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DISCRETE-TIME POSICAST PID×$(n-2)$ STAGE PD CASCADE CONTROLLERS FOR UNSTABLE SYSTEM

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Abstract. *This paper aims to present a discrete-time controller design for a two-degree-of-freedom (2-DOF) control system connected with Posicast function to stabilize an unstable third-order or higher-order plant. The continuous-time controllers used in the 2-DOF control system, the proportional-integral-derivative (PID)×$(n-2)$ stage proportional-derivative (PD) cascade controllers and the forward controller, are designed by using the Kitti's method. The Posicast function is used to eliminate an overshoot in step response. Based on the continuous-time controllers designed in the s-domain, the discrete-time controllers with closed-form expression in the z-domain are then calculated by using the bilinear transformation. The magnetic levitation system is employed as an illustrative case study of the unstable plant to be controlled. By comparing the discrete-time system with the continuous-time system, the effectiveness of the proposed discrete-time controller design is verified by MATLAB simulation results.*
Keywords: PID×$(n-2)$ stage PD cascade controllers, Forward controller, Posicast, 2-DOF, Unstable system, Discrete time, Kitti's method, Bilinear transformation

1. **Introduction.** In order to effectively control an n^{th}-order plant, a technique to design the proportional-integral-derivative (PID) and $(n-2)$ stage proportional-derivative (PD) controllers in cascade connection has been presented [1]. This proposed technique, later referred to as 'Kitti's method', is based on root locus approach to determine controller zero locations for approximating the overall system to be second-order system as well as for satisfying both transient and steady state responses. The structure of a two-degree-of-freedom (2-DOF) control system combining the forward controller and the PID×$(n-2)$ stage PD cascade controllers together to reduce the amount of overshoot in transient response has been also introduced [1,2]. Therefore, the proposed PID×$(n-2)$ stage PD cascade controllers can be utilized instead of a classical PID controller without depending upon any tuning methods. For achieving these cascade controllers in digital form, the design techniques based on the Kitti's method have been suggested [3-5]. With discrete-time controller design techniques proposed in [3,4], the continuous-time control system is initially determined in the s-domain. The structure of discrete-time cascade controllers then can be calculated by using the zero-order hold (ZOH) [3] or the bilinear transformation (so called Tustin's method) [4] for discretization. There are four unknown parameters to be defined when designing the discrete-time cascade controllers. The Kitti's method is applied in the z-domain for determining the controller zero locations and solving other unknown parameters to complete the digital controller design. However, especially for too short sampling time (or too high sampling rate), it is quite difficult to determine

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the controller zero locations in the z-domain. With the ZOH and bilinear transformation methods used in [3,4], respectively, using bilinear transformation provides the better step-response characteristic than using the ZOH at the same sampling time. In order to reduce the complexity in discrete-time controller design, an easier technique based on the Kitti's method for determining the controller zero locations in the s-domain has been presented [5]. Based on the continuous-time cascade controllers designed in the s-domain, the discrete-time cascade controllers expressed by closed-form formulas can be obtained by using the bilinear transformation for discretization. There are only two unknown controller parameters to be solved in s-domain. Recently, an application of Posicast function to the 2-DOF control system for nonlinear unstable plant like a magnetic levitation system has been introduced [6]. This proposed technique is a combined approach of the Posicast function, the forward controller, and the PID$\times(n-2)$ stage PD cascade controllers to stabilize the nonlinear dynamic system with no overshoot. The forward controller is used to provide the target overshoot in step responses, and the Posicast function is utilized to eliminate the overshoot. Thus, the system transient response that contains no overshoot can be obtained. In order to be more useful, the aim of this paper is to present a digital controller design method to achieve a discrete-time version of the existing continuous-time Posicast PID$\times(n-2)$ stage PD cascade controllers proposed in [6]. The performance of the proposed design technique was studied through the MATLAB simulation results.

The rest of the paper is structured as follows. Section 2 explains the proposed discrete-time controller design for controlling the magnetic levitation system [7], which is an example of unstable nonlinear plant. Section 3 shows the simulation results to demonstrate the effectiveness of the proposed technique. Section 4 concludes this article with future work.

2. **Proposed Controller Design.** Generally, there are three concepts for designing digital controller as depicted in Figure 1: discrete design concept shown in route (1), sampled-data design concept shown in route (2), and digital redesign concept shown in route (3) [8]. The proposed controller design is based on the indirect digital redesign concept by extending the continuous-time design procedures to the case of discrete-time design procedures. The details of the proposed controller design can be explained as follows.

