ICIC Express Letters
Part B: Applications
Volume **9**, Number **1**, January **2018**

ICIC International ©2018 ISSN 2185-2766
pp. 23–30

ECG CHARACTERIZATION OF SINUS BRADYCARDIA AND VENTRICULAR FLUTTER USING MALTHUSIAN PARAMETER AND RECURRENCE PLOT

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Received July 2017; accepted September 2017

Abstract. *We have recently proposed the estimation method of the decay and growth rates using the Malthusian parameter and applied it to extracting the features of pathological cardiovascular rhythms using the PhysioBank databases. In this paper, we use the mathematical model using the Van der Pol oscillators and the PhysioBank databases as the sources of ECG signals and focus on the irregular rhythms: sinus bradycardia and ventricular flutter. We compare the mathematical simulation data with the real data in the PhysioBank databases using the Malthusian parameter estimates and the recurrence plots.*
Keywords: Electrocardiogram (ECG), Sinus bradycardia (SB), Ventricular flutter (VF), Adaptive observer

1. **Introduction.** The Electrocardiogram (ECG) is used for diagnosing heart conditions by recording the small electric waves generated during heart activity. Early detection of heart diseases/abnormalities enhances the quality of living through appropriate treatment. Numerous research and work analyzing the ECG signals have been reported [1, 2]. Computer-based analysis and classification of ECG signals can be helpful in such ubiquitous health monitoring system [3]. There are two approaches to analyze ECG signals: one is a model-based one, and the other is a non-model-based one via time series data. An example of the former method is a mathematical model to describe heart rhythms considering three-coupled Van der Pol oscillators. Gois and Savi indicated that the heart rhythms of cardiac diseases can be shown by changing three coupling parameters [4]. In the non-model-based approach of a dynamical system from an experimental time series such as ECG signals, there has been an increasing interest in applying chaos theory [1, 2]. Goldberger [1] pointed out that ventricular fibrillation and related tachyarrhythmias are relatively periodic, not chaos, in contrast, the healthy heart beat shows chaotic dynamics. Thus, sudden death may be viewed as a bifurcation out of chaos. A positive largest Lyapunov exponent indicates chaos. Casaleggio et al. [6, 7] reported that the use of caution is necessary in a diagnosis based on Lyapunov exponent, because the calculation of the Lyapunov exponent strongly depends on the various working parameters. Übeyli [5] proposed the recurrent neural networks using Lyapunov exponents for classification of the ECG signal.

The Lyapunov exponent is one of the measures that estimate the decay and growth rates of nonlinear dynamics. We have recently proposed the estimation method of the decay and growth rates using the instantaneous Lyapunov exponent [9] and the Malthusian parameter. The Malthusian parameter is a kind of the instantaneous decay and growth rate. Moreover, we applied it to extracting the features of pathological cardiovascular rhythms using the PhysioBank databases [8].

In this paper, we apply the adaptive estimation method of the time-varying parameter to the Malthusian parameter estimation. Moreover, the validity of the estimation method is demonstared using ECG signals. We use the mathematical model using Van der Pol oscillator and the PhysioBank databases as the sources of ECG signals. In particular, we focus on the irregular rhythms: sinus bradycardia (SB) and ventricular flutter (VF). SB is a sinus rhythm with a rate that is lower than normal and under 60 beats per minute. VF is a tachyarrhythmia characterized by a high ventricular rate (180 $\sim$ 250 beats per minute) with a regular rhythm. We compare the mathematical simulation data with the real data in the PhysioBank databases of the irregular rhythms using the Malthusian parameter estimates and the recurrence plots. This comparison shows that the mathematical model is enough or not to approximate the irregular rhythms and the proposed indices can or cannot distinguish between SB and VF.

2. **Malthusian Parameter and Adaptive Estimator.** In this chapter, we introduce the Malthusian parameter and propose the estimator for the Malthusian parameter.

2.1. **Malthusian parameter and Lyapunov exponent.** A general solution $x(t) = cf(t) \neq 0$; c : constant satisfies the following first-order scalar differential equation:

$$\frac{dx(t)}{dt} = \frac{f'(t)}{f(t)} x(t) \tag{1}$$

where t is a time. The coefficient $M = f'(t)/f(t)$ is called the Malthusian parameter. When M is a constant, the solution of (1) is $x(t) = e^{Mt}x(0)$. Thus, the Malthusian parameter is a decay/growth rate and is closely related to the Lyapunov exponent as a decay/growth rate in local area.

