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PARAMETERS IDENTIFICATION METHOD OF MAGIC FORMULA TIRE MODEL BASED ON GENETIC ALGORITHM

Tao Chen[1], Shuai Hou[1], Lingzhi Meng[2], Wangfang Yuan[1] and Lang Wei[1]

[1]Key Laboratory of Automotive Transportation Safety Techniques of Ministry of Transport
Chang'an University
Middle-section of Nan'er Huan Road, Xi'an 710064, P. R. China
chentao@chd.edu.cn

[2]Great Wall Motor Co., Ltd.
No. 2266, Chaoyang South Street, Baoding 071000, P. R. China
475349429@qq.com

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Abstract. *Aiming at the problem of the conversion of friction between tires and pavement, a method of parameter identification of the magic formula tire model based on genetic algorithm is proposed. Based on the data of the AW-2D road skid resistance test, the genetic algorithm is used to identify the parameters of the magic formula in the pure longitudinal slip condition. The 14 parameters of the tire model are obtained, and the conversion results of the tire/road friction characteristics are obtained. Through the comparison, the error between the conversion results and the test results is less than 10%; the dynamic simulation of the testing process of the test vehicle was carried out by ADAMS/tire test rig, and the average error between simulation results and test results is less than 6.0% which proves the validity of the parameters identification of MF tire model based on genetic algorithm.*
Keywords: Magic formula, Tire model, Genetic algorithm, Parameter identification

1. **Introduction.** The tire/road friction characteristics are of great importance to the vehicle's safety, comfort and economy. The analysis of the tire/road friction characteristics is the key point to study the vehicle dynamics. As the characteristics of road slip resistance are mainly to evaluate the road performance, and the connection to the friction characteristic between the road and tires is needed to study the vehicle's driving performance. So the study of transformation of tire/road friction characteristics which is based on the road skid resistance test is very necessary.

In 1989, Bakker et al. [1] proposed a tire model, with a combination of trigonometric functions to represent tire longitudinal force, lateral force and self-aligning torque. The model parameters are identified by the experimental data, and all the parameters form the corresponding MF tire model, which can then calculate the force of the tire under various conditions [2]. For the magic formula, usually use the optimization algorithm to identify parameters [3]. Compared with the gradient descent and least squares optimization methods, the genetic algorithm does not require the model information of the object when solving the nonlinear problem, while avoiding the local minimum, so the adaptability is wide and the robustness is strong [4]. Therefore, this paper uses the genetic algorithm to identify the magic formula parameters.

In [5], based on the data of lateral tire force from TRUCKSIM 7.0, the characteristic parameters of the relationship between the lateral force and the lateral angle of the tire in the MF model are identified by the constructed identification method. And in this paper, based on the data of the AW-2D road skid resistance test, the genetic algorithm is used to identify the parameters of the magic formula in the pure longitudinal slip condition.

Then the dynamic simulation of the testing process of the test vehicle was carried out, and the simulation results are compared with the test results of the test vehicle.

2. Tire Model.

2.1. Magic formula tire model. The MF tire model is based on test data, and it has a good precision within the test range and the limits, so the magic formula can extend the limited working conditions and guarantee a higher confidence level. The equation of the magic formula is expressed as follows [6,7]:

$$Y(x) = D\sin\left\{C\arctan\left[Bx - E\left(Bx - \arctan(Bx)\right)\right]\right\} \tag{1}$$

where $Y(x)$ is the longitudinal force F_x, the lateral force F_y or self-aligning torque M_z. x is the longitudinal slip rate, side angle, camber or vertical load. B is stiffness factor. C is shape factor. D is peak value. E is curvature factor.

