ICIC Express Letters
Part B: Applications
Volume **9**, Number **11**, November **2018**

ICIC International ©2018 ISSN 2185-2766
pp. 1101–1107

THE ASSESSMENT OF QUALITY PERFORMANCE OF LIFETIME INDEX OF EXPONENTIAL PRODUCTS WITH FUZZY DATA UNDER PROGRESSIVELY TYPE II RIGHT CENSORED SAMPLE

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Received May 2018; accepted August 2018

Abstract. *Process capability analysis has been developed for assessing quality performance. In practice, lifetime performance index is a popular means to assess larger-the-better type quality performance. The exponential distribution has been recognized as a useful model for the analysis of lifetime data. In life testing experiments, progressive censoring is quite useful in many practical situations where budget constraints are in place or there is a demand for rapid testing. Moreover, observations with coarse scales, and measurement error is not quantified accurately. Therefore, progressive censored samples and imprecise data may arise in practice. This study constructs a fuzzy statistical estimator of lifetime performance index under the exponential distribution with the progressively type II right censored sample. The fuzzy statistical estimator of lifetime performance index is then utilized to develop a new fuzzy statistical hypothesis testing procedure. Finally, one practical example is utilized to illustrate the use of the new fuzzy statistical hypothesis testing procedure.*
Keywords: Process capability analysis, Progressive censoring, Fuzzy statistical hypothesis testing

1. **Introduction.** Montgomery [1] (or Kane [2]) proposed the larger-the-better process capability index C_L (or C_{PL}) for evaluating the lifetime performance of electronic components. Tong et al. [3] constructed a statistical estimator of C_L under an exponential distribution. Moreover, a statistical hypothesis testing procedure is developed to assess the lifetime performance of electronic components.

The exponential distribution has been recognized as a useful model for the analysis of lifetime data. Let X denote the lifetime of such a product and X has the exponential distribution with the probability density function (p.d.f.) is

$$f_X(x,\theta) = \theta \exp(-\theta x),\ x > 0,\ \theta > 0, \tag{1}$$

where θ is the unknown scale parameter.

In life testing experiments, progressive censoring is quite useful in many practical situations where budget constraints are in place or there is a demand for rapid testing. A schematic illustration of progressively type II right censoring is depicted, where $x_{1,n}, x_{2,n}, \ldots, x_{m,n}$ denote the observed failure times and $R_1, R_2, \ldots, R_m$ denote the corresponding numbers of units removed (withdrawn) from the test. Let m be the number of failures observed before termination and $x_{1,n} \le x_{2,n} \le \cdots \le x_{m,n}$ be the observed

DOI: 10.24507/icicelb.09.11.1101

ordered lifetimes. Let R_i denote the number of units removed at the time of the ith failure, $0 \leq R_i \leq n - \sum_{j=1}^{i-1} R_j - i$, $i = 2, 3, \ldots, m-1$ with $0 \leq R_1 \leq n-1$ and $R_m = n - \sum_{j=1}^{m-1} R_j - m$, where R_i's and m are pre-specified integers (see [4,5]).

In life testing experiments, observations have coarse scales, measurement error that is not quantified accurately, and imprecise specification limits. The best description of such data is by so-called imprecise numbers. Such observations are also called fuzzy (see [6]). Many studies were done to combine statistical methods and fuzzy set theory. For example, Buckley [7] used a set of confidence intervals producing a triangular shaped fuzzy number for the estimator, and then used this fuzzy estimator in hypothesis testing producing a fuzzy test statistic and fuzzy critical values in fuzzy hypothesis testing. Kaya and Kahraman [8] proposed fuzzy process capability analyses with fuzzy normal distribution when specification limits are either triangular or trapezoidal fuzzy numbers. Lee et al. [9] applied the large sample theory to constructing a fuzzy statistical estimator of C_{pk} under the normal distribution with the type II right censored sample and imprecise data, and then this fuzzy statistical estimator of C_{pk} is utilized for a new fuzzy statistical hypothesis testing procedure. Hryniewicz [10] proposed different approaches for the calculation of the p-value for fuzzy statistical tests about the mean in the normal distribution with known standard deviation, and a certain type of fuzziness (both in data and tested hypotheses).

