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JOINTS POSITION APPROACH FOR SATELLITE ANTENNA ATTITUDE STABILIZATION SYSTEM BASED ON CONSTANT ROTATION RATIO

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Abstract. *The antenna stabilization system with four degrees of freedom is a redundant manipulator system for attitude control. In order to solve the problem of the inverse kinematics in the process of tracking satellites, a joints position approach based on constant rotation ratio (CRR) is proposed. Firstly, the relationship between the antenna stabilization system, rigid vehicle and the tracking satellite is established by using their attitude transition matrix. Then the method of calculating the expected angular velocity value of the antenna motion is given and the optimal solution of the joint space rotational velocity is obtained by using the weighted M-P inverse, then joints position algorithm on constant rotation ratio is proposed. Finally, under the condition of typical motion input, the joint position obtained by weighted M-P inverse velocity integral and joints position approach based on constant rotation ratio are simulated, respectively. Further the simulation results of attitude error described by Frobenius norm values are also presented. Simulation results show that the proposed approach can guarantee the desired accuracy of the antenna attitude in the tracking process and the effectiveness of the algorithm is verified.*

Keywords: Redundant manipulators, Kinematics, Motion planning, Numerical solution, Constant rotation ratio (CRR), Inverse kinematics, Joints position, Attitude stabilization

1. **Introduction.** When a ship sails on the sea, the communication between ships and command center on land is needed to rely on geostationary communications satellites. When the signal of uplink and downlink is transmitted, the main lobe of the antenna is required to point to the satellite, while the antenna polarization angle and the polarization angle of the satellite launching signal need to be consistent [1]. The advanced antenna frame system used to have three azimuth servo axes and a polarization angle servo axis. Based on this system, an algorithm of inverse kinematics for series redundant degree of freedom is proposed.

When the degree of freedom of the system is 4 (as the system in Figure 1), the system is an attitude redundancy degree of freedom system. The redundant degree of freedom system can achieve some performance requirements, such as the joint avoidance, overcoming the singularity and completing the second task, which concludes achieving better flexibility of the robot [2,3], avoiding obstacles [4-7], preventing joint movement beyond the community [8-11], avoiding singularity [12-14], reducing joint movement speed and optimizing the dynamic performance [15-17] and so on. The above methods, in addition to the geometric method and [18-20], mainly focus on the velocity solution of the joint space. The research on the position of joint space is relatively less now.

The algorithms for $\dot{\Theta}$ have been given in the literature, where $\dot{\Theta}$ is the angular rotation speed of these joints. The position of joints Θ is worked out by $\Theta = \int_0^t \dot{\Theta} dt$. The inverse