2.2. Calculation of longitudinal force. In the case of pure longitudinal slip, the longitudinal force F_{x0} of the tire is a function of the slip rate κ and the vertical load F_z.

$$F_{x0}(\kappa, F_z) = D_x\sin\{C_x\arctan[B_x\kappa_x - E_x(B_x\kappa_x - \arctan(B_x\kappa_x))]\} + S_{Vx} \tag{2}$$

$$\kappa_x = \kappa + S_{Hx} \tag{3}$$

$$C_x = p_{Cx1}\lambda_{Cx} \tag{4}$$

$$D_x = \mu_x F_z \tag{5}$$

$$\mu_x = \left(p_{Dx1} + p_{Dx2}df_z\right)\lambda_{\mu x} \tag{6}$$

$$E_x = \left(p_{Ex1} + p_{Ex2}df_z + p_{Ex3}df_z^2\right)\cdot\left[1 - p_{Ex4}\mathrm{sgn}(\kappa_x)\right] \tag{7}$$

$$\kappa_x = F_z\left(p_{Kx1} + p_{Kx2}df_z\right)\exp\left(p_{Kx3}df_z\right)\lambda_{Kx} \tag{8}$$

$$B_x = K_x/(C_xD_x) \tag{9}$$

$$S_{Hx} = \left(p_{Hx1} + p_{Hx2}df_z\right)\lambda_{Hx} \tag{10}$$

$$S_{Vx} = F_z\left(p_{Vx1} + p_{Vx2}df_z\right)\lambda_{Vx}\lambda_{\mu x} \tag{11}$$

where λ_{Cx}, $\lambda_{\mu x}$, λ_{Kx}, λ_{Hx}, λ_{Vx}, $\lambda_{F_{z0}}$ are the scale factors, and the value is 1. D_x is the peak factor, indicating the maximum value of the curve. C_x is the curve shape factor, indicating whether the curve is like lateral force, longitudinal force or self-aligning torque. B_x is the stiffness factor. E_x is the curvature of the curve, indicating the shape of the curve near the maximum. S_{Vx} is the vertical shift of the curve. S_{Hx} is the horizontal shift of the curve [8].

In the longitudinal slip conditions, a total of 14 parameters are to be identified. Parameters include shape factor (p_{Cx1}), peak factor (p_{Dx1}, p_{Dx2}), curvature factor (p_{Ex1}, p_{Ex2}, p_{Ex3}, p_{Ex4}), stiffness factor (p_{Kx1}, p_{Kx2}, p_{Kx3}), the horizontal drift factor of the curve (p_{Hx1}, p_{Hx2}), and the vertical drift factor of the curve (p_{Vx1}, p_{Vx2}).

3. Parameter Identification of Tire Model Based on Genetic Algorithm.

The genetic algorithm was proposed by Professor Holland in 1975. Genetic algorithm is different from other general optimization and search procedures, and the main features are reflected in the following four aspects: firstly, the genetic algorithm is used to process the encoded parameter set, rather than the parameter set itself; secondly, the genetic algorithm searches the optimal solution from the set of points, rather than the single point search; thirdly, the genetic algorithm uses the objective function to search the optimization results; fourthly, it uses probabilistic conversion rules rather than deterministic conversion rules.

In order to identify the parameters of the MF tire model, the initial population is a set of parameter sets of magic formulas, which are randomly generated in the search space, and then finally obtain the optimal solution through the process of crossing and mutation [5]. The identification process of tire model based on genetic algorithm is as follows.

(1) Determine the decision variables and constraints of the magic formula.

(2) Determine the parameter conversion coding method. After determining the mathematical Equation (1) of the parameter, it is necessary to start the coding work and convert the variable into a binary string. If the range of variables is $[a_j, b_j]$, the precision is the last four digits of the decimal point, that is, each variable is divided into about $(b_j - a_j) \times 10^4$ parts. Use the following formula to calculate the number of bits of a binary string for a variable (expressed by m_j):

$$2^{m_j-1} < (b_j - a_j) \times 10^4 \leq 2^{m_j-1} \tag{12}$$

(3) Determine the decoding method. Use the following formula to get an actual value from the binary string:

$$x_j = a_j + decimal(substring_j) \times \frac{b_j - a_j}{2^{m_j} - 1} \tag{13}$$

where $decimal(substring_j)$ is the decimal value of the variable x_j.

The initial population in binary form can be generated randomly as follows: $U_1, U_2, \ldots$.

