

ON FLOW RATE OF “UONOGAWA” FROM RAINFALL WITH TANK MODEL

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ABSTRACT. *The tank model was proposed in 1972 to calculate the flow rate of a river from rainfall. The Japan Meteorological Agency (JMA) currently uses it as a prediction method of the criteria for the warning information of sediment-related disaster and flood. In the tank model, there are some important parameters to predict the flow rate, such as the outflow coefficient, penetration coefficient, and height of outflow hole. Therefore, many researches have estimated such parameters of various rivers. Based on these researches, JMA has also decided the parameters that can be applied to general rivers. On the other hand, it is realistic that these parameters are different in each river. In order to obtain more accurate flow rate of each river, it is natural to consider the difference of such parameters in each river. Therefore, in this paper, taking the “Uonogawa”, which is the largest tributary from the central “Shinano river”, as a case study, we investigate the optimum parameters of the “Uonogawa” by the simulation of the tank model with the data of past rainfall and flow rate. By using the obtained parameters of our simulation, we found that the actual flow rate could be almost reproduced.*

Keywords: Tank model, Simulation, River basin area, Flow rate model

1. Introduction. It is very famous that there are very rainy and snowy days at Niigata Prefecture in Japan, i.e., 150 days of rain and snow in most years and 190 days in the rainy and snowy years. Even compared to other prefectures in Japan, there are many rainy and snowy days (its ranking of Niigata Prefecture is fifth or higher in Japan). The precipitation is recently also increasing. In addition, even from viewpoint of the nationwide, there are many rivers and the length of the river is long, including the “Shinano river” represented in Niigata Prefecture. From these facts, Niigata Prefecture is faced to be affected by heavy rain disasters. As the heavy rain disasters, the river flooding and landslide disasters are conceivable. These disasters cause the huge damage to the people and their homes.

The warning information of sediment-related disaster and flood, etc. is very important that is involved in the human life. If we can predict the flow rate of a river in advance from the rainfall, it is considered that the damage can be suppressed as much as possible. The tank model was proposed by Sugawara [1] in 1972 to calculate the flow rate of a river from rainfall. The tank model is used as a method of calculating the flow rate of a river according to the amount of rain. Currently, the Japan Meteorological Agency (JMA) uses it as a prediction method of the criteria for the warning information of sediment-related disaster and flood, etc. [2]. In the tank model, there are some important parameters to predict the flow rate, such as the outflow coefficient, penetration coefficient, and height of outflow hole. Therefore, many researches have estimated such parameters of a river [3, 4].

For example, Ishihara and Kobatake [5] pointed out that three-stage tank was suitable for the outflow model of flood prediction.

Based on these researches, JMA has also decided the parameters that can be applied to general rivers. On the other hand, it is realistic that these parameters are different in each river. If we can estimate more accurate flow rate for each river, we can notify more precise warning information of sediment-related disaster and flood, etc. Moreover, to estimate the accurate flow rate for each river is important for the criteria for notifying the warning information. In order to obtain the accurate flow rate of each river, it is natural to consider the difference of parameters in each river. Some researches tried to estimate the parameters of particular river, e.g., Hong et al. [6] simulated the groundwater levels in Kumamoto city, Japan and Suntoro and Sayama [7] developed a flow rate model for “Bogowonto” river in Indonesia. Following these studies, taking the “Uonogawa”, which is the largest tributary from the central “Shinano river”, as a case study, we investigate the optimum parameters of the “Uonogawa” by the simulation of the tank model with the data of past rainfall and flow rate.

The paper is organized as follows. In Section 2, we give an overview of tank model and the data of past rainfall and flow rate, which we use in this paper. In Section 3, the results of simulation of the tank model with the data of past rainfall and flow rate and the discussion are presented. Section 4 is devoted to a summary.

