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MODIFIED FUZZY DELAY-DEPENDENT CONTROL CRITERION FOR SYSTEMS

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Abstract. *A modified evolved fuzzy method for the control of dynamic systems in various states is considered in this paper. For this purpose, a two-level strategy is proposed to first decompose the large-scale system into several interconnected subsystems. Then modified fuzzy control inputs are obtained in the form of states and interaction feedback. The relaxed delay-dependent stability criterion obtained from the energy function and linear matrix inequality (LMI) based optimum is proposed and used to coordinate the solutions and guarantee the asymptotic stability of the overall system.*
Keywords: Intelligent information, System control, Artificial intelligence

1. **Introduction.** Time delay is commonly encountered in various engineering systems. For example, systems with computer control have time delays, as it takes time for the computer to execute numerical operations. The introduction of the time-delay factor is often a source of instability and generally complicates the analysis. Fuzzy-rule-based modeling has become an active research field because of its unique merits in solving complex nonlinear systems. Therefore, in this study we consider a multiple time-delay system based on the so-called Takagi-Sugeno fuzzy model. One critical property of control systems is stability and considerable reports have appeared in the literature on how to handle the stability problem for fuzzy dynamic systems (see, for example, [1-6] and the references therein). Hence, a stability criterion for multiple time-delay fuzzy large-scale systems is quite important and has garnered much attention. The rest of this study is organized as follows. First, the system description is given. Next, based on Lyapunov approach, a stability condition is derived to guarantee the asymptotic stability of multiple time-delay fuzzy large-scale systems. Finally, an example is given to demonstrate the result, followed by some conclusions.

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2. **Problem Statement and Preliminaries.** Consider a time-delay system $\boldsymbol{F}$ described by the following equations:

$$\dot{x}_j(t) = \sum_{i=1}^{r_j} h_{ij}(t) \left(A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj}) \right) + \sum_{\substack{n=1 \\ n\neq j}}^{J} C_{nj}x_n(t), \tag{1}$$

where A_{ij} is a constant matrix with appropriate dimensions, $x_j(t)$ is the state vector, and C_{nj} is the interconnection with the j interconnected plants. Equation (1) is represented by a Takagi-Sugeno fuzzy model of the following form:

$$\text{IF } x_{1j}(t) \text{ is } M_{i1j} \cdots \text{ and } x_{gj}(t) \text{ is } M_{igj}; \text{ THEN } \dot{x}_j(t) = A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj}), \tag{2}$$

where $x_j^T(t) = [x_{1j}(t), x_{2j}(t), \ldots, x_{gj}(t)]$, M_{ipj} $(p = 1, 2, \ldots, g)$ are the fuzzy sets and $x_{1j}(t) \sim x_{gj}(t)$ are the premise variables. The final output of the Takagi-Sugeno fuzzy model is inferred as follows:

$$\dot{x}_j(t) = \sum_{i=1}^{r_j} h_{ij}(t) \left[A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj}) \right]. \tag{3}$$

Theorem 2.1. *The multiple time-delay fuzzy large-scale system is asymptotically stable, if there exist positive constants α_j and β, $j = 1, 2, \ldots, J$ are chosen to satisfy*

$$\lambda_m(Q_{ij}) + \overline{H}_j(\tau_j) < 0 \text{ for } i = 1, 2, \ldots, r_j. \tag{4}$$

Proof: Let the Lyapunov function be defined as [8]