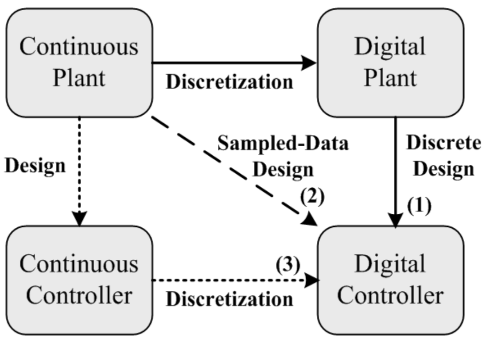


FIGURE 1. Three concepts for designing digital controller [8]

2.1. **Continuous plant under control [7].** The magnetic levitation system shown in Figure 2 is used as the unstable nonlinear plant to be controlled in this paper. The characteristic equation of this control system including the transfer functions of the controller and plant, $K(s)$ and $G(s)$, respectively, in the s-domain can be written as

$$F(s) = 1 + G(s)K(s) = x_0Lms^3 + x_0Rms^2 - ki_0Ls - ki_0R + K(s)kx_0B = 0, \quad (1)$$

where

$$k = 2C\frac{i_0}{x_0^2}. \quad (2)$$

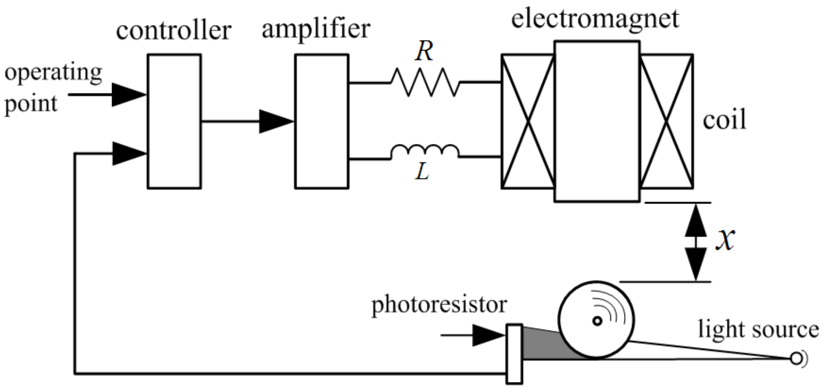


FIGURE 2. Magnetic levitation control system [7]

TABLE 1. Measured parameters of the magnetic levitation system in Figure 2 [7]

Parameter	Description	Measured Value
x_0	Distance between electromagnet and ball bearing of the operating point	0.008 m
i_0	Coil current of the operating point	0.76 A
m	Mass of ball bearing	0.068 kg
R	Coil resistance	28 Ω
L	Coil inductance	0.483 H
C	Constant	7.39×10^{-5} N·m^2/A^2
B	Constant	1.14×10^3 V/m
k	Specified parameter in (2)	1.756 N/A

From (1), it can also be stated as

$$\begin{aligned} F(s) &= 1 + \frac{K(s)kx_0B}{x_0Lms^3 + x_0Rms^2 - ki_0Ls - ki_0R} \\ &= 1 + \frac{K(s)\frac{kB}{mL}}{\left(s + \sqrt{\frac{ki_0}{mx_0}}\right)\left(s - \sqrt{\frac{ki_0}{mx_0}}\right)\left(s + \frac{R}{L}\right)} = 0. \end{aligned} \tag{3}$$

Substituting the parameters given in Table 1 into (3), it can be approximately given by

$$F(s) = 1 + \frac{K(s)60990}{(s + 49.5)(s - 49.5)(s + 58)} = 1 + K(s)G(s) = 0. \tag{4}$$

2.2. **Continuous-time controller design [6].** Figure 3 shows the 2-DOF control system including the forward controller $K_f(s)$, the cascaded PID×$(n-2)$ stage PD controller $K(s)$, and the plant $G(s)$. From (4), it is seen that the plant under control is the third-order process, so the plant transfer function can be rewritten in general form as

$$\begin{aligned} G(s) &= \frac{K_n}{s^N(T_1s + 1)(T_2s + 1)\cdots(T_ps + 1)} \\ &= \frac{60990}{(s + 49.5)(s - 49.5)(s + 58)}; \quad n = 3,\ N = 0. \end{aligned} \tag{5}$$