2. Tank Model. In this section, we briefly review the tank model and the data of past rainfall and flow rate.

2.1. Flow rate of simple one tank. Let us consider the simple one tank case shown in Figure 1. If a kind of liquid such as water is in the tank, the flow velocity v [mm/s] flowing from the bottom valve is simply calculated from the energy conservation law:

$$v = \sqrt{2gh}, \quad (1)$$

where h [mm] is the difference in height of the object’s surface and g [mm/s²] is the gravitational acceleration. Then, the outflow rate of the tank q [mm/s] can be expressed as:

$$q = \frac{1}{\mu}v, \quad (2)$$

where μ is the valve resistance. In other word, the outflow rate of the tank q can be simply determined by the resistance μ and the tank water level h .

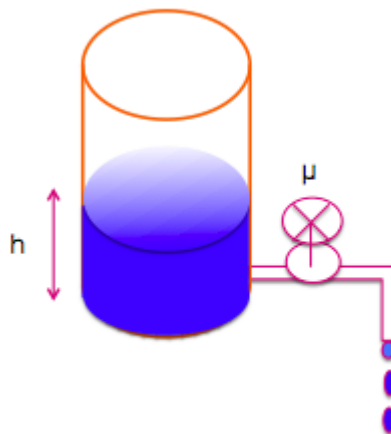


FIGURE 1. Simple one tank case

2.2. **Model of the rain water flows of a river.** If we consider the rain precipitation into the river, the schematic view that rain flows into the river and/or penetrates into the basement is shown in the left side of Figure 2 [7, 8]. The rain is stored in each layer. If the storage size of each layer is over, the outflow is started. The outflow flowing into the river is generally divided into three kinds of outflows in each layer: the surface outflow, surface penetration outflow and groundwater outflow. Moreover, the surface outflow is divided into two kinds of surface outflow: the flow within the layer and the flow seeping from surface.

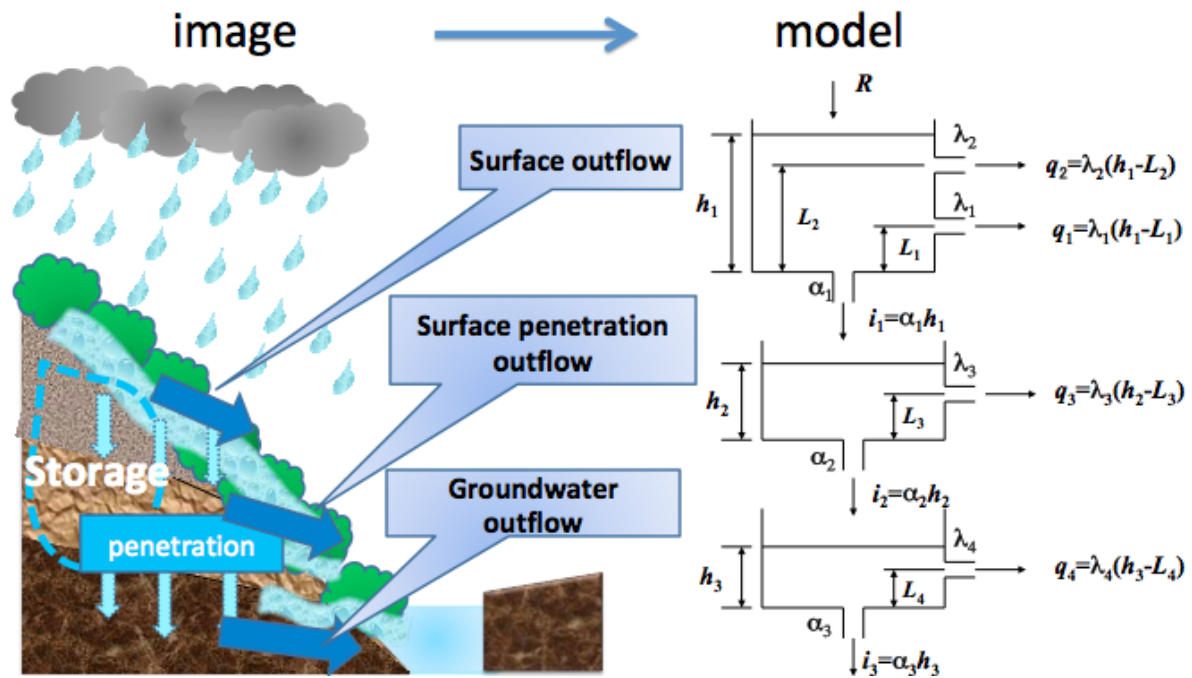


FIGURE 2. Schematic view that rain flows into the river and three layers of tank model