There are many progressively censored samples and imprecise data in the product of life-testing and reliability experiments. It is important to handle the progressively censored samples and imprecise data in the quality assessment. In order to utilize the process capability index C_L in assessing the quality performance of products more generally and accurately under the progressively censored samples and imprecise data. Therefore, this study proposed a new approach of analyzing exponential, progressive censored and imprecise data in the quality assessment. The main aim of our study is to construct a fuzzy statistical estimator of C_L under the exponential distribution with the progressively type II right censored sample and imprecise data. The fuzzy statistical estimator of C_L is then utilized to develop a new fuzzy statistical hypothesis testing procedure. Moreover, the managers can then employ the new fuzzy statistical hypothesis testing procedure to assess the lifetime performance of products.

The rest of this study is organized as follows. Section 2 discusses the relationship between the lifetime performance index and conforming rate. Section 3 constructs a fuzzy statistical estimator of C_L under the exponential distribution with the progressively type II right censored sample and imprecise data. Section 4 utilizes the fuzzy statistical estimator of C_L to develop a new fuzzy statistical hypothesis testing procedure. One practical example and concluding remarks are made in Section 5 and Section 6, respectively.

2. **The Lifetime Performance Index and Conforming Rate.** Let X denote the lifetime of such a product and X has the exponential distribution with the p.d.f. as given by Equation (1). The lifetime is generally required to exceed L to both be economically profitable and investors. If the lifetime of a product X exceeds the lower specification limit L, then the product is defined as a conforming product. To assess the lifetime performance of products, C_L can be defined as the lifetime performance index. Under X having the exponential distribution, there are several important properties, as follows.

- The lifetime performance index C_L can be rewritten as:

$$C_L = \frac{\mu - L}{\sigma} = \frac{1/\theta - L}{1/\theta} = 1 - \theta L, \; C_L < 1, \tag{2}$$

  where the process mean $\mu = E(X) = 1/\theta$, the process standard deviation $\sigma = \sqrt{VAR(X)} = 1/\theta$, and L is the lower specification limit.

- The conforming rate

$$P_r = P(X \geq L) = \int_L^{\infty} \theta \exp(-\theta x) dx = \exp(C_L - 1), \quad C_L < 1. \tag{3}$$

Since the conforming rate P_r and the lifetime performance index C_L is one-to-one function and strictly increasing relationship. The lifetime performance index C_L can be a flexible and effective tool, not only evaluating product quality, but also for estimating the conforming rate P_r.

3. **Fuzzy Statistical Estimator of C_L.** We apply Buckley's estimation method to constructing fuzzy statistical estimator of C_L. Basically, we employ all confidence intervals, from the 99% to the 0%, placing them one on top of another, to produce a triangular shaped fuzzy number as our estimator. The estimator uses far more information than just a point estimate (see [7]). In order to add more sensitiveness to process capability analysis including more information and flexibility. We also apply Kaya and Kahraman's estimation method to constructing fuzzy statistical estimator of C_L with the fuzzy specification limit (see [8]).

3.1. **Fuzzy statistical estimator of C_L with the crisp specification limit.** Suppose that the lifetime of products may be modeled by the exponential distribution. Let X denote the lifetime of such a product and X has the exponential distribution with the p.d.f. as Equation (1). With progressively type II right censoring, n units are placed on life-testing. Consider that $X_{1,n} \leq X_{2,n} \leq \cdots \leq X_{m,n}$ is the corresponding progressively type II right censored sample, with censoring scheme $R = (R_1, R_2, \ldots, R_m)$. The likelihood function is given by $L(\theta) = c^* \theta^m \exp\left[-\theta \sum_{i=1}^m (1+R_i) x_{i,n}\right]$, where $c^* = n(n - R_1 - 1) \cdots (n - R_1 - R_2 - \cdots - R_{m-1} - m + 1)$. Let $\frac{\partial}{\partial \theta} \ln L(\theta) = 0$, and we can attain the maximum likelihood estimator (*MLE*) of θ is $\hat{\theta} = m / \sum_{i=1}^m (1+R_i) X_{i,n}$, where R_i and m are the above definition (see [4,5]). By using Equation (2) and the invariance of *MLE* (see [11]), the *MLE* of C_L can be written as given by