From the desired specifications for unit step input in terms of the percent overshoot $(P.O.) \leq 5\%$ and the settling time $(t_s) \leq 0.1$ sec, then one of the dominant closed-loop poles at $s_d = -42.354 + j44.416$ is obtained [6]. Based on the Kitti's method, the

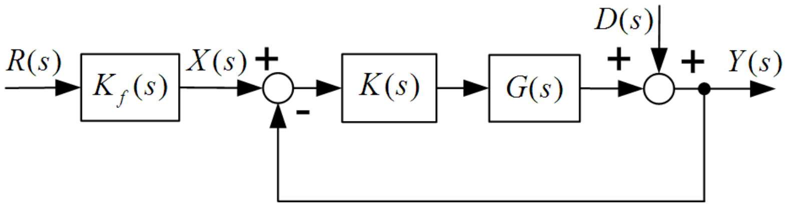


FIGURE 3. 2-DOF control system [6]

open-loop transfer function is

$$KG(s) = \frac{\overbrace{K_{pid}(s+z_1)(s+z_2)}^{\text{PID Controller}} \times \overbrace{K_{pd}(s+z_{pd})\cdots}^{(n-2)\text{PD}} K_n}{s \cdot \underbrace{s^N(s+p_1)(s+p_2)\cdots(s+p_p)}_{n\text{th order plant}}}$$
$$= \frac{K\overbrace{(s+49.6)(s+58.1)}^{\text{Pre-assigned}}(s+z_{pd})60990}{s \cdot \underbrace{(s+49.5)(s+58)(s-49.5)}_{\text{3rd order plant}}}. \tag{6}$$

The angle and location of the zero of $(s + z_{pd})$ can be determined from the root locus angle condition, which are

$$\measuredangle(s+z_{pd}) = 108.073°, \quad -z_{pd} = -27.86. \tag{7}$$

The controller gain at s_d, $K = 2.195 \times 10^{-3}$, is determined from the root locus magnitude condition. Hence, the closed-loop transfer function can be approximately expressed as

$$\frac{Y(s)}{X(s)} \approx \frac{K(s+z_{pd})60990}{s(s-49.5)+K(s+z_{pd})60990}. \tag{8}$$

To minimize the overshoot caused by adding the zero of $(s + z_{pd})$ to the open-loop transfer function $KG(s)$, the following forward controller is employed.

$$K_f(s) = z_{pd}/(s+z_{pd}). \tag{9}$$

The overall control system then can be approximated as a standard second-order system, which is

$$\frac{Y(s)}{R(s)} \approx \left(\frac{z_{pd}}{\cancel{s+z_{pd}}}\right)\left(\frac{K\cancel{(s+z_{pd})}60990}{s(s-49.5)+K(s+z_{pd})60990}\right)$$
$$\approx \frac{3.73\times 10^3}{s^2 + 2\cdot\underbrace{0.691}_{\zeta}\cdot\underbrace{61.073}_{\omega_n}s + \underbrace{3.73\times 10^3}_{\omega_n^2}}. \tag{10}$$

From the unit-step input response of the standard second-order system, the maximum overshoot M_p is dependent on the damping ratio ζ only, which can be stated as

$$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.05, \quad \zeta = 0.691. \tag{11}$$

The maximum overshoot can occur at the first peak time, which is

$$t_p = \pi \Big/ \omega_n\sqrt{1-\zeta^2} = 0.071 \text{ secs.}, \quad \omega_n = 61.073 \text{ rad/sec.} \tag{12}$$

To eliminate the overshoot in transient response, the Posicast function is used to rescale the unit-step input $R(s)$ before applying to the forward controller $K_f(s)$ (see Figure 3).

The input $R(s)$ is rescaled by the factor $(1+M_p)$ in two parts as follows:

$$\underbrace{\frac{1}{1+M_p}}_{❶}+\underbrace{\frac{M_p}{1+M_p}e^{-t_p s}}_{❷}=\overbrace{\underbrace{1-\frac{M_p}{1+M_p}}_{❶}+\underbrace{\frac{M_p}{1+M_p}e^{-t_p s}}_{❷}}^{\text{Posicast}=1+P(s)}. \tag{13}$$

From (13), the Posicast function can be simulated by the SIMULINK diagram as shown in Figure 4 [9], where $d = M_p$ and $e^{-t_p s}$ = transport delay are defined.