These three kinds of outflows are modeled by three layers of tanks shown in the right side of Figure 2 [9]. The definition of the variables and parameters is shown in Table 1. The storages $h_1(t)$, $h_2(t)$ and $h_3(t)$ of each tank at certain time t show the storages of each layer at certain time t , respectively. The outflow rates $q_1(t)$ and $q_2(t)$ from the side hole of the first tank correspond to two kinds of surface outflow rate: the flow within the layer and flow seeping from surface, respectively. The outflow $i_1(t)$ of the bottom hole of the first tank shows the penetration rate into the second layer. Moreover, the outflow rate $q_3(t)$ from the side hole of the second tank corresponds to the surface penetration

TABLE 1. Definition of variables and parameters

Variable	1st tank	2nd tank	3rd tank
Storage of tank [mm]	$h_1(t)$	$h_2(t)$	$h_3(t)$
Outflow rate from the side hole of tank [mm/s]	$q_1(t)$ and $q_2(t)$	$q_3(t)$	$q_4(t)$
Outflow rate from the bottom hole of tank [mm/s]	$i_1(t)$	$i_2(t)$	$i_3(t)$
Parameter			
Coefficient of outflow from the side hole of tank [1/s]	λ_1 and λ_2	λ_3	λ_4
Coefficient of outflow from the bottom hole of tank [1/s]	α_1	α_2	α_3
Height of the side hole of tank [mm]	L_1 and L_2	L_3	L_4

outflow rate and the outflow rate of the bottom hole of the second tank $i_2(t)$ shows the penetration rate into the third layer. Similarly, the outflow rate $q_4(t)$ from the side hole of the third tank corresponds to the groundwater outflow rate and the outflow rate of the bottom hole of the third tank $i_3(t)$ shows the penetration rate into the deeper layer.

From the analogy of the simple one tank model discussed in Section 2.1, the outflow rates $i_1(t)$, $i_2(t)$ and $i_3(t)$ of the bottom hole of three tanks are proportional to the storage amount of each tank:

$$i_1(t) = \alpha_1 h_1(t), \quad i_2(t) = \alpha_2 h_2(t), \quad i_3(t) = \alpha_3 h_3(t). \quad (3)$$

The outflow rates $q_1(t)$, $q_2(t)$, $q_3(t)$ and $q_4(t)$ from the side hole of each tank are also proportional to the value of subtracting the height of each side hole from the storage amount of each tank:

$$\begin{aligned} q_1(t) &= \lambda_1(h_1(t) - L_1), & q_2(t) &= \lambda_2(h_1(t) - L_2), \\ q_3(t) &= \lambda_3(h_2(t) - L_3), & q_4(t) &= \lambda_4(h_3(t) - L_4). \end{aligned} \quad (4)$$

Moreover, the summation of all outflow rates means the flow rate to the river:

$$q(t) = q_1(t) + q_2(t) + q_3(t) + q_4(t). \quad (5)$$

From the relationship between the inflow rate $R(t)$ [mm/s] which means the amount of rain (rainfall precipitation) and the outflow rates of the side and bottom holes, the change of the storage of the first tank can be expressed by:

$$\text{First tank : } \frac{dh_1(t)}{dt} = R(t) - i_1(t) - q_1(t) - q_2(t). \quad (6)$$

In the second and third tanks, the penetration outflow rate of the upper tank becomes the inflow rate. Thus, the change of the storage of the second and third tanks can be expressed by:

$$\text{Second tank : } \frac{dh_2(t)}{dt} = i_1(t) - i_2(t) - q_3(t), \quad (7)$$

$$\text{Third tank : } \frac{dh_3(t)}{dt} = i_2(t) - i_3(t) - q_4(t), \quad (8)$$

respectively.