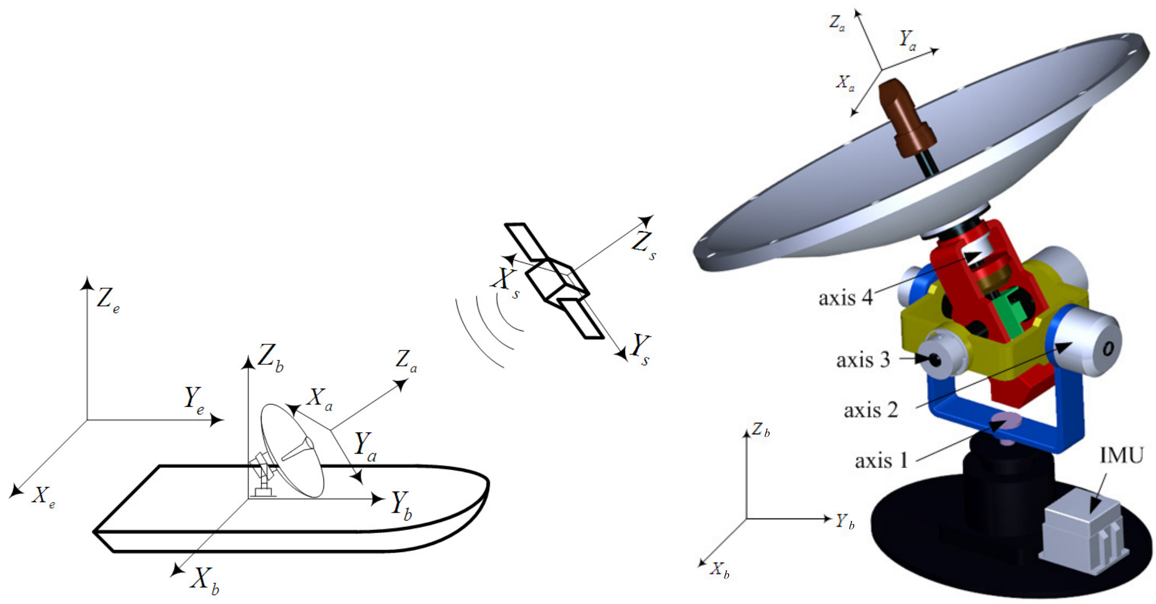

FIGURE 1. Description of the relationship between the carrier, antenna and satellite coordinate system

kinematics of joint angular velocity can introduce cumulative error in the calculation of joint angular position, which results in the divergence of the effectors error. It is necessary to feedback the effectors velocity to eliminate the cumulative error of effectors. In some systems, accurate measurements of the effectors cannot be achieved or costly (e.g., additional inertial measurement units).

In this paper, the kinematic description of the antenna system is given firstly, and then a constant rotation ratio (CRR) algorithm is proposed to solve joint position based on the weighted pseudo inverse velocity. The algorithm is used to calculate the angular position Θ in the joint space directly. Finally, the simulation is carried out on the system. Simulation results show that the divergence of the effectors error will be eliminated by the algorithm.

2. **Description of the System Attitude.** In Figure 1, the local geodetic coordinate system is established by East-North-Up coordinate system $X_eY_eZ_e$. The carrier coordinate system is the front right upper coordinate system $X_bY_bZ_b$ with which it is fixed. The coordinate system $X_aY_aZ_a$ is fixed with the antenna, where Z_a is the antenna receiving the main lobe direction, Y_a is the direction of polarization angle. $X_sY_sZ_s$ coordinate system is established and it is solid connected with satellites where Z_s is the opposite direction of main lobe of the satellite transmitting signal and Y_s is the direction of polarization of the satellite transmitting antenna.

The antenna attitude stabilization system has four orthogonal axes intersecting at one point, and the attitude of the carrier is measured by the inertial measurement unit (IMU), which is installed on the base of the system. The structure of the four axis antenna attitude stabilization system is shown in Figure 1.

According to the position of the carrier, the three parameters of satellite can be obtained: azimuth angle, pitch angle and polarization angle. The Z_s axis of the coordinate of the satellite is defined as the direction of the signal, the Y_s axis is defined as the direction of polarization angle. As Figure 1, in the case of achieving tracking, the coordinate system $X_aY_aZ_a$ needs to be aligned with the coordinate system $X_sY_sZ_s$.

There are three satellite attitude parameters: A_e is the azimuth angle of the satellite; E_e is the pitch angle of the satellite; σ_e is the polarization angle of signal. In this paper, the local rotation angle of the satellite is negative. Then the expression of the satellite in the carrier coordinate system can be obtained. The transformation matrix ${}^bT_s$ is as follows:

$$\begin{aligned}
{}^bT_s &= {}^bT_e \cdot {}^eT_s \\
&= \begin{bmatrix} \cos\gamma & 0 & -\sin\gamma \\ 0 & 1 & 0 \\ \sin\gamma & 0 & \cos\gamma \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&\cdot \begin{bmatrix} CE_eS\sigma_e - C\sigma_eSA_eSE_e & CE_eC\sigma_e + SA_eSE_eS\sigma_e & CE_eSA_e \\ -SA_eS\sigma_e - CA_eC\sigma_eSE_e & -C\sigma_eSA_e + CA_eSE_eS\sigma_e & CA_eCE_e \\ CE_eC\sigma_e & -CE_eS\sigma_e & SE_e \end{bmatrix} \\
&= N(t)
\end{aligned} \tag{1}$$

where ${}^bT_s$ is the transformation matrix between the carrier and satellite; ${}^bT_e$ is the transformation matrix between the carrier and geodetic; ${}^eT_s$ is the transformation matrix between the geodetic and satellite; the attitude angles of the carrier are: ψ, ϕ, γ, and $CE_e = \cos E_e$ and so on.

