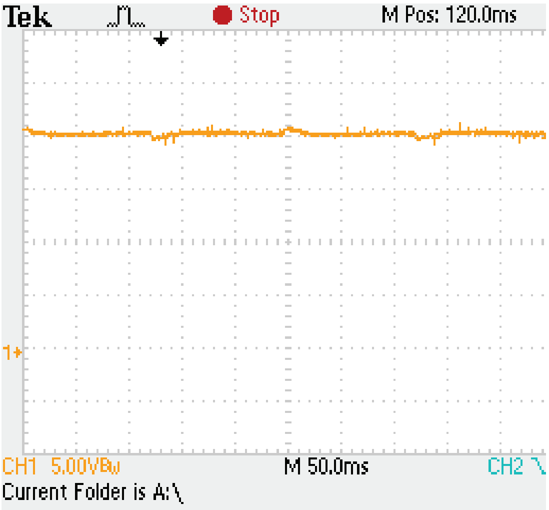

FIGURE 4. Output response (5 V/div, 50 ms/div) with the load resistor R varying periodically stepwise between 1000 Ω and 500 Ω

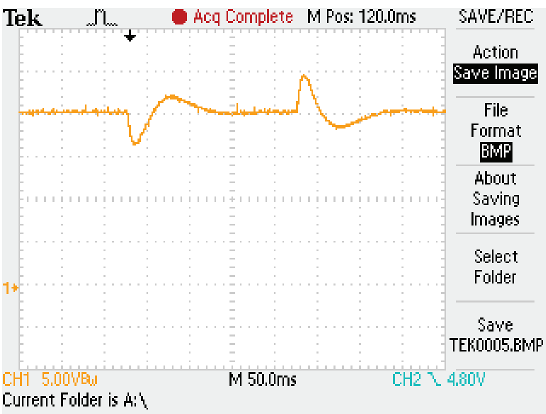

FIGURE 5. Output response (5 V/div, 50 ms/div) with the input voltage E undergoing a step change from 6 V to 4 V then back to 6 V

stepwise periodically between 1000 Ω and 500 Ω. Shown in Figure 5 is the robustness of the controller to the external input voltage change where E undergoes a step variation from 6 V to 4 V and then back to 6 V. These experimental results show the excellent features in terms of robustness and recovery of the SAC to input voltage and load step variations.

5. **Conclusion.** The SAC has been shown to be suited to the output regulation of quadratic boost converters. Its implementation requires only the output voltage measurements in contrast to most of the studies that also require the measurement of the first inductor current. Experimental results show that the SAC is robust to sudden input voltage source and load variations. In future work, the SAC scheme will be compared experimentally with other robust controllers developed in the context of output regulation of quadratic boost converters.

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