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LINEARIZATION FOR UNCERTAIN SYSTEMS BY TAKAGI-SUGENO FUZZY INTELLIGENT MODEL CONTROL WITH LYAPUNOV ENERGY FUNCTIONS

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Abstract. *In this paper, the modified TS (Takagi-Sugeno) fuzzy model intelligent control for nonlinear uncertain system represented by a continuous TS model with bounded disturbance via Lyapunov energy function is studied. By properly designing the constraints in optimization problem, a high frequency signal which is derived via Lyapunov energy function is simultaneously introduced to stabilize the system. A numerical example is given to illustrate the effectiveness of the proposed algorithm.*
Keywords: Linearization, Uncertain system, Fuzzy model, Intelligent control

1. **Introduction.** There has been rapidly growing interest in Fuzzy Logic Control (FLC) of nonlinear system in recent years. Various fuzzy models have been proposed in recent years; see, for example ([1-7] and the reference therein). From [1-3], the TS (Takagi-Sugeno) fuzzy model was firstly used to control systems by systematic control rules. Then [4] provided the improvement of the control rules in stability analysis. Recently, [5-7] extended the control rules in the practical application. Based on the previous research results, the Takagi-Sugeno (TS) fuzzy model is used to model uncertain nonlinear systems. Furthermore, we injected a high frequency signal to improve its performance. A rigorous analysis of stability in a general nonlinear system with a dither control was given in [8].

Hence, this paper proposes an auxiliary of the controller injected into the uncertain nonlinear system. In summary, the first section proposed a novel modified intelligent fuzzy modeling framework based on the controller design of high frequency injection to overcome the chaotic and uncertainty systems. Then we utilized the machine learning algorithms and the third section is the modified fuzzy system for modeling an unknown

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nonlinear distributed parameter system. Section four aims at the example simulation and conclusions are provided in Section five.

2. **Problem Statement and Preliminaries.** Consider an uncertain nonlinear system represented by the following equation:

$$\dot{x} = f(x,u) + \Delta f(x),$$

where x is the state, u is the input and $\Delta f(\cdot)$ is the uncertainty. $f(\cdot)$ is a vector-valued function which satisfies those assumptions of general continuity and boundedness given in [8] as follows.

There exists an open set V in Euclidean n-space E_n, a closed set $A \subset V$ and, for all u in U and $0 < t < T$, the following conditions hold: (I) $f(x,t,U)$ is continuous in t uniformly in x and u for all x in V; (II) there exists a positive constant M such that norm $f(x,t,u) < M$ for all x in V; (III) $f(x,t,U)$ is continuous in (x,t) uniformly in u for all x in A; and (IV) $f(x,t,U)$ is continuous in u for all x in V.

Furthermore, the ith rule of TS fuzzy model is of the following form:

$$\begin{aligned}&\text{IF } z_1(t) \text{ is } M_{i1} \text{ and } \cdots \text{ and } z_k(t) \text{ is } M_{ik}\\ &\text{THEN } \dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t).\end{aligned} \tag{1}$$

Then the overall fuzzy model with uncertainty can be rewritten as follows:

$$\dot{x}(t) = \sum_{i=1}^{g}\sum_{\psi=1}^{s_\psi}\sum_{\upsilon=1}^{s_\upsilon} h_i(z(t))\lambda_{i\psi}(x(t))\eta_{i\upsilon}(x(t))\left\{(A_i + A_{i\psi})\,x(t) + (B_i + B_{i\upsilon})\,u(t)\right\}. \tag{2}$$

Notice that $h_i(z(t))$, $A_{i\psi}$ and $B_{i\upsilon}$ are known. However, $\lambda_{i\psi}(x(t))$ and $\eta_{i\upsilon}(x(t))$ are unknown.

The concept of the Parallel-Distributed-Compensation (PDC) scheme is utilized to design a fuzzy controller:

$$\text{Rule } i: \text{ IF } z_1(t) \text{ is } M_{i1} \text{ and } \cdots \text{ and } z_k(t) \text{ is } M_{ik} \text{ THEN } u(t) = -F_i x(t). \tag{3}$$

The closed-loop form

$$\dot{x}(t) = \sum_{i=1}^{g}\sum_{j=1}^{g}\sum_{\psi=1}^{s_\psi}\sum_{\upsilon=1}^{s_\upsilon} e_{ij\psi\upsilon}(z(t))\left\{A_i - B_iF_j + A_{i\psi} - B_{i\upsilon}F_j\right\}x(t), \tag{4}$$

where

$$e_{ij\psi\upsilon}(z(t)) = h_i(z(t))h_j(z(t))\lambda_{i\psi}(x(t))\eta_{i\upsilon}(x(t)). \tag{5}$$