(4) Identify individual evaluation methods. The fitness of the chromosomes formed by the binary string of variables is evaluated:

(a) The chromosome string for anti-coding, converted into a real value, that is, the binary string to the actual value

$$p^k = \left(p^k_{Cx1}, p^k_{Dx1}, \ldots, p^k_{Vx2}\right), \quad k = 1, 2, \ldots \tag{14}$$

(b) Evaluate the objective function $f(x_k)$. Then calculate the size of the objective function as fitness. In this paper, the size of fitness is equal to the objective function value

$$eval(U_K) = f\left(p^k\right), \quad k = 1, 2, \ldots \tag{15}$$

(5) Use the roulette selection operator to select the operation. The new population is selected using the probability allocation based on the selection operation, as follows:

(a) The fitness $eval(U_k)$ for each chromosome U_k is calculated:

$$eval(U_K) = f\left(x^k\right), \quad k = 1, 2, \ldots \tag{16}$$

(b) The sum F of the population fitness is calculated:

$$F = \sum_{k=1}^{pop\text{-}size} eval(U_k) \tag{17}$$

(c) The selection probability P_k for each chromosome U_k is calculated:

$$P_k = \frac{eval(U_k)}{F} \tag{18}$$

(d) The cumulative probability Q_k for each chromosome U_k is calculated:

$$Q_k = \sum_{j=1}^{k} P_j, \quad j = 1, 2, \ldots \tag{19}$$

Rotate N times with a roulette, and pick a chromosome in a new population at a time.

(6) Crossover operation is performed using a single-point crossover operator. The nodes of the chromosome string are randomly selected, and then the right part of the nodes of the two parents is exchanged to produce the younger generation.

The crossover probability is defined as $P_0 = M\%$, that is, the $M\%$ of the chromosome to cross the operation. The cross procedure is as follows:

(a) start $k \leftarrow 0$, when $k \leq N$, $r_k \leftarrow [0, 1]$ between the random number;

(b) if $r_k < (M \times 10^{-2})$, select U_k as a parent of the cross, end; $k \leftarrow k+1$, return (a).

(7) Use the basic bit mutation operator to perform the mutation operation. After the completion of the mutation, the final generation of the population is obtained. At this point, the first generation of genetic algorithm has been completed. With the evolution of the chromosome evolution process, the elimination of less adaptive chromosomes, more and more high fitness chromosomes are left down, and ultimately search for the best.

4. Identification Method Application.

4.1. Identification process.

The AW-2D road friction coefficient test vehicle was used for the test. The obtained sample data is: slip rate of 15% to 25% range corresponding to the longitudinal force. The genetic algorithm is used to fit the parameters of the magic formula in the pure longitudinal slip condition, and then get the parameters of the tire model.

(1) The constraints are based on other literature related to SRTT tires, as shown in Table 1.

TABLE 1. Decision variable constraints

Decision variables	Restrictions	Decision variables	Restrictions
p_{Cx1}	$-3 \le p_{Cx1} \le 12.1$	p_{Kx1}	$10.8 \le p_{Kx1} \le 29.7$
p_{Dx1}	$4.1 \le p_{Dx1} \le 5.8$	p_{Kx2}	$-2.3 \le p_{Kx2} \le 3.9$
p_{Dx2}	$-0.5 \le p_{Dx2} \le -0.1$	p_{Kx3}	$-6.3 \le p_{Kx3} \le 7.6$
p_{Ex1}	$-3.8 \le p_{Ex1} \le 2.9$	p_{Hx1}	$-4.1 \le p_{Hx1} \le 8.8$
p_{Ex2}	$-3.8 \le p_{Ex2} \le 4.4$	p_{Hx2}	$-3.1 \le p_{Hx2} \le 3.9$
p_{Ex3}	$-5.4 \le p_{Ex3} \le 4.4$	p_{Vx1}	$-5.6 \le p_{Vx1} \le 3.8$
p_{Ex4}	$-9.8 \le p_{Ex4} \le 6.1$	p_{Vx2}	$-7.1 \le p_{Vx2} \le 2.9$