Thus, if the parameters shown in Table 1 and the rainfall precipitation $R(t)$ are given, then we can estimate the flow rate to the river $q(t)$ by solving Equations (6)-(8).

2.3. Data of the rainfall and flow rate. We collect the data of the rain precipitation and flow rate every hour from the open data of Shinanogawa River Office, Hokuriku Regional Development Bureau, Ministry of Land, Infrastructure, Transport and Tourism, Japan [10].

To decide all parameters of the tank model shown in Table 1, it is known that we have to consider the heavy rainfall case. Thus, we use the data of Niigata-Fukushima torrential rain in July 2011. This torrential rain is the concentrated rainstorms occurring in the three regions: “Chuetsu” and “Kaetsu” regions in Niigata Prefecture and “Aizu” region in Fukushima Prefecture, from 26th to 30th, July 2011. Because the embankment of the river broke down, the major flood damage was happened in Niigata Prefecture [11]. In this paper, as a case study, we use the data of rainfall precipitation and flow rate of the “Uonogawa” on July 28, 2011, when it rained the most in this area. This data was observed at “Horinouchi Observatories” in Uonuma City in Niigata Prefecture, Japan. “Uonogawa” is the river which is the largest branch of “Shinano river”. The basin area is 1503.6 km², and the net distance is 68.4 km.

Figure 3 shows the observed flow rate per hour [m³/hour] and the observed rainfall precipitation per hour [mm/hour] of the “Uonogawa” on July 28, 2011.

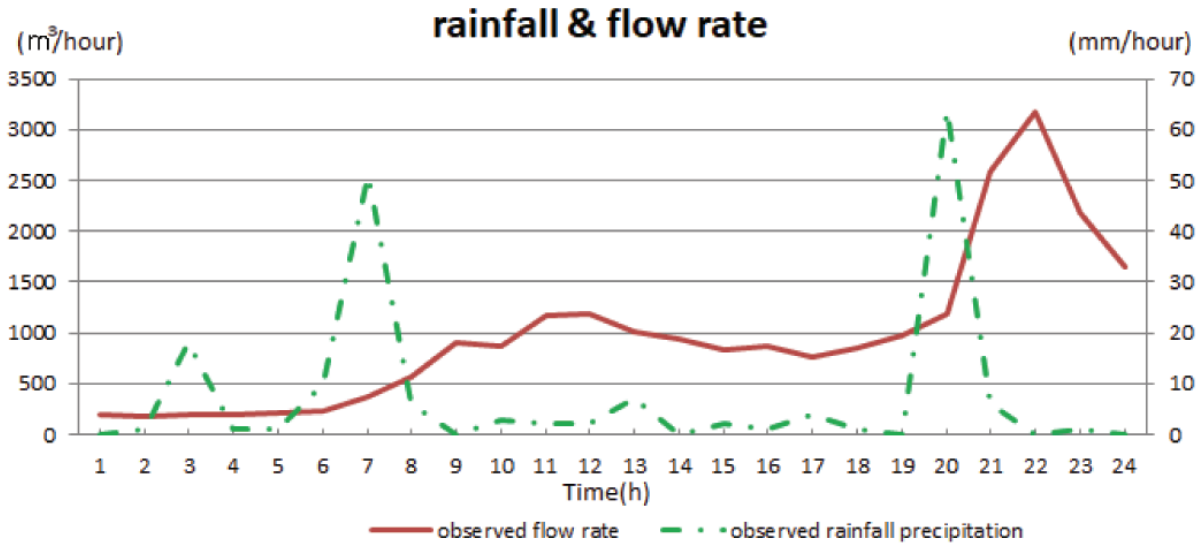


FIGURE 3. The observed flow rate per hour [m³/hour] (solid line) and the observed rainfall precipitation per hour [mm/hour] (dashed dotted line) of the “Uonogawa” on July 28, 2011

If we fix the parameters shown in Table 1, by using the observed rainfall precipitation $R(t)$ and discussion in Section 2.2, we can estimate the flow rate to the river in Equation (5). In other word, we can search for the optimal parameters of the tank model to reproduce the observed flow rate to the river.