Lemma 2.1. [9] *An uncertain system* $\dot{x}(t) = \{A_0 + D\Delta(t)E\}\,x(t)$ (*where* D *and* E *are known real matrices which characterize the structure of the uncertainty*

$$\Delta(t) = diag[\vartheta_1(t) \quad \vartheta_2(t) \quad \cdots \quad \vartheta_q(t)] \tag{6}$$

and

$$\sigma_i \le \vartheta_i(t) \le \delta_i, \quad i = 1, 2, \ldots, q) \tag{7}$$

is quadratically stable if and only if the following conditions hold.

$$\textit{i) } A_0 + DOE \textit{ is a stable matrix,} \quad \textit{ii) } \left\|E(sI - A_0 - DOE)^{-1}DQ\right\|_\infty < 1. \tag{8}$$

Let G be a matrix which has desired eigenvalues and it is independent of i, j and then (2) can be rewritten as follows.

$$\begin{aligned}\dot{x}(t) &= Gx(t) + \sum_{i=1}^{g}\sum_{j=1}^{g}\sum_{\psi=1}^{s_\psi}\sum_{\upsilon=1}^{s_\upsilon} e_{ij\psi\upsilon}(z(t))\Delta G_{ij\psi\upsilon}x(t)\\ &= Gx(t) + \sum_{i=1}^{g}\sum_{\psi=1}^{s_\psi}\sum_{\upsilon=1}^{s_\upsilon} e_{ii\psi\upsilon}(z(t))\Delta G_{ii\psi\upsilon}x(t)\end{aligned}$$

$$+\sum_{i<j}^{g}\sum_{\psi=1}^{s_\psi}\sum_{\upsilon=1}^{s_\upsilon} e_{ij\psi\upsilon}(z(t))\Delta T_{ij\psi\upsilon}x(t). \tag{9}$$

Using the singular value decomposition technique, we obtain

$$\Delta G_{ii\psi\upsilon} = U_{ii\psi\upsilon}S_{ii\psi\upsilon}V_{ii\psi\upsilon}^T \qquad \Delta T_{ij\psi\upsilon} = U_{ij\psi\upsilon}S_{ij\psi\upsilon}V_{ij\psi\upsilon}^T. \tag{10}$$

Lemma 2.2. [9] *The closed-loop uncertain fuzzy system $F(C;0)$ is quadratically stable if and only if the following conditions hold*

i) $G + \overline{D}\,\overline{O}\,\overline{E}$ *is a stable matrix,* *ii)* $\left\|\overline{E}\left(sI - G - \overline{D}\,\overline{O}\,\overline{E}\right)^{-1}\overline{D}\,\overline{Q}\right\|_\infty \equiv \rho < 1.$ (11)

3. TS Fuzzy Relaxed Model and Stability Analysis.

An uncertain nonlinear system with an added dither is normally called an uncertain dithered system, and it is described by

$$\dot{x} = f(x,u,d) + \Delta f(x). \tag{12}$$

Notice that the dither signal is injected just in front of the nonlinearity of the uncertain nonlinear system. The algorithm for constructing a dither signal is given as follows [8]. The time interval $[0,T]$ is divided into an arbitrary number φ of equal subintervals. The beginning of the first interval, the end of the first interval, the end of the second interval and the end of the φth interval are denoted by t_0, t_1, t_2 and t_φ respectively. Hence, the repetition frequency, shape and amplitude of dither can be determined by regulating the parameters φ, $\alpha_m(t_p)$ and $\beta_m(t_p)$.

The TS fuzzy model with uncertainty of the uncertain relaxed model is reconstructed described by

$$\text{IF } z_{R1}(t) \text{ is } M_{Ri1}(\alpha_m,\beta_m) \text{ and } \cdots \text{ and } z_{Rk}(t) \text{ is } M_{Rik}(\alpha_m,\beta_m) \tag{13}$$

$$\text{THEN } \dot{x}_R(t) = (A_i(\alpha_m,\beta_m) + \Delta A_i)x_R(t) + (B_i(\alpha_m,\beta_m) + \Delta B_i)u_R(t),$$

$$i = 1,2,\ldots,g. \tag{14}$$

Similarly, the overall fuzzy relaxed model with uncertainty is described as follows:

$$\begin{aligned}\dot{x}_R(t) = &\sum_{i=1}^{g}\sum_{\psi=1}^{s_\psi}\sum_{\upsilon=1}^{s_\upsilon} h_i(z_R(t),\alpha_m,\beta_m)\lambda_{i\psi}(x_R(t))\eta_{i\upsilon}(x_R(t))\\ &\{(A_i(\alpha_m,\beta_m) + A_{i\psi})\,x_R(t) + (B_i(\alpha_m,\beta_m) + B_{i\upsilon})\,u_R(t)\}.\end{aligned} \tag{15}$$