3. The Optimal Solution of Velocity Based on Weighted Pseudo Inverse of Jacobian Matrix.

3.1. Creating a Jacobian matrix.

The kinematics can be attributed to the trajectory tracking problem. The initial moment is t_0 with matrix of the antenna $N(t_0)$. With the movement of the carrier, the attitude of the antenna should be updated to $N(t_1)$ under the carrier coordinates system at t_1, and the attitude matrix is $N(t_n)$ at any time.

$$f(\Theta_n) = N(t_n) \tag{2}$$

where $\Theta_n = [\theta_1(t_n) \;\; \theta_2(t_n) \;\; \theta_3(t_n) \;\; \theta_4(t_n)]^T$ is the angular position in the joint space at t_n. The tracking problem can be regarded as working out Θ_n, which is based on $N(t_n)$:

$$\Theta_n = f^{-1}(N(t_n)) \tag{3}$$

The optimal solution at velocities can be obtained by using the generalized inverse of Moore-Penrose (M-P) of the system

$$\dot{X} = J(\Theta) \cdot \dot{\Theta} \tag{4}$$

where $\dot{X} = [\omega_x \;\; \omega_y \;\; \omega_z]^T$ represents the velocity vector of the desired antenna coordinate system in the vehicle coordinate system and $\dot{\Theta}$ represents the joint velocity vector corresponded by $\dot{X}$ at this time. $J(\Theta)$ is a Jacobian matrix when joint angle vector is Θ. $J(\Theta)$ is simplified to J.

3.2. Rotational velocity of antenna coordinate system.

The description matrix ${}^eT_s$ of satellite attitude has been obtained by satellite parameters, and the Euler angle of the carrier attitude at the moment of t is obtained by the inertial measurement units, and then ${}^eT_b$ is obtained. Continuously varying matrix sequence $N(t)$ can be obtained by (1). For any adjacent time of $N(t_n)$ and $N(t_{n+1}) = N(t_n + \Delta t) = Rot(q, d\theta)N(t_n)$.

$$Rot(q, d\theta) = Rot(x, \delta x)Rot(y, \delta y)Rot(z, \delta z) \tag{5}$$

where $Rot(q, d\theta)$ represents the coordinate system rotating over angle $d\theta$ about an axis q, Δt is the time interval between two measurements; $Rot(x, \delta x)$ is a differential motion that rotates over δx about the x-axis of the current coordinate system. Accordingly, $Rot(y, \delta y)$ and $Rot(z, \delta z)$ are the differential motion around the other two axes. When δx, δy and δz

are relatively small, ignore the second order small quantity. Ignoring second order small quantity, we have:

$$\begin{bmatrix} 1 & -\omega_z & \omega_y \\ \omega_z & 1 & -\omega_x \\ -\omega_y & \omega_x & 1 \end{bmatrix} = \frac{N(t_n + \Delta t) \cdot N^T(t_n)}{\Delta t} \tag{6}$$

The rotational angular velocity of the antenna coordinate system $X_aY_aZ_a$ between Δt can be obtained directly from (6). Then $\dot{X}$ is formed.

3.3. **To solve the optimal velocity of instantaneous joints.** The CRR algorithm is based on the inverse solution of joint velocity. For the four-axis stabilization system, the energy consumed from the first axis to the fourth axis decreases gradually. In accordance with the principle of gradually decrease of the weights coefficient, we distribute rotation weights for rotation of the joints, establishing a weight vector:

$$W = [\ w_1 \quad w_2 \quad w_3 \quad w_4\ ]^T, \quad \dot{\Theta} = W^{-1}J^T\left(J \cdot W^{-1} \cdot J^T\right)^{-1}\dot{X} \tag{7}$$

where W is the weight vector of the frame system, w_i, $i = 1, \ldots, 4$ ($w_1 > w_2 > w_3 > w_4$) represents the motion weights of the four axes, respectively, e.g., $W = [10 \quad 5 \quad 2 \quad 1]^T$. Then, $\dot{\Theta}$ can be obtained by using (7).

4. Updating Algorithm of Constant Rotation Ratio in Joints Spaces.

4.1. **The characteristics of the attitude tracking system.** The attitude tracking system has a higher requirement for the attitude accuracy of effector. On the other hand, in the case of given a desired attitude, the system is expected to achieve update with the fastest speed.

The attitude measurement unit updates the attitude information 20-50 times per second, and each updating attitude information corresponds to a matrix in the attitude matrix sequence $N(t)$. The focus is on the joint angle position Θ_n corresponding to each desired attitude.