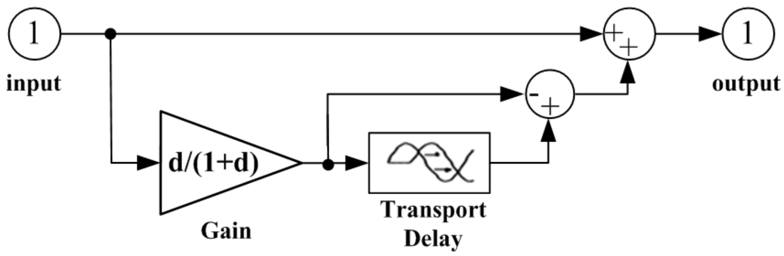


FIGURE 4. SIMULINK diagram [9]

2.3. **Discrete-time controller design.** The transfer function of the PID×$(n-2)$ stage PD cascade controllers, $K(s)$, still is not proper to be converted from continuous time into discrete time by using 'c2dm' MATLAB command, because the number of zeros in the numerator is greater than the number poles in the denominator. Based on the discrete-time controller design proposed in [5], the transfer function $K(s)$ can be rewritten in polynomial form for ease of discretization by using bilinear transformation as

$$\begin{cases} K(s)=\dfrac{K(s+z_1)(s+z_2)(s+z_{pd})}{s}, \Rightarrow K(s)=\dfrac{c_3s^3+c_2s^2+c_1s+c_0}{s}, \\ c_0=Kz_1z_2z_{pd},\ c_1=K(z_1z_2+z_1z_{pd}+z_2z_{pd}),\ c_2=K(z_1+z_2+z_{pd}),\ c_3=K. \end{cases} \tag{14}$$

The bilinear transformation can be defined as

$$s=\frac{2}{T}\left(\frac{z-1}{z+1}\right), \tag{15}$$

where T denotes the sampling time (sec/samples). By transforming the continuous-time coefficients into discrete-time coefficients, then the discrete-time controller with closed-form expression can be given by

$$K(z)=\frac{d_3z^3+d_2z^2+d_1z+d_0}{(z+1)(z+1)(z-1)}, \begin{bmatrix} d_3\\ d_2\\ d_1\\ d_0 \end{bmatrix}=\frac{1}{2T^2}\begin{bmatrix} 2T^2 & T^3 & 4T & 8\\ 2T^2 & 3T^3 & -4T & -24\\ -2T^2 & 3T^3 & -4T & 24\\ -2T^2 & T^3 & 4T & -8 \end{bmatrix}\begin{bmatrix} c_1\\ c_0\\ c_2\\ c_3 \end{bmatrix}. \tag{16}$$

At the sampling time of $T = 1/500$ sec/samples, the continuous-time controller $K(s)$ and the discretized controller $K(z)$ can be, respectively, expressed in

$$\begin{cases} K(s)=\dfrac{K(s+z_1)(s+z_2)(s+z_{pd})}{s}=\dfrac{2.195\times10^{-3}(s+49.6)(s+58.1)(s+27.86)}{s}, \\ K(z)=2.506\times10^{3}\dfrac{(z-0.905)(z-0.89)(z-0.946)}{(z-1)(z+1)^2}. \end{cases} \tag{17}$$

From the plant transfer function in (5), the discretized plant is

$$G(z)=5.779\times10^{-5}\frac{(z+1)^3}{(z-0.906)(z-1.104)(z-0.89)}. \tag{18}$$

Then, the open-loop transfer function can be approximately given by

$$KG(z) \approx \frac{0.145(z - 0.946)(z + 1)}{(z - 1)(z - 1.104)}. \tag{19}$$

The closed-loop transfer function can be approximately expressed by

$$\begin{aligned} \frac{Y(z)}{X(z)} &\approx \frac{0.145(z - 0.946)(z + 1)}{(z - 1)(z - 1.104) + 0.145(z - 0.946)(z + 1)} \\ &\approx \frac{0.145(z - 0.946)(z + 1)}{1.145z^2 - 2.096170z + 0.966830}. \end{aligned} \tag{20}$$

The transfer function of the forward controller is

$$K_f(z) = \frac{0.027(z + 1)}{(z - 0.946)}. \tag{21}$$

From (20) and (21), the transfer function of the overall 2-DOF control system can be approximately given by