The closed-loop uncertain fuzzy relaxed system is represented as follows:

$$\dot{x}_R(t) = \sum_{i=1}^{g}\sum_{j=1}^{g}\sum_{\psi=1}^{s_\psi}\sum_{\upsilon=1}^{s_\upsilon} \widetilde{e}_{ij\psi\upsilon}(z_R(t),\alpha_m,\beta_m)\left\{A_i - B_iF_j + A_{i\psi} - B_{i\upsilon}F_j\right\}x_R(t). \tag{16}$$

Hereafter, we are concerned with stability of the closed-loop fuzzy relaxed reduced system instead of discussing that of the closed-loop dithered system. A stability criterion is presented in the following theorem.

Theorem 3.1. *The closed-loop uncertain fuzzy relaxed system is quadratically stable if and only if the following conditions hold*

i) $G(\alpha_m,\beta_m) + \overline{D}(\alpha_m,\beta_m)\overline{O}(\alpha_m,\beta_m)\overline{E}(\alpha_m,\beta_m)$ *is a stable matrix,* (17a)

ii) $\left\|\overline{E}(\alpha_m,\beta_m)\left(sI - G(\alpha_m,\beta_m) - \overline{D}(\alpha_m,\beta_m)\overline{O}(\alpha_m,\beta_m)\overline{E}(\alpha_m,\beta_m)\right)^{-1}\overline{D}(\alpha_m,\beta_m)\overline{Q}(\alpha_m,\beta_m)\right\|_\infty$

$\equiv \rho(\alpha_m,\beta_m) < 1.$ (17b)

From the stability conditions above, the parameters α_m and β_m can be chosen to fulfill the requirements in Theorem 3.1. In other words, an appropriate dither may be chosen to guarantee that the closed-loop fuzzy relaxed reduced system is stable.

4. **A Numerical Example.** Consider an uncertain nonlinear system

$$\ddot{\overline{x}} = 0.5\overline{x} + c(t)\overline{x} + 1.2\dot{\overline{x}} + \overline{x}^3 u. \tag{18}$$

where $\overline{x}$ is the state variable; u is the input variable and $c(t)\overline{x}$ is an uncertain term ($c(t) \in [c_3 \quad c_4]$ where c_3 and c_4 are the lower bound and the upper bound of $c(t)$). Assume that $c(t) \in [-0.4 \quad 0.4]$. The nonlinear term of the uncertain nonlinear system is $\overline{x}^3 u$, which satisfies

$$-27u \leq \overline{x}^3 u \leq 27u \text{ for } \overline{x} \in [-3 \quad 3]. \tag{19}$$

The above equation proves that the nonlinear term can be confined by an upper bound and a lower bound. The nonlinear term can be represented by the following equation:

$$\text{IF } \overline{x}(t) \text{ is } M_1 \quad \text{THEN } \dot{x}(t) = (A_1 + \Delta A_1)\, x(t) + (B_1 + \Delta B_1)\, u(t), \tag{20}$$

$$\text{IF } \overline{x}(t) \text{ is } M_2 \quad \text{THEN } \dot{x}(t) = (A_2 + \Delta A_2)\, x(t) + (B_2 + \Delta B_2)\, u(t), \tag{21}$$

where

$$\Delta A_i = \sum_{\psi=1}^{2} \lambda_{i\psi}(\overline{x}(t)) A_{i\psi},\ \Delta B_i = \sum_{\upsilon=1}^{2} \eta_{i\upsilon}(\overline{x}(t)) B_{i\upsilon}, \quad x(t) = \left[\dot{\overline{x}}(t) \quad \overline{x}(t)\right]^T, \tag{22}$$

$$\dot{x}(t) = \sum_{i=1}^{2}\sum_{\psi=1}^{2}\sum_{\upsilon=1}^{2} M_i(\overline{x}(t))\, \lambda_{i\psi}(\overline{x}(t))\, \eta_{i\upsilon}(\overline{x}(t)) \left\{(A_i + A_{i\psi})\, x(t) + (B_i + B_{i\upsilon})\, u(t)\right\}. \tag{23}$$