3. Results and Discussion. Our simulation of the tank model is conducted by S^4 simulation system [12]. The differential equations (Equations (6)-(8)) are solved by using the explicit fifth order Runge-Kutta method.

3.1. Simulation 1. We perform the simulation by using the parameters which are actually used by JMA’s tank model [2]. These parameters are shown in Table 2.

TABLE 2. The parameters which are actually used by JMA’s tank model [2]

1st tank	2nd tank	3rd tank
$\lambda_1 = 2.78 \times 10^{-5}$ and $\lambda_2 = 4.17 \times 10^{-5}$	$\lambda_3 = 1.39 \times 10^{-5}$	$\lambda_4 = 2.78 \times 10^{-6}$
$\alpha_1 = 3.34 \times 10^{-5}$	$\alpha_2 = 1.39 \times 10^{-5}$	$\alpha_3 = 2.78 \times 10^{-6}$
$L_1 = 15$ and $L_2 = 60$	$L_3 = 15$	$L_4 = 15$

In Figure 4, we plot the simulated flow rate to the river in Equation (5), which is multiplied by the basin area [13]. We also plot the observed flow rate, which is the same shown in Figure 3. We can find that the simulated flow rate is larger than the observed one during most of all period. We can also find that there is a difference in the time when the flow rate increases. We guess that the JMA’s tank model is suitable for the alert purpose because it aims to make some prediction in advance and issue alerts of evacuation etc. in advance.

On the other hand, since JMA decided the parameters that can be applied to general rivers, in the case of “Uonogawa”, we might overestimate the simulated flow rate: the mean squared error (MSE) between the observed and simulated flow rates is 5.8×10^5 . Thus, in the next subsection, we try to find the optimal parameters in the case of “Uonogawa” to get an accurate simulated flow rate.