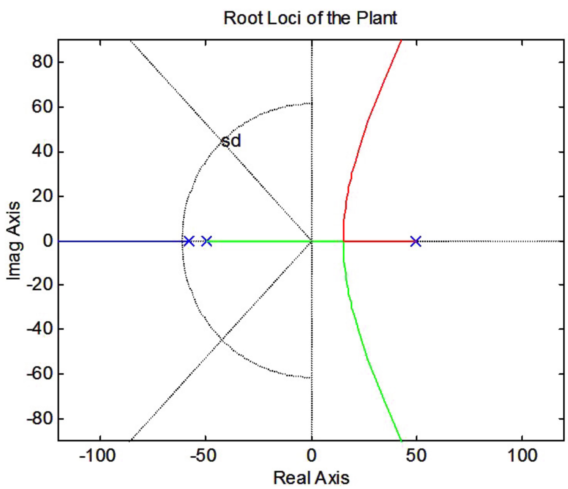
$$\frac{Y(z)}{R(z)} \approx \frac{3.915 \times 10^{-3}(z + 1)^2}{1.145z^2 - 2.096170z + 0.966830}. \tag{22}$$

To prove that (22) is the standard second-order system, discretizing the standard second-order transfer function can be stated as

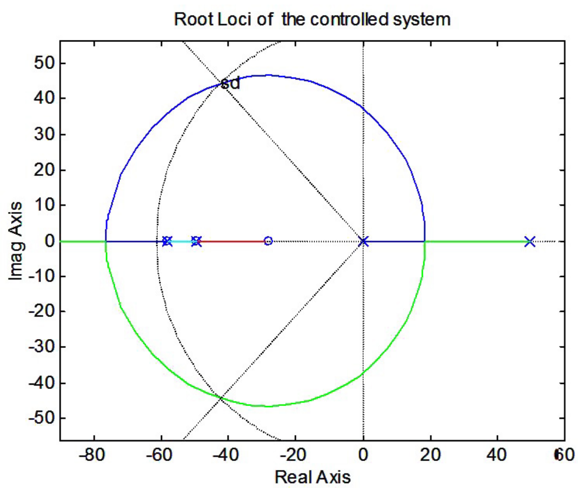
$$\left\{ G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \Rightarrow G(z) = \frac{\omega_n^2 T^2 (z + 1)^2}{\alpha_2 z^2 + \alpha_1 z + \alpha_0}. \right. \tag{23}$$

By comparing (22) with (23), the constant coefficients are $\alpha_2 = 4.352$, $\alpha_1 = -7.97$, and $\alpha_0 = 3.687$. It is confirmed that the proposed design technique using the Kitti's method can approximate the studied system by the second-order system.

3. Simulation Results. The effectiveness of the proposed discrete-time controller design for controlling unstable plant was studied through the MATLAB simulation results. Figures 5(a) and 6(a) show the root loci plots of the studied magnetic levitation system without any control in the s-plane and z-plane, respectively. It is evident that the uncontrolled system is unstable, because one real pole is in the right half of the s-plane, and one pole is located outside the unit circle in the z-plane. Figures 5(b) and 6(b) display the root loci plots of the controlled magnetic levitation system by using the continuous-time Posicast PID×$(n - 2)$ stage PD cascade controllers proposed in [6] and by utilizing