The feedback gains are

$$F_1 = [-1.0824 \quad -1.0187], \quad F_2 = [0.7428 \quad 0.9316]. \tag{24}$$

We can derive the closed-loop uncertain fuzzy system:

$$\begin{aligned} \dot{x}(t) = & \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{\psi=1}^{2}\sum_{\upsilon=1}^{2} M_i(\overline{x}(t))\, M_j(\overline{x}(t))\, \lambda_{i\psi}(\overline{x}(t))\, \eta_{i\upsilon}(\overline{x}(t)) \big\{A_i - B_i F_j \\ & + A_{i\psi} - B_{i\upsilon} F_j\big\} x(t). \end{aligned} \tag{25}$$

Subsequently, we attempt to improve the stability of the closed-loop uncertain nonlinear system by injecting a periodic symmetrical square-wave dither $d(t)$ with sufficiently high frequency. Notice that the dither signal is injected just in front of the nonlinearity. The uncertain dithered system is described with

$$\ddot{\overline{x}} = 0.5\overline{x} + 1.2\dot{\overline{x}} + c(t)\overline{x} + (d + \overline{x})^3 u. \tag{26}$$

By using M_{R1} and M_{R2}, the uncertain relaxed model can be represented by the following TS fuzzy relaxed model with uncertainty:

$$\text{IF } \overline{x}_R(t) \text{ is } M_{R1} \quad \text{THEN } \dot{x}_R(t) = (A_1 + \Delta A_1)\, x_R(t) + (B_1 + \Delta B_1)\, u_R(t), \tag{27a}$$

$$\text{IF } \overline{x}_R(t) \text{ is } M_{R2} \quad \text{THEN } \dot{x}_R(t) = (A_2 + \Delta A_2)\, x_R(t) + (B_2 + \Delta B_2)\, u_R(t). \tag{27b}$$

We obtain amplitude from Simulation in Figure 1 of the closed-loop fuzzy relaxed system with the dash line of 1. The closed-loop uncertain dithered system is approximated by its corresponding closed-loop uncertain fuzzy relaxed model and the approximation improves as the frequency of dither increases.

5. **Conclusions and Future Research.** This paper combines fuzzy theory and the parallel-distributed-compensation scheme to design a fuzzy controller to stabilize an uncertain nonlinear system. A frequency is injected into the uncertain nonlinear system to improve the stability of nonlinear system. Simulation results display the stability by choosing appropriate parameters. Further study could focus on the tunes of the parameters.

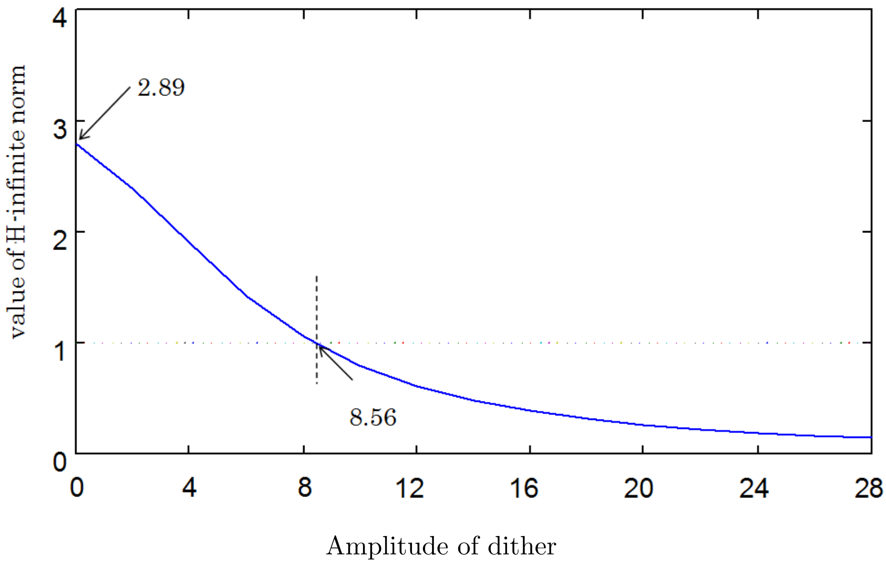

Figure 1. Simulation results of the dither

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