$$\hat{C}_L = 1 - \frac{mL}{\sum_{i=1}^m (1+R_i) X_{i,n}}. \tag{4}$$

By Buckley's method (see [7]), the fuzzy statistical estimator $\overline{C}_L$ of C_L can be attained as follows:

$$\overline{C}_L[\alpha] = \left[1 - \frac{\left(1-\hat{C}_L\right)\chi^2_{(2m),\alpha/2}}{2m}, 1 - \frac{\left(1-\hat{C}_L\right)\chi^2_{(2m),1-\alpha/2}}{2m}\right], \text{ for } 0.01 \leq \alpha \leq 1, \tag{5}$$

and

$$\overline{C}_L[\alpha] = \left[1 - \frac{\left(1-\hat{C}_L\right)\chi^2_{(2m),0.005}}{2m}, 1 - \frac{\left(1-\hat{C}_L\right)\chi^2_{(2m),0.995}}{2m}\right], \text{ for } 0 \leq \alpha < 0.01, \tag{6}$$

where $\chi^2_{(2m),\alpha/2}$ represents the upper $100(\alpha/2)$th percentile of $\chi^2_{(2m)}$.

3.2. **Fuzzy statistical estimator of C_L with the fuzzy specification limit.** Suppose that the lower specification limit L is defined as triangular fuzzy numbers $\overline{L} = (l_1/l_2/l_3)$. The α-cut of triangular fuzzy number L is as follows:

$$\overline{L}[\alpha] = [(l_2 - l_1)\alpha + l_1, (l_2 - l_3)\alpha + l_3]. \tag{7}$$

The α-cut of fuzzy statistical estimator $\overline{C}_L$ is

$$\overline{C}_L[\alpha] = \left[1 - \frac{[(l_2 - l_3)\alpha + l_3]\chi^2_{(2m),\frac{\alpha}{2}}}{2W}, 1 - \frac{[(l_2 - l_1)\alpha + l_1]\chi^2_{(2m),1-\frac{\alpha}{2}}}{2W}\right], \quad 0.01 \le \alpha \le 1, \tag{8}$$

and

$$\overline{C}_L[\alpha] = \left[1 - \frac{[(l_2 - l_3) \times 0.01 + l_3]\chi^2_{(2m),0.005}}{2W}, 1 - \frac{[(l_2 - l_1) \times 0.01 + l_1]\chi^2_{(2m),0.995}}{2W}\right], \quad 0 \le \alpha < 0.01, \tag{9}$$

where $W = \sum_{i=1}^{m}(1 + R_i)X_{i,n}$ and $\chi^2_{(2m),\alpha/2}$ represents the upper $100(\alpha/2)$th percentile of $\chi^2_{(2m)}$.

4. Fuzzy Statistical Hypothesis Testing Procedure for C_L.

Assuming that the required index value of quality performance is larger than c, where c denotes the value of capability requirement, the null hypothesis $H_0 : C_L \le c$ (process is not capable) and the alternative hypothesis $H_1 : C_L > c$ (process is capable) are constructed. In the fuzzy statistical hypothesis testing procedure, the test statistics is a triangular shaped fuzzy number, and the critical values of test also are triangular shaped fuzzy numbers (see [7]).

4.1. Crisp hypothesis statistical testing procedure.

For crisp data, by using the *MLE* $\hat{C}_L$ of C_L as the test statistic, given the specified significance level β, the rejection region can be expressed to $\left\{\hat{C}_L > 1 - \frac{2m(1-c)}{\chi^2_{(2m),\beta}}\right\}$, where the *MLE* $\hat{C}_L$ of C_L can be calculated by (4), c, β and m denote the target value, the specified significance level and the number of observed failures before termination, respectively, and $\chi^2_{(2m),\beta}$ which represents the upper $100(\beta)$th percentile of $\chi^2_{(2m)}$.

The decision rule of crisp hypothesis testing procedure is provided as follows.