(2) Encode parameters. Variables p_{Cx1} and p_{Dx1} can be converted to the following binary strings:

$$[12.1 - (-3.0)] \times 10000 = 151000, \quad 2^{17} < 151000 \le 2^{18}, \quad m_1 = 18;$$
$$(5.8 - 4.1) \times 10000 = 17000, \quad 2^{14} < 17000 \le 2^{15}, \quad m_2 = 15.$$

The binary string of the first two variables of the chromosome string should be 33 ($15+18 = 33$) bits. Similarly, the other 12 parameters can be converted into the corresponding binary string, a total of 14 variables of the binary string to form a chromosome string, and the chromosome string is calculated as 167 bits.

(3) Decode the parameters. The initial population in binary can be randomly generated as follows: U_1, U_2, U_3, ..., U_{100}. The corresponding decimal value is:

$$U_1 = [p_{Cx1}, p_{Dx1}, \ldots, p_{Vx2}] = [-2.6880, 5.3617, \ldots, 1.9842]$$
$$\cdots$$
$$U_{100} = [p_{Cx1}, p_{Dx1}, \ldots, p_{Vx2}] = [11.4463, 4.1719, \ldots, 3.7881]$$

(4) Evaluate individuals. The fitness of the chromosomes formed by the binary string of 14 variables is evaluated.

(a) The chromosome string for anti-coding, converted into a real value, that is, the binary string to the actual value.

(b) Evaluate the objective function $f(x_k)$. In this paper, the research on the conversion of tire/road friction characteristics belongs to the maximum value category, and the size of fitness is equal to the objective function value. The fitness of chromosomes is as follows:

$$eval(U_1) = f(-2.6880, 5.3617) = 19.8051$$
$$\cdots$$

$$eval(U_{100}) = f(11.4463, 4.1719) = 10.2525$$

Calculated the initial population of chromosomes, the most robust is U_{35}, and the weakest is U_{11}.

(5) Use the roulette selection operator to select the operation. In the process of parameter conversion, the sum of the population's fitness is: $F = \sum_{k=1}^{100} eval(U_k) = 1767.1354$.

The selection probability p_k corresponding to each chromosome U_k ($k = 1, 2, \ldots, 100$) is as follows: $P_1 = 0.0112$, $P_2 = 0.0098$, $P_3 = 0.0138$, ..., $P_{100} = 0.0058$.

The cumulative probability Q_k corresponding to each chromosome U_k ($k = 1, 2, \ldots, 100$) is as follows: $Q_1 = 0.0112$, $Q_2 = 0.0210$, $Q_3 = 0.0348$, ..., $Q_{100} = 1$.

Rotate 100 times with a wheel to select a chromosome in a new population. Finally, the new population consists of the following chromosomes: U_2, U_3, U_{35}, ..., U_{47}.

(6) Crossover operation. In the process of fitting the genetic algorithm, the nodes of the chromosomal string were randomly selected at the 48th gene, and the crossover probability is defined as $P_0 = 25\%$. In the cross operation, U_5, U_7, ..., U_{91}, U_{96}, U_{97}, U_{99} are selected as crossover generations.

(7) Perform a mutation operation. The probability of mutation is defined as $P_m = 0.01$. In this study, for a total of 16700 ($167 \times 100 = 16700$) genes, hoping to have 167 mutations in each generation of genes, the probability of each gene mutation is equal. After completing the mutation, the final next generation population was obtained: U_1^*, U_2^*, $U_3^*, \ldots, U_{100}^*$. At this point, the first generation of genetic algorithm has been completed.

In parameter conversion, the termination algebra is set to 100. In the 42nd generation, the best chromosome is: $U^* = [1111110000000111000111101001010110\ldots]$. In the final search to the optimal point U^*, the corresponding fitness value is: $eval(U^*) = 62.3449$, so the maximum value of the objective function is $f\left(p_{Cx1}^*, p_{Dx1}^*, \ldots, p_{Vx2}^*\right) = 62.3449$, as shown in Figure 1.