4.2. **The algorithm of constant rotation ratio for joints position.** The core idea of the CRR algorithm is to convert the instantaneous optimum joints rotation speed to a base rotation speed and a set of rotation speed ratio, while the base speed is assumed to be corrected, and the rotation speed ratio remains constant during the update cycle. It is called a constant rotation ratio (CRR) position algorithm.

This algorithm is applied to the antenna attitude stabilization system. The steps to solve the joint angular position are as follows.

Step 1: According to the parameters of the satellite, calculate the matrix ${}^eT_s$.

Step 2: Attitude measurement unit fixed with the carrier detects the motion attitude of the carrier. ${}^bT_e$ can be obtained by using the Euler angle ψ, ϕ and γ which can describe the heading angle, pitch angle and roll angle. Combined with ${}^eT_s$, the attitude matrix of the satellite in the vehicle coordinate system can be obtained, which can be defined as $N(t_n)$.

Step 3: The motion differential extraction is carried out in the workspace. The difference between two successive pose rotations, that is the attitude transfer matrix between two poses, is calculated as follows:

$$N(t_{n+1}) = Rot(q, d\theta)N(t_n) \tag{8}$$

According to (4), the average velocity $\dot{X}_n = [\omega_x \ \ \omega_y \ \ \omega_z]^T$ of the change of the attitude of the rotation process can be obtained.

Step 4: The optimal solution for the instantaneous velocity of the structural kinematics of the redundant degree of freedom. The differential motion vector obtained from (6) is taken into (7) to calculate the optimal solution $\dot{\Theta}_n = \begin{bmatrix} \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 & \dot{\theta}_4 \end{bmatrix}^T$ under the weighted minimum multiplication, where $\dot{\Theta}_n$ is the instantaneous optimal velocity corresponding to measuring time t_n.

$$\dot{\Theta}_n = W^{-1} J_n^T \left(J_n \cdot W^{-1} \cdot J_n^T\right)^{-1} \dot{X}_n \tag{9}$$

where J_n is the corresponding Jacobian matrix, and J_n^T is the transpose of the matrix.

Step 5: According to the relationship between the joint space variable and the workspace, at the time of the nth attitude measurement is $f(\Theta_n) = N(t_n)$, and then:

$$f(\Theta_{n+1}) = f(\Theta_n + \Delta\Theta) = f(\Theta_n + \Phi \cdot \Delta t) = N(t_{n+1}) \tag{10}$$

Here, using the four order Runge Kutta method to calculate the expression, the calculation of the four order Runge Kutta method is as follows, where $\tilde{K} \in R^{n\times 1}$:

$$\tilde{K} = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) \tag{11}$$

$\Delta\Theta$ is the increment of the joint space whose value is to be solved for this problem; Δt is the time interval for attitude measurement; Φ is the average velocity vector of the joint space motion with $\Phi = [\xi_1 \;\; \xi_2 \;\; \xi_3 \;\; \xi_4]^T$, where ξ_i is an average velocity corresponding to the joint in Δt.

Step 6: Establish the relationship of constant rotation ratio:

$$\frac{\tilde{\dot{\theta}}_1(t_n)}{k_1} = \frac{\tilde{\dot{\theta}}_2(t_n)}{k_2} = \frac{\tilde{\dot{\theta}}_3(t_n)}{k_3} = \frac{\tilde{\dot{\theta}}_4(t_n)}{k_4} = \xi \tag{12}$$

Using the first axis to indicate the velocity of the rest:

$$\tilde{\dot{\theta}}_1(t_n) = k_1 \cdot \xi \cdot \Delta t, \quad \tilde{\dot{\theta}}_2(t_n) = k_2 \cdot \xi \cdot \Delta t, \quad \tilde{\dot{\theta}}_3(t_n) = k_3 \cdot \xi \cdot \Delta t, \quad \tilde{\dot{\theta}}_4(t_n) = k_4 \cdot \xi \cdot \Delta t \tag{13}$$

where k_1, k_2, k_3, k_4 are the proportional coefficients of joint velocity and remain constant during the calculation of the single joint position that is maintaining a constant rotation ratio. Incremental expression of the joint can be written as:

$$\Delta\Theta = [\; k_1 \quad k_2 \quad k_3 \quad k_4 \;]^T \cdot \xi \cdot \Delta t \tag{14}$$

Step 7: Substituting (14) into (10) and setting the initial value as Θ, then ξ can be obtained by Newton's iteration method. Substituting ξ into (14), $\Delta\Theta$ can be obtained.