$$\begin{aligned}
\dot{V} &= \sum_{j=1}^{J} \dot{v}_j(t) = \sum_{j=1}^{J} \left[\dot{x}_j^T(t)P_jx_j(t) + x_j^T(t)P_j\dot{x}_j(t) \right] \\
&= \sum_{j=1}^{J} \left[\sum_{i=1}^{r_j} h_{ij}(t) \left((A_{ij})x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj}) \right) + \phi_j(t) \right]^T P_jx_j(t) \\
&\quad + x_j^T(t)P_j \left[\sum_{i=1}^{r_j} h_{ij}(t) \left((A_{ij})x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj}) \right) + \phi_j(t) \right] \\
&\leq \sum_{j=1}^{J}\sum_{i=1}^{r_j} h_{ij}(t)x_j^T(t) \left[(A_{ij})^TP_j + P_j(A_{ij}) \right] x_j(t) + \sum_{j=1}^{J} \left[\phi_j^T(t)P_jx_j(t) + x_j^T(t)P_j\phi_j(t) \right] \\
&\quad + \sum_{j=1}^{J}\sum_{i=1}^{r_j} h_{ij}(t) \sum_{k=1}^{N_j} \left[\alpha_jx_j^T(t)x_j(t) + \alpha_j^{-1}x_j^T(t-\tau_{kj})A_{ikj}^TP_jP_jA_{ikj}x_j(t-\tau_{kj}) \right] \\
&= D_1 + D_2 + D_3,
\end{aligned}$$

where

$$\begin{aligned}
D_1 &\equiv \sum_{j=1}^{J}\sum_{i=1}^{r_j} h_{ij}(t)x_j^T(t) \left[(A_{ij})^TP_j + P_j(A_{ij}) + \alpha_jN_jI \right] x_j(t), \\
D_2 &\equiv \sum_{j=1}^{J} \left[\phi_j^T(t)P_jx_j(t) + x_j^T(t)P_j\phi_j(t) \right] \\
&\leq \sum_{j=1}^{J}\sum_{n=1}^{J} x_j^T(t) \left[\beta \left(\frac{J-1}{J} \right) I + \beta^{-1}P_jC_{nj}C_{nj}^TP_j \right] x_j(t),
\end{aligned}$$

$$
\begin{aligned}
D_3 &\equiv \sum_{j=1}^{J}\sum_{i=1}^{r_j} h_{ij}(t)\sum_{k=1}^{N_j}\alpha_j^{-1}x_j^T(t-\tau_{kj})A_{ikj}^T P_j P_j A_{ikj}x_j(t-\tau_{kj}) \\
&\le \sum_{j=1}^{J}\sum_{i=1}^{r_j} h_{ij}(t)\sum_{k=1}^{N_j}\lambda_M\left(\overline{Q}_{ikj}\right)\|x_j(t-\tau_{kj})\|^2 .
\end{aligned}
$$

That means

$$
\begin{aligned}
\dot{V} &\le \sum_{j=1}^{J}\sum_{i=1}^{r_j} h_{ij}(t)x_j^T(t)\left[(A_{ij})^T P_j + P_j(A_{ij}) + \alpha_j N_j I\right]x_j(t) \\
&\quad + \sum_{j=1}^{J}\sum_{n=1}^{J} x_j^T(t)\left[\beta\left(\frac{J-1}{J}\right)I + \beta^{-1}P_j C_{nj}C_{nj}^T P_j\right]x_j(t) \\
&\quad + \sum_{j=1}^{J}\sum_{i=1}^{r_j} h_{ij}(t)\sum_{k=1}^{N_j}\alpha_j^{-1}x_j^T(t-\tau_{kj})A_{ikj}^T P_j P_j A_{ikj}x_j(t-\tau_{kj}) \\
&\le \sum_{j=1}^{J}\left\{\sum_{i=1}^{r_j} h_{ij}(t)x_j^T(t)Q_{ij}x_j(t) + \sum_{i=1}^{r_j} h_{ij}(t)\sum_{k=1}^{N_j}\lambda_M\left(\overline{Q}_{ikj}\right)\|x_j(t-\tau_{kj})\|^2\right\} \\
&\le \sum_{j=1}^{J}\left\{\sum_{i=1}^{r_j} h_{ij}(t)x_j^T(t)\lambda_m(Q_{ij})x_j(t)\right\} + \sum_{j=1}^{J}\sum_{i=1}^{r_j} h_{ij}(t)\overline{H}_j(\tau_j)\|x_j(t)\|^2 \\
&= \sum_{j=1}^{J}\sum_{i=1}^{r_j} h_{ij}(t)x_j^T(t)\left(\lambda_m(Q_{ij}) + \overline{H}_j(\tau_j)\right)x_j(t).
\end{aligned}
$$

Therefore, the Lyapunov derivative is negative if $\lambda_m(Q_{ij}) + \overline{H}_j(\tau_j) < 0$.