The results of the parameters calculated in the MF tire model are shown in Table 2. From the results, the objective function in the 42 generation tends to the maximum value, and gets the highest fitness chromosome. The result of the parameter is brought into Equations (2)-(19) to obtain the coefficients B, C, D and E in Formula (1). Get $B = 0.178$, $C = 1.55$, $D = 328$, $E = 0.432$.

It is known that the normal force of the test tire is 1280N. Based on the above calculated longitudinal force-slip rate relations, the curve relation between the road adhesion coefficient and the slip rate is obtained, as shown in Figure 2.

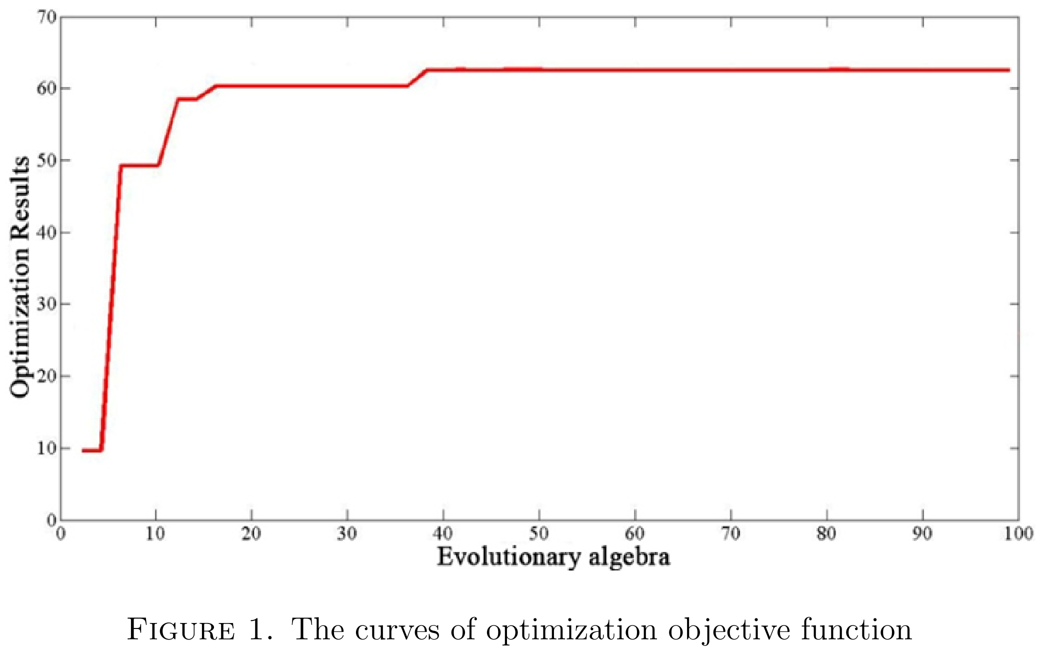

FIGURE 1. The curves of optimization objective function

TABLE 2. Calculation results of genetic algorithm for tire model parameters

Parameter	Calculated	Parameter	Calculated
p_{Cx1}	1.5587	p_{Kx1}	18.7330
p_{Dx1}	4.8712	p_{Kx2}	0.0934
p_{Dx2}	−0.7930	p_{Kx3}	0.1249
p_{Ex1}	0.3140	p_{Hx1}	−0.0017
p_{Ex2}	0.1023	p_{Hx2}	0.0009
p_{Ex3}	0.0749	p_{Vx1}	0.0246
p_{Ex4}	−0.0003	p_{Vx2}	0.0192