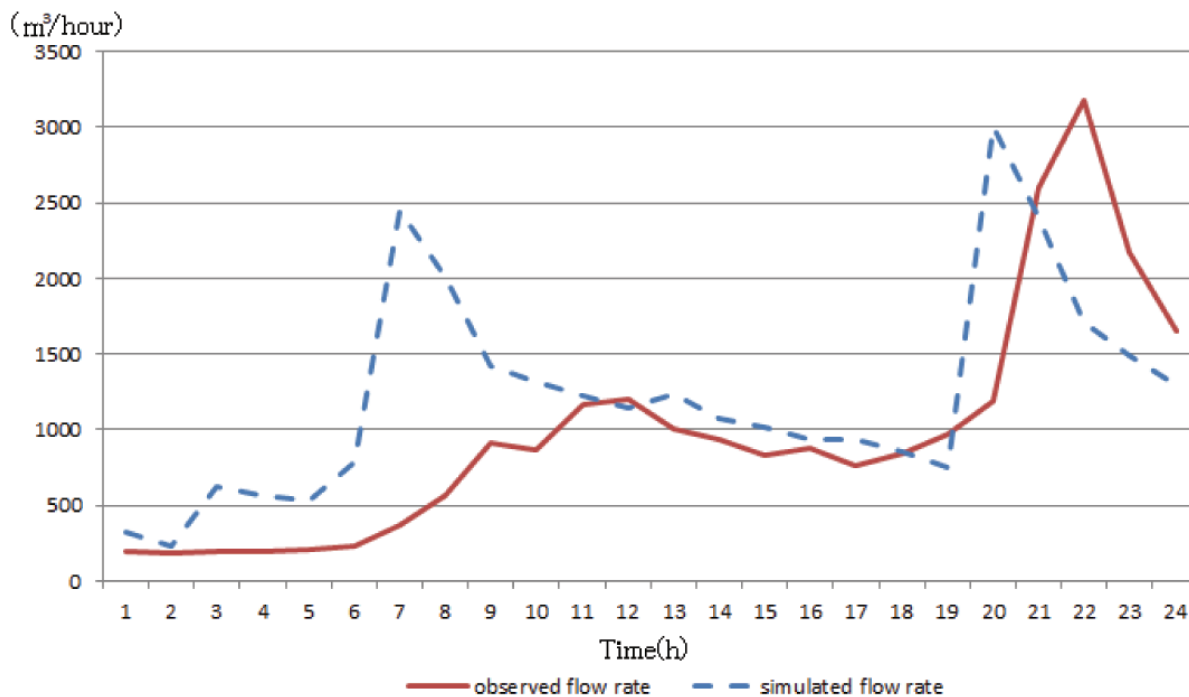


FIGURE 4. Comparison of the observed (solid line) and simulated (dashed line) flow rates to the river in the case of JMA's parameters. The mean squared error (MSE) between the observed and simulated flow rates is 5.8×10^5 .

3.2. **Simulation 2.** As a case study, we aim to investigate the optimum parameters of the “Uonogawa” by the simulation of the tank model with the data of past rainfall and flow rate.

In the parameters of JMA's tank model discussed in Section 3.1, the simulated flow rate is larger than the observed one. To make the simulated flow rate closer to the observed flow rate, the improvement points are examined as follows. From Figure 4, the simulated flow rate shows that the flow rate increases at the time when the rainfall precipitation increases. On the other hand, the observed flow rate was increasing after several hours when the rainfall precipitation was increased. Thus, we focus on the accumulation in the tank by decreasing the outflow coefficient to strengthen the delay element. Then, the simulated flow rate tends to increase after the rainfall precipitation increases.

Considering these improvement points, to get an accurate simulated flow rate, we search for the parameters sequentially as follows:

- $L_1 \sim L_4$ set the same values in Table 2. Since these parameters are related to the structure of river in the tank model, we should not change these parameters.
- Other parameters are searched to get the smallest MSE.
 - The search range of the parameters is between one third and triple of the parameters in Table 2.
 - The search interval is $\sim 0.02 \times 10^{-5}$.

As the result of search, we obtain the parameters shown in Table 3 and the result of simulated flow rate shown in Figure 5. We also plot the observed flow rate, which is the same shown in Figure 3.

Compared with the results in the case of JMA's parameters, we can confirm that the difference of the observed and simulated flow rates is small, i.e., the MSE between the observed and simulated flow rates is 1.7×10^5 . This value is 3 times smaller than that of JMA's tank model (Figure 4). The observed flow rate was rapidly increased around 21 o'clock. This might be because the rainfall precipitation accumulated so far had become

TABLE 3. Parameters after improvement

1st tank	2nd tank	3rd tank
$\lambda_1 = 1.20 \times 10^{-5}$ and $\lambda_2 = 1.90 \times 10^{-5}$	$\lambda_3 = 7.00 \times 10^{-6}$	$\lambda_4 = 1.00 \times 10^{-6}$
$\alpha_1 = 3.70 \times 10^{-5}$	$\alpha_2 = 1.10 \times 10^{-5}$	$\alpha_3 = 3.00 \times 10^{-6}$
$L_1 = 15$ and $L_2 = 60$	$L_3 = 15$	$L_4 = 15$