If $\hat{C}_L > 1 - \frac{2m(1-c)}{\chi^2_{(2m),\beta}}$, it is concluded that the quality performance index of product meets the required level.

4.2. Fuzzy statistical hypothesis testing procedure.

For imprecise data, use the fuzzy statistical estimator $\overline{C}_L$ as the test statistic. Since the test statistic is fuzzy, the critical value will also be fuzzy. Letting the fuzzy critical value be $\overline{CV}$, given the specified significance level β, the following α-cut of $\overline{CV}$ can be derived:

$$\overline{CV}[\alpha] = \left[1 - \frac{(1-c)\chi^2_{(2m),\alpha/2}}{\chi^2_{(2m),\beta}}, 1 - \frac{(1-c)\chi^2_{(2m),1-\alpha/2}}{\chi^2_{(2m),\beta}}\right], \tag{10}$$

where c and β denote the value of capability requirement and the specified significance level, and $\chi^2_{(2m),\beta}$ represents the upper $100(\beta)$th percentile of $\chi^2_{(2m)}$.

The vertex of $\overline{C}_L$ is at $1 - \frac{(1-\hat{C}_L)\chi^2_{(2m),0.5}}{2m}$ and the vertex of $\overline{CV}$ is at $1 - \frac{(1-c)\chi^2_{(2m),0.5}}{\chi^2_{(2m),\beta}}$. A_T represents the total area under the graph of $\overline{C}_L$, A_R is the area under the graph of $\overline{C}_L$, but to the right of the vertical line through $1 - \frac{(1-c)\chi^2_{(2m),0.5}}{\chi^2_{(2m),\beta}}$, A_R and A_T are as follows respectively:

$$A_R = \begin{cases} \displaystyle\int_{1-\frac{(1-c)\chi^2_{(2m),0.5}}{\chi^2_{(2m),\beta}}}^{1-\frac{(1-\hat{C}_L)\chi^2_{(2m),0.995}}{2m}} 2\left[F_{\chi^2_{(2m)}}\left(\frac{2m(1-x)}{1-\hat{C}_L}\right)\right]dx, \\ \text{when } 1-\dfrac{\left(1-\hat{C}_L\right)\chi^2_{(2m),0.5}}{2m} < 1-\dfrac{(1-c)\chi^2_{(2m),0.5}}{\chi^2_{(2m),\beta}}. \\ \displaystyle\int_{1-\frac{(1-c)\chi^2_{(2m),0.5}}{\chi^2_{(2m),\beta}}}^{1-\frac{(1-\hat{C}_L)\chi^2_{(2m),0.5}}{2m}} 2\left[1-F_{\chi^2_{(2m)}}\left(\frac{2m(1-x)}{1-\hat{C}_L}\right)\right]dx \\ \displaystyle +\int_{1-\frac{(1-\hat{C}_L)\chi^2_{(2m),0.5}}{2m}}^{1-\frac{(1-\hat{C}_L)\chi^2_{(2m),0.995}}{2m}} 2\left[F_{\chi^2_{(2m)}}\left(\frac{2m(1-x)}{1-\hat{C}_L}\right)\right]dx, \\ \text{when } 1-\dfrac{\left(1-\hat{C}_L\right)\chi^2_{(2m),0.5}}{2m} \geq 1-\dfrac{(1-c)\chi^2_{(2m),0.5}}{\chi^2_{(2m),\beta}}, \end{cases} \tag{11}$$

and

$$\begin{aligned} A_T = & \int_{1-\frac{(1-\hat{C}_L)\chi^2_{(2m),0.005}}{2m}}^{1-\frac{(1-\hat{C}_L)\chi^2_{(2m),0.5}}{2m}} 2\left[1-F_{\chi^2_{(2m)}}\left(\frac{2m(1-x)}{1-\hat{C}_L}\right)\right]dx \\ & +\int_{1-\frac{(1-\hat{C}_L)\chi^2_{(2m),0.5}}{2m}}^{1-\frac{(1-\hat{C}_L)\chi^2_{(2m),0.995}}{2m}} 2\left[F_{\chi^2_{(2m)}}\left(\frac{2m(1-x)}{1-\hat{C}_L}\right)\right]dx, \end{aligned} \tag{12}$$

where $F_{\chi^2_{(2m)}}(\cdot)$ is the c.d.f. of $\chi^2_{(2m)}$ distribution. This is a difficult integration to compute A_R and A_T, and we will use the "trapezoidal rule" to solve the numerical integration (see [7]).