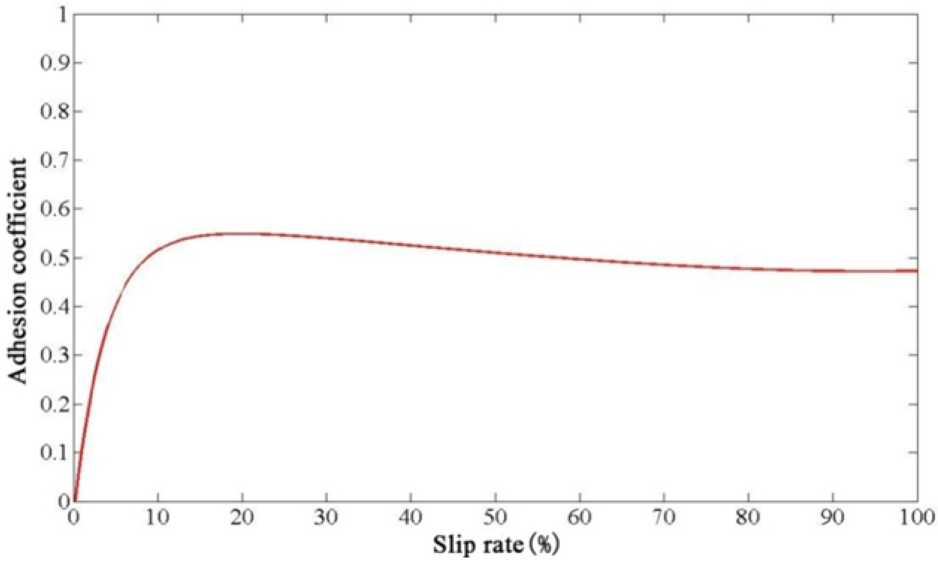

FIGURE 2. The conversion results of tire/road friction characteristics

TABLE 3. Results comparison

Slip rate (%)	Conversion value	Test value	Error (%)
15	0.522	0.53	−1.51
17.5	0.541	0.50	8.20
20	0.524	0.54	−3.01
22.5	0.499	0.51	−2.16
25	0.495	0.47	5.32

According to the calculated parameters, the expression formula of the magic formula is derived, and the adhesion coefficient under the partial slip rate is calculated by the relationship between the road adhesion coefficient and the slip rate. By comparing with the skid resistance test results under the same slip rate, the error is less than 10%, as shown in Table 3. It shows that the conversion of tire/road friction characteristics is accurate, which also reflects the accuracy of the parameters identification of MF tire model based on genetic algorithm.

4.2. **Simulation.** According to the calculated parameters, the tire attribute file is established, and the attribute file is imported into the ADAMS/tire test rig. Then the dynamic simulation of the testing process is carried out. The ADAMS reprocessing program draws the relationship between longitudinal force and time, adhesion coefficient and time.

(1) The relationship between longitudinal force and time. In the simulation, the vehicle speed is set to 40km/h, the sampling frequency is 100Hz, the vertical static load is 2560N, the simulation end time is 0.05s, and the slip rate is 16.2%. Through the simulation results and the skid resistance test results of the road, we get the relationship between longitudinal force and time of two kinds of results in a certain measuring point. The curve indicates the simulation results, and the dots indicate the test results as shown in Figure 3. The average error between the test results and the simulation results is 4.3%, which indicates that the experimental results and the simulation results are in good agreement with the time trend.

(2) The relationship between adhesion coefficient and time. By comparing the results of ADAMS simulation with the skid resistance test results of the road, the relationship between adhesion coefficient and time of the two results is obtained. The curve indicates the simulation results, and the dots indicate the test results as shown in Figure 4. The average error between the test results and the simulation results is 5.4%, which indicates that the test results are in good agreement with the simulation results.