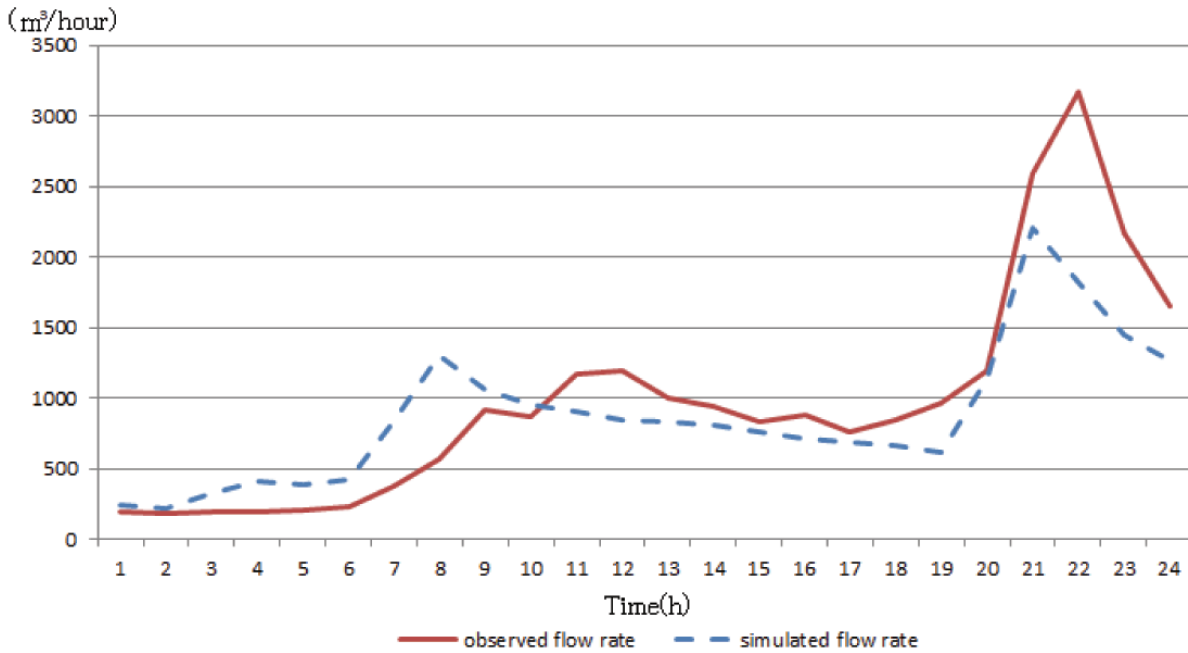


FIGURE 5. Comparison of the observed (solid line) and simulated (dashed line) flow rates to the river in the case of the simulation with the searched parameters. The MSE between the observed and simulated flow rates is 1.7×10^5 . This value is 3 times smaller than that of JMA’s tank model (Figure 4).

the limit and it flowed out suddenly. On the other hand, the simulated flow rate does not increase much.

Since we set the lowered outflow coefficient from the side hole λ_i ($i = 1, 2, 3, 4$) significantly, we did not change the outflow coefficient from the bottom hole α_j ($j = 1, 2, 3$) so much. Therefore, the outflow from the bottom hole (penetration) increased, then it seems that less water was accumulated in the tank.

Note that we got the flow rate of other day with some errors by using the obtained parameter. To suppress these errors and get more general parameters, we need to find the parameters for the other day and average it. The errors also might be caused by searching the parameters with coarse grid. We need to search for the parameters with fine grid.

Though this is only one case study, we emphasize that there is a possibility that we can obtain more accurate flow rate of each river to consider the difference in parameters in each river.

4. Summary. The Japan Meteorological Agency (JMA) currently uses the tank model as a prediction method of the criteria for the warning information of sediment-related disaster and flood. In the tank model, there are some important parameters to predict the flow rate. JMA has decided the parameters that can be applied to general rivers. On the other hand, it is realistic that these parameters are different in each river. In this paper, by using the tank model of three layers, we tried to search for the parameters of

the “Uonogawa” with the data of past rainfall and flow rate. As the result, we could confirm that the difference of the observed and simulated flow rates was small, i.e., the mean squared error between the observed flow rate and the simulation flow rate with our obtained parameters was 3 times smaller than that with JMA’s parameters. This result suggested that there was a possibility that we could obtain a more accurate flow rate of each river to consider the difference in parameters in each river.

It is necessary to strictly set the basin area. Since we took it into account for only non-urban areas, where 3 layers tank model could be used, we estimated less than the actual flow rate. To obtain more accurate result, it is necessary to make the tank model for urban areas and to obtain respective basin areas. As the result, more accurate flow rate of the river can be obtained by estimating the outflow amount from non-urban and urban areas. Moreover, the flow rate of the river is considered to be nonlinear behavior. The tank model is the linear model. We will need to develop a model that the nonlinear effects are included.

We also emphasize that our result is only one case study. So we will have to confirm the case of different rivers.

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