We choose a value of $\gamma \in (0, 1)$ and our decision rule is: (1) if $A_R/A_T \geq \gamma$, then reject H_0; (2) otherwise do not reject H_0. Buckley [7] suggested $\gamma < 0.5$, let us in this paper use only one value for γ and we choose $\gamma = 0.3$ and the decision rule of fuzzy statistical test is provided as follows.

If $A_R/A_T \geq \gamma$, then reject H_0 and it is concluded that the quality performance index of product meets the required level.

5. **Numerical Example.** Nelson [12, pp.105, Table 1.1] presents the results of a life-test experiment in which specimens of a type of electrical insulating fluid were subject to a constant voltage stress. The length of time until each specimen failed (or broke down) was observed. The $n = 19$ observations recorded at 34 Kv are 0.19, 0.78, 0.96, 1.31, 2.78, 3.16, 4.15, 4.67, 4.85, 6.50, 7.35, 8.01, 8.27, 12.06, 31.75, 32.52, 33.91, 36.71, 72.89. In analyzing the $n = 19$ observations, Nelson assumed a scaled Weibull distribution for the times to breakdown (from the 90% confidence interval $[0.459, 1.381]$ that he determined for the shape parameter, it is quite clear that an exponential model is also appropriate for the $n = 19$ observations). In the practical example, a progressively type II censored sample is generated from the $n = 19$ observations recorded at 34 kV. The vector of observed failure times and the progressively censoring scheme are given as follows:

$$(x_{1,n}, x_{2,n}, \ldots, x_{m,n}) = (0.19, 0.78, 0.96, 1.31, 2.78, 4.85, 6.50, 7.35),$$
$$R = (0, 0, 3, 0, 3, 0, 0, 5),\ m = 8 \text{ and } n = 19.$$

(I) We also state the proposed fuzzy statistical estimator of C_L as follows:

Specification limit for the electrical insulating fluid is set to the lower specification limit $L = 1.04$. The $\overline{C}_L$ index can be derived by using Equations (5) and (6), the membership functions $\overline{C}_L = \left(1 - \frac{(1-\hat{C}_L)\chi^2_{(16),0.005}}{16} \Big/ 1 - \frac{(1-\hat{C}_L)\chi^2_{(16),0.5}}{16} \Big/ 1 - \frac{(1-\hat{C}_L)\chi^2_{(16),0.995}}{16}\right) = (0.75486/0.89027/0.96321)$, where $m = 8$, $L = 1.04$, $W = \sum_{i=1}^{m}(1+R_i)x_{i,n} = 72.69$, the $\hat{C}_L = 1 - \frac{8\times1.04}{72.69} = 0.885541$ can be found by Equation (4), $\chi^2_{(16),0.005} = 34.267187$, $\chi^2_{(16),0.5} = 15.338500$ and $\chi^2_{(16),0.995} = 5.142206$.

We assume that the lower specification limit L is defined as approximately 1.04. The triangular fuzzy numbers are as $\overline{L} = (1.039/1.04/1.041)$. The $\overline{C}_L$ indices can be derived by using Equations (8) and (9). The membership function of $\overline{C}_L$ is determined as follows:

$$\begin{aligned}\overline{C}_L = &\left(1 - \frac{[(l_2-l_3)\times 0.01 + l_3]\chi^2_{(2m),0.005}}{2W} \Big/ 1 - \frac{l_2\chi^2_{(2m),0.5}}{2W} \Big/ 1\right.\\ &\left. - \frac{[(l_2-l_1)\times 0.01 + l_1]\chi^2_{(2m),0.995}}{2W}\right)\\ = &\,(0.75463, 0.89027, 0.96325),\end{aligned}$$

where $l_1 = 1.039$, $l_2 = 1.04$, $l_3 = 1.041$, $m = 8$, $\chi^2_{(16),0.5} = 15.33850$, $\chi^2_{(16),0.995} = 5.14221$ and $W = \sum_{i=1}^{m}(1+R_i)x_{i,n} = 72.69$.