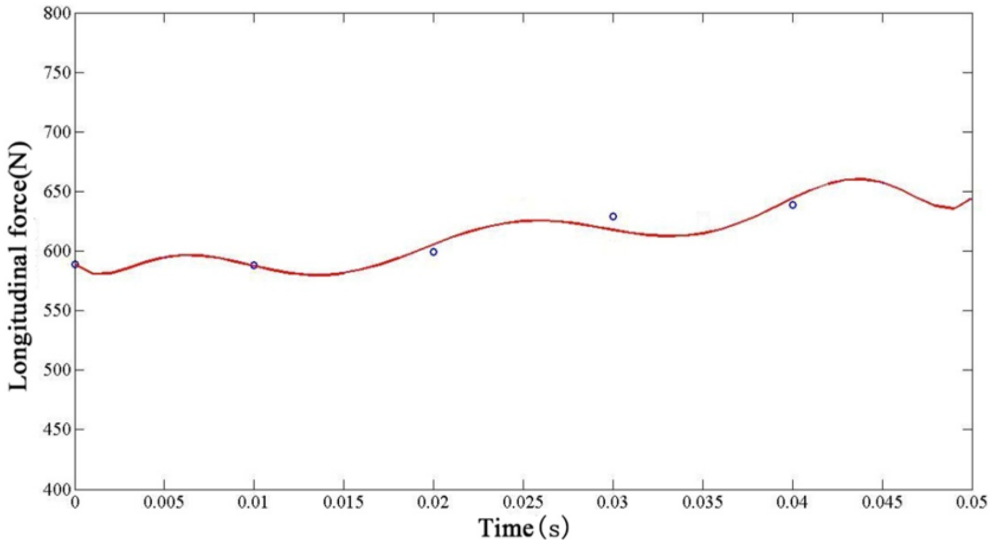

Figure 3. Relationship between longitudinal force and time

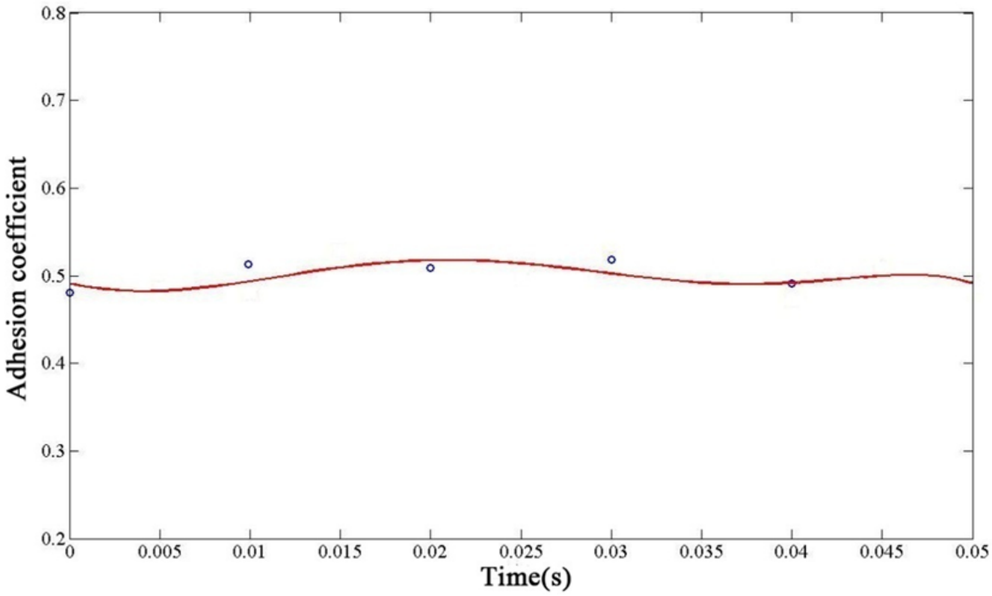

Figure 4. Relationship between adhesion coefficient and time

5. **Conclusions.** Based on the data of the AW-2D road skid resistance test, the genetic algorithm is used to identify the parameters in the magic formula under the pure longitudinal slip condition. Through the analysis and comparison, the error between the conversion results and the test results is less than 10%. In addition, the average error between simulation results and test results is less than 6.0%, proving that the simulation of tire test rig can accurately simulate the test process of test car, and showing that the genetic algorithm is an effective method to realize the multi-parameter identification of the MF tire model. Although this paper has achieved some results, there are still some shortcomings needed to do further research. In the process of parameter identification, the selection probability, crossover probability and mutation probability need to be further considered, and the conversion effect needs further study. In addition, when using the ADAMS/tire test rig for dynamic simulation, the parameter setting is more ideal, but cannot fully reflect the test car test process, which needs to be further studied.

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