In the above searching optimal inequalities, the evolved bat algorithm (EBA) is proposed based on the bat echolocation fuzzy complex system in the natural world. The operation of EBA can be summarized by initialization and movement. The artificial agent is moved using the random walk process $x_i^t = x_i^{t-1} + D$ proposed by [7].

3. **Example.** Consider a two-subsystem multiple time-delay system as follows.

Rule 1: If $x_{11}(t)$ is about M_{111}; Then $\dot{x}_1(t) = A_{11}x_1(t) + \sum_{k=1}^{3} A_{1k1}x_1(t-\tau_{k1})$,

Rule 2: If $x_{11}(t)$ is about M_{211}; Then $\dot{x}_1(t) = A_{21}x_1(t) + \sum_{k=1}^{3} A_{2k1}x_1(t-\tau_{k1})$

with $x_1^T(t) = [x_{11}(t)\ \ x_{21}(t)]$, $\tau_{11} = 0.3$ (sec), $\tau_{21} = 0.5$ (sec), $\tau_{31} = 0.7$ (sec),

$$
A_{11} = \begin{bmatrix} -9 & 1 \\ 3 & 2 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} -35 & -4 \\ 5 & -34 \end{bmatrix}, \quad A_{111} = \begin{bmatrix} 1.2 & -0.3 \\ 0.6 & 0.8 \end{bmatrix},
$$

$$
A_{121} = \begin{bmatrix} 0.9 & 0.3 \\ -0.3 & 0.9 \end{bmatrix}, \quad A_{211} = \begin{bmatrix} 1 & -0.1 \\ 0.4 & 0.6 \end{bmatrix}, \quad A_{221} = \begin{bmatrix} 0.7 & 0.1 \\ -0.1 & 0.7 \end{bmatrix}.
$$

Rule 1: If $x_{12}(t)$ is about M_{112}; Then $\dot{x}_2(t) = A_{12}x_2(t) + \sum_{k=1}^{3} A_{1k2}x_2(t-\tau_{k2})$,

Rule 2: If $x_{12}(t)$ is about M_{212}; Then $\dot{x}_2(t) = A_{22}x_2(t) + \sum_{k=1}^{3} A_{2k2}x_2(t-\tau_{k2})$

with $x_2^T(t) = [x_{12}(t)\ \ x_{22}(t)]$, $\tau_{12} = 0.4$ (sec), $\tau_{22} = 0.6$ (sec), $\tau_{32} = 0.8$ (sec),

$$
A_{12} = \begin{bmatrix} -10 & 1 \\ 1 & 3 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -34 & -4 \\ 3 & -33 \end{bmatrix}, \quad A_{112} = \begin{bmatrix} 0.8 & 0.2 \\ -0.5 & 0.5 \end{bmatrix},
$$

$$
A_{122} = \begin{bmatrix} 1 & 0.3 \\ 0.2 & 2.4 \end{bmatrix}, \quad A_{212} = \begin{bmatrix} 0.7 & 0.1 \\ -0.4 & 0.4 \end{bmatrix}, \quad A_{222} = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.7 \end{bmatrix}
$$

with the interconnection matrices $C_{21} = \begin{bmatrix} 0.3 & 0.1 \\ -1.2 & 1 \end{bmatrix}$, $C_{12} = \begin{bmatrix} 5.1 & -1.4 \\ 1.6 & 3 \end{bmatrix}$.