We can compare the fuzzy estimator $\overline{C}_L$ with the crisp specification limit to the fuzzy estimator $\overline{C}_L$ with the fuzzy specification limit. The fuzzy estimator $\overline{C}_L$ with the fuzzy specification limit is wider than the fuzzy estimator $\overline{C}_L$ with the crisp specification limit.

(II) We also state the proposed fuzzy hypothesis testing procedure about C_L as following.

Step 1. Specification limit for the electrical insulating fluid is set to the lower specification limit $L = 1.04$, i.e., if the lifetime of an electrical insulating fluid exceeds 1.04 hours, then the electrical insulating fluid is defined as a conforming product. The conforming rate P_r of operational performance is required to exceed 80 percent. By Equation (3), the C_L value operational performance is required to exceed 0.80. Thus, the performance index value is set at $c = 0.8$. The testing hypothesis $H_0 : C_L \leq 0.8$ (process is not capable) v.s. $H_1 : C_L > 0.8$ (process is capable) is constructed.

Step 2. Specify a significance level $\beta = 0.05$.

Step 3. We can compute A_R and A_T by Equations (11) and (12) as follows respectively:

We can compute $A_R = 0.029606$ and $A_T = 0.063191$, according to $m = 8$, $\hat{C}_L = 0.885541$, the performance index value $c = 0.8$, $\chi^2_{(2m),0.5} = 15.338500$, $\chi^2_{(2m),\beta} = 26.296228$ for the specified significance level $\beta = 0.05$, $\chi^2_{(2m),0.995} = 5.142205$ and $F_{\chi^2_{(2m)}}(\cdot)$ is the c.d.f. of $\chi^2_{(2m)}$ distribution.

Step 4. Given $\gamma = 0.3$.

Step 5. Because of $A_R/A_T = 0.46852 \geq 0.3$, we do reject to the null hypothesis $H_0 : C_L \leq 0.8$. Thus, we can conclude that the lifetime performance index of electrical insulating fluid meets the required level.

In addition, by using the crisp hypothesis testing procedure, according to $m = 8$, $\hat{C}_L = 0.885541$, the performance index value $c = 0.8$, $\chi^2_{(2m),\beta} = 26.296228$ for the specified significance level $\beta = 0.05$, we find that $\hat{C}_L = 0.885541 > 1 - \frac{2m(1-c)}{\chi^2_{(2m),\beta}} \left(= 1 - \frac{16(1-0.80)}{26.296228} = 0.878310\right)$, so the lifetime performance index of electrical insulating fluid does meet the required level. Hence, our decision result of the fuzzy hypothesis testing procedure agrees with the result stated in the crisp hypothesis testing procedure.

6. **Conclusions.** The progressively censored sample and imprecise data may arise in practice. It is very important to handle the progressive censored sample, imprecise and non-normal data in the quality assessment. There is an important contribution that a new fuzzy statistical hypothesis testing procedure of analyzing exponential, progressively censored sample and imprecise data is proposed in this study. The fuzzy statistical hypothesis testing procedure of past studies only can handle the complete sample and the type II right sample in the quality assessment. The new fuzzy statistical hypothesis testing procedure of this study not only can handle the complete sample and the type II right sample but also can handle the progressive censored sample in the quality assessment. The new fuzzy statistical hypothesis testing procedure can handle exponential, progressively type II right censored sample and imprecise data in the lifetime performance of products. Moreover, the new fuzzy statistical hypothesis testing procedure is utilized to determine whether the lifetime performance of electrical insulating fluid adheres to the required level. In future research on this problem, it would be interesting to deal with the Weibull products based on the progressively type II right censored sample or the record values.

Acknowledgment. The authors are very much grateful to the Editor-in-Chief and reviewers for their suggestions and helpful comments which led to the improvement of this paper. This research was partially supported by the Ministry of Science and Technology, Taiwan (Plan Nos.: MOST 107-2221-E-415-012, MOST 106-2221-E-415-010, MOST 106-2221-E-158-001).

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