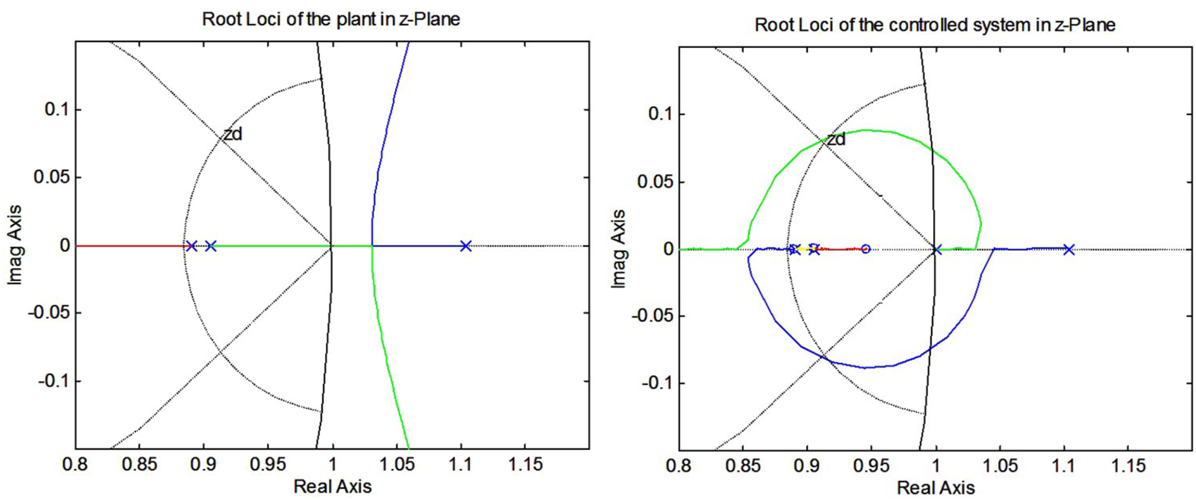


(a) For uncontrolled system



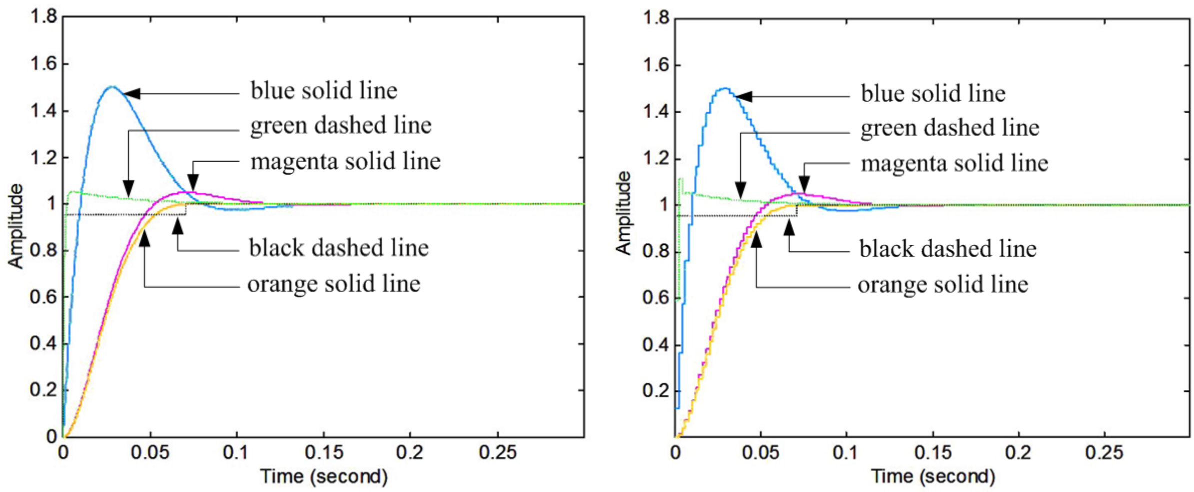
(b) For controlled system

FIGURE 5. Root loci plots for the uncontrolled and controlled systems in s-plane



(a) For uncontrolled system (b) For controlled system

Figure 6. Root loci plots for the uncontrolled and controlled systems in z-plane



(a) Continuous-time system (b) Discrete-time system

Figure 7. Unit step responses of the studied control systems

the discrete-time Posicast PID$\times(n-2)$ stage PD cascade controllers proposed in this paper, respectively. It is seen that the pole from the integral term at the origin can bring the unstable pole to move across the imaginary axis towards the left half of the s-plane. Figures 7(a) and 7(b) illustrate the unit step responses of the continuous-time system controlled by the controllers proposed in [6] and the discrete-time system controlled by the controllers proposed in this paper, respectively. The blue solid line, magenta solid line, and green dashed line represent the responses of the studied plant controlled by the PID$\times(n-2)$ stage PD cascade controllers only, by the forward controller in combination with the PID$\times(n-2)$ stage PD cascade controllers, and by the forward controller in combination with the cascade controllers by increasing the controller gain to ten times of the designed value, respectively. The black dashed line shows the Posicast command for splitting the step input into two parts, and the orange solid line represents the response of the studied plant controlled by all controllers used in connection with the Posicast function. It is shown that the discrete-time system yields the same desired control performance as the continuous-time system. In addition, as shown in the green dashed and orange solid line graphs on both Figures 7(a) and 7(b), it appears that the forward controller with increased gain of ten times can not only reduce the overshoot but also provide fast response, and the Posicast function can eliminate the overshoot in the response to the step input, respectively.

4. **Conclusions.** The digital controller design technique by extending the continuous-time controller design previously proposed in [6] for the 2-DOF control system connected with the Posicast function has been presented. The procedures of the proposed design method for controlling the unstable third-order magnetic levitation plant in the case study have been described. The performance of the discrete-time Posicast PID$\times(n-2)$ stage PD cascade controllers designed by the proposed technique has been confirmed by comparing their MATLAB simulation results with those of the continuous-time controllers. An evaluation of effects of different sampling time periods on the discrete-time controller performance is the future research of this paper.

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