For the purpose of fulfilling the stability conditions of Theorem 2.1, selecting the proper parameters becomes the key problem to be dealt with. The fitness function is formulated by EBA algorithm with LMI optimization positive definite matrixes $P_1 = \begin{bmatrix} 0.88 & 0.22 \\ 0.22 & 0.65 \end{bmatrix}$, $P_2 = \begin{bmatrix} 0.8 & 0.22 \\ 0.22 & 0.5 \end{bmatrix}$. Simulation results are illustrated in Figures 1 and 2 with arbitrary conditions. From Figures 1 and 2, we can observe the arbitrary initial conditions will converge to be asymptotically stable which thus prove the reliability and effectiveness of the proposed theorem in this paper. In the meanwhile, the numerical simulation is used

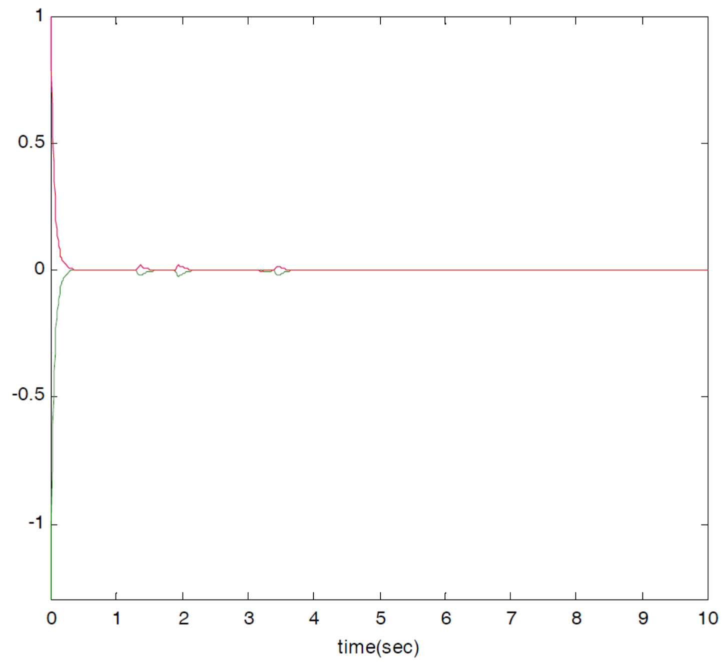

FIGURE 1. The state response of system 1

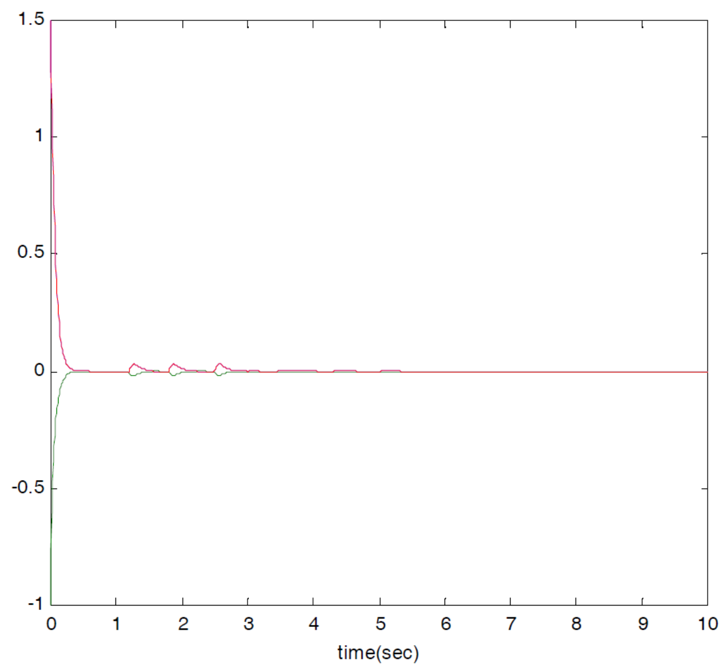

FIGURE 2. The state response of system 2

to illustrate the design concept and feasibility of the proposed controller design is straight and easy to be demonstrated in practical engineering problems.

4. **Conclusions and Future Work.** A stability criterion is derived in this paper from Lyapunov's direct method for time-delay systems. The proposed stability conditions are conceptually simple and straightforward. An example is given to demonstrate the feasibility of the result. As can be seen from the simulated results, the anticipated automated viewpoints will be achieved and the instant control laws of the system state variables will also be simulated to be converged to zero.

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