5. **Simulation Results of the System.**

5.1. **Simulation conditions.** To observe the movement of the joint space and the error of the effector attitude, the simulation interval is $\Delta T = 0.01$s.

Make the following assumptions about the tracking target: target azimuth angle: 223.49°, pitch angle: 60°, polarization angle: −28.8°. According to the modeling rules in this paper, the joint angles corresponding to the target at the initial position are:

$$\Theta_0 = [\; 136.51 \quad -60 \quad 0 \quad -28.8 \;]^T, \quad \psi = 40 \cdot \sin(2\pi/30000 \cdot t) + 0.01 \cdot t \tag{15}$$

$$\phi = 40 \cdot \sin(2\pi/800 \cdot t), \quad \gamma = 45 \cdot \sin(2\pi/1000 \cdot t) \tag{16}$$

The Frobenius norm of the difference matrix is calculated as the measure index of the accuracy of the algorithm.

5.2. **Simulation results of pseudo inverse method.** By using $\dot{X} = [\omega_x \quad \omega_y \quad \omega_z]^T$ and M-P inverse directly, the angular velocity of each joint is calculated directly, and the joint angular position is obtained by directly integrating the angular velocity to update the Jacobian matrix. The obtained continuous angular velocity is shown in Figure 2.

The angular position of the joint is obtained by integrating the angular velocity directly as shown in Figure 3. Each calculated joint angle is taken into the kinematics equation to obtain terminal attitude matrix. The difference between terminal attitude matrix and desired attitude matrix is used to calculate the Frobenius norm value. It is shown in Figure 4.

The following calculation uses the constant rotation ratio approach. Then the results are updated for each step of the joint angle, and it can be obtained directly from the curves varying of the joint angle position. It is shown in Figure 5.

This paper focuses on the calculation of the position of the joint space angle without considering the joint limited problem. The M-P inverse velocity calculated at each step by using the constant rotation ratio approach is shown in Figure 6. The Frobenius norm

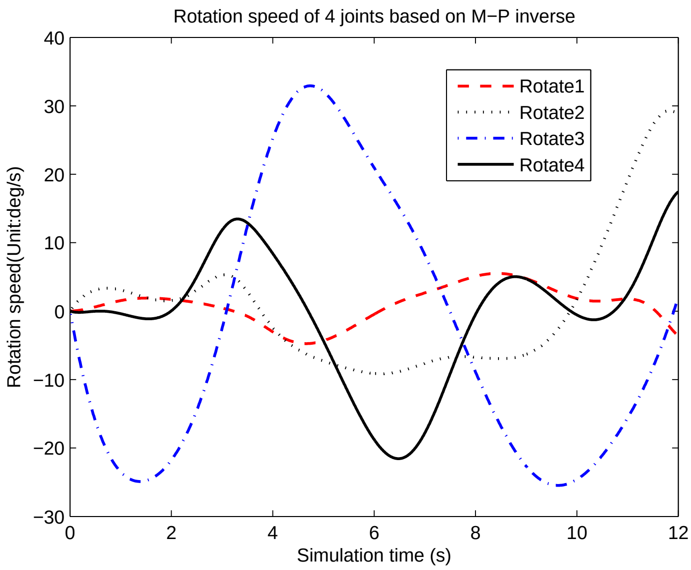

FIGURE 2. The solution of joints velocity based on the M-P inverse

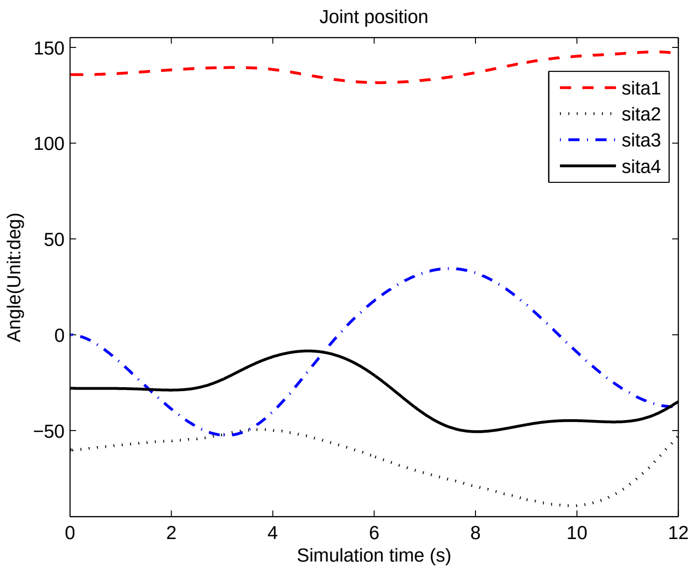

FIGURE 3. The joints values obtained by using M-P inverse velocity integral

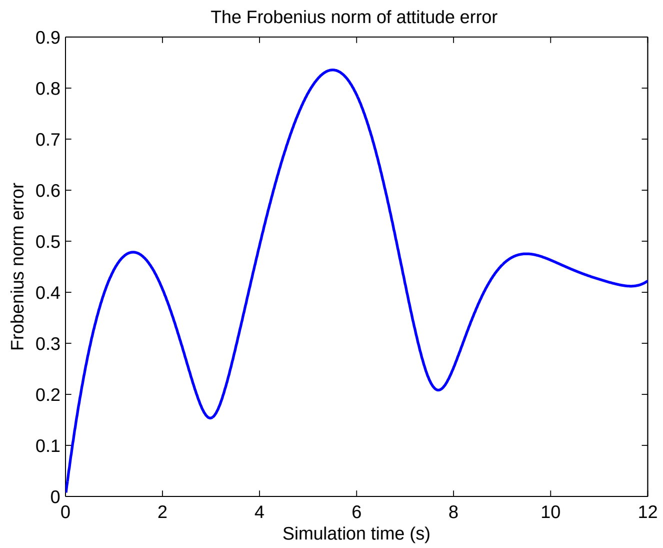

Figure 4. Frobenius norm values of attitude error matrix

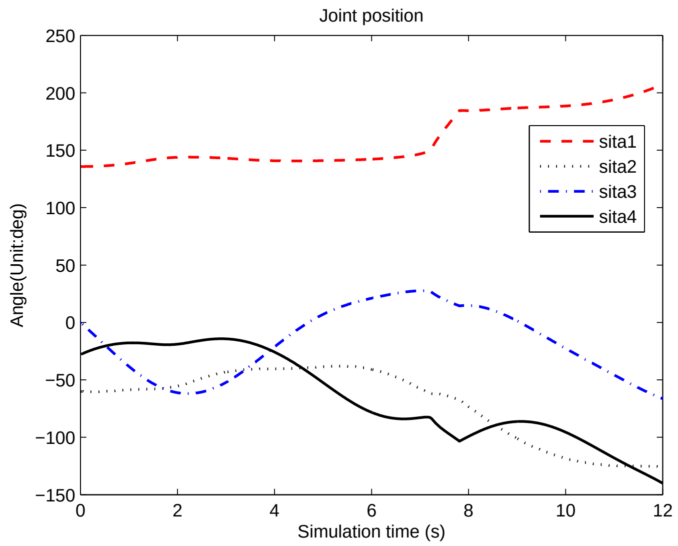

Figure 5. The joints values obtained by using RCC approach

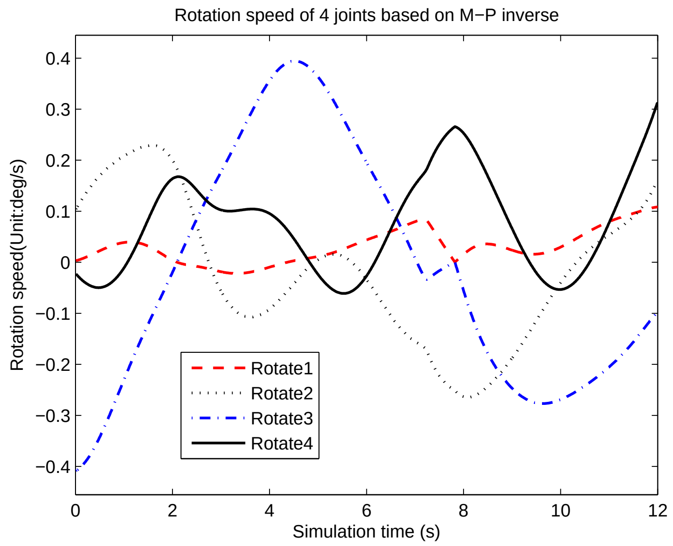

Figure 6. The M-P inverse velocity of joints in RCC approach

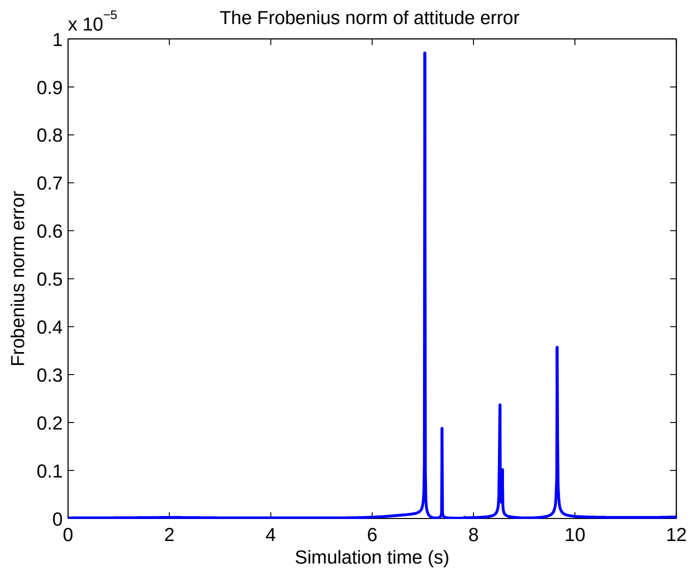

FIGURE 7. Frobenius normal values of attitude error matrix

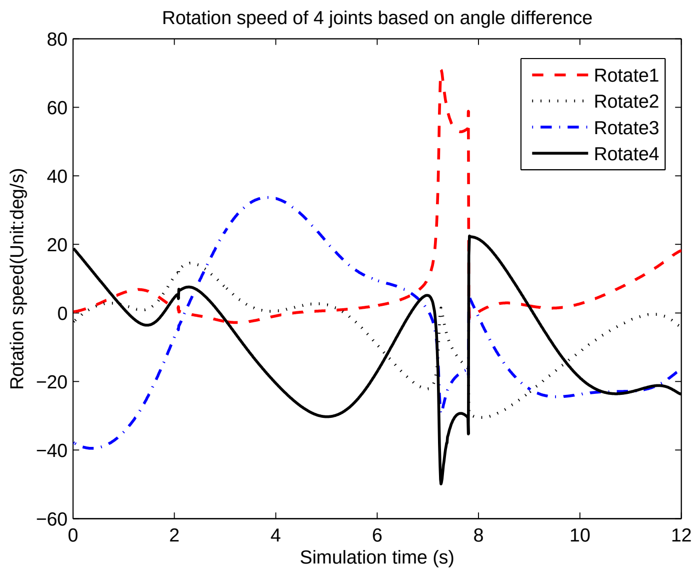

FIGURE 8. The velocity of joints obtained by position difference in RCC approach

curve of the error matrix is shown in Figure 7. Because the position value is directly obtained by using the constant rotation ratio approach, the velocity value of the joint angle is the average velocity value obtained by the position difference. The difference results are shown in Figure 8.

The attitude accuracy depends on the accuracy of the Newton iteration in the algorithm; in other words, according to the different requirements of the terminal attitude, we can flexibly choose the accuracy of Newton iterative method.

6. Conclusion.

(1) The motion model of the antenna stabilization system with four degrees of freedom has been established. It gives the relationship between the antenna movement, the carrier movement and the target satellite, and further gives the calculation method of the antenna attitude matrix sequence.

(2) The paper has analyzed the disadvantages of the existing redundant degree of freedom kinematics inverse method in trajectory tracking. It proposes an algorithm of the joint position based on constant rotation ratio.

(3) The method of solving the velocity based on M-P inverse and the method proposed in this paper have been simulated and verified. The simulation results show that the joint angle obtained by the constant rotation ratio approach ensures that the terminal attitude of the system has sufficient accuracy in the process of motion.

(4) The algorithm will be extended to a redundant robot with 7 or more degrees of freedom, in the future research. Future research results will be unified planning and control of all joints of redundant DOF robot, which will make redundant robot with more excellent kinematics performance.

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