2.2. **Estimation of malthusian parameter.** We propose a Malthusian parameter estimator by using the adaptive observer. Given the signal $x(t)$ generated by

$$\dot{x}(t) = \theta(t)x(t) \tag{2}$$

we have to estimate the Malthusian parameter $\theta(t)$ for each time. The Malthusian parameter is assumed to be small as:

$$|\dot{\theta}(t)| < \epsilon_*$$

The adaptive observer that estimates the state $x(t)$ and the Malthusian parameter $\theta(t)$ is given by

$$\dot{\hat{x}}(t) = -k(\hat{x}(t) - x(t)) + \hat{\theta}(t)x(t) \tag{3}$$

where $\hat{x}(t)$ is a state estimate, $\hat{\theta}(t)$ is a Malthusian parameter estimate and k is a positive constant. Defining the observer error as $e(t) = \hat{x}(t) - x(t)$, we obtain the error system as

$$\dot{e} = -ke + \tilde{\theta}(t)x(t) \tag{4}$$

where $\tilde{\theta}(t) = \hat{\theta}(t) - \theta(t)$.

The parameter update laws are selected as

$$\dot{\hat{\theta}}(t) = -\gamma e(t)x(t), \quad \gamma > 0 \tag{5}$$

We can prove the following lemma.

Lemma 2.1. *Consider the error system (4). The parameter update law guarantees the stability of the origin of the error system as follows: If* $\left|\dot{\theta}(t)\right| < \epsilon_*$ *and* $\left|\tilde{\theta}(t)\right| < \delta$*, then the output error tends to the region* $M = \left\{e : |e| \leq \sqrt{\frac{\epsilon_* \delta}{k\gamma}}\right\}$.

Proof: The whole system can be written by

$$\dot{e} = -ke + \tilde{\theta}(t)x(t) \qquad \dot{\tilde{\theta}} = -\gamma e x(t) - \dot{\theta}(t).$$

Define a Lyapunov-like function as

$$V = \frac{1}{2}\left(e^2(t) + \frac{1}{\gamma}\tilde{\theta}^2(t)\right).$$

Its time derivative is given by

$$\begin{aligned}
\dot{V} &= e\dot{e} + \frac{1}{\gamma_1}\tilde{\theta}\dot{\tilde{\theta}} \\
&= e\left(-ke + \tilde{\theta}(t)x(t)\right) - \frac{1}{\gamma}\tilde{\theta}\left(\gamma e x(t) + \dot{\theta}\right) \\
&< -ke^2 + \frac{1}{\gamma}|\tilde{\theta}||\dot{\theta}| \\
&\leq -ke^2 + \frac{\epsilon_* \delta}{\gamma}.
\end{aligned}$$

Thus, for $e \in R - M$, it is guaranteed that $\dot{V} < 0$. We can make the region M small by selecting large γ and k. □

3. **ECG, Mathematical Model, and Database.** In this chapter, we review the ECG signal and the heart diseases: SB and VF. Next, we summarize the mathematical model of the ECG and the database: Physionet.

3.1. **Electrocardiogram (ECG).** In this section, we quote the description on the ECG from Gois and Savi [4]. Cardiac electric signals on an intracellular level may be recorded with a microelectrode, which is inserted inside a cardiac muscle cell. The ECG is a measure of the extra-cellular electric behavior of the cardiac muscle tissue. The propagation wavefront of the cardiac electrical signal through the body presents a very complicated shape. In general, the signal contains the following waves.

- *P-Wave*: It is the first wave registered in the ECG, representing the atrium activation just after the sinus stimulation. It normally lasts between 60 and 90 ms in adults, having a round shape with maximal amplitude between 0.25 and 0.30 mV.
- *PR-Interval*: It is measured from the start of the P-wave to the start of the QRT-Complex and lasts 90 ms.
- *QRS-Complex*: It corresponds to the ventricular activation and is measured from the start of the first wave (no matter if it is Q- or R-Wave), to the last wave (R- or S-Wave). In normal adults, the complex lasts about 80 ms and presents a sharp shape because of the high frequencies of the signal. Its shape varies a lot, depending on the lead system used.
- *ST-Interval*: It lasts from the end of the QRS-complex to the start of the T-Wave and corresponds to the ventricular repolarization process.
- *T-Wave*: It represents the ventricular activation, which has a round shape with amplitude about 0.60 mV.

3.2. **Ventricular flutter (VF) and sinus bradycardia (SB).** A heart rate of 60 to 100 beats per minute while at rest is considered normal. The common arrhythmias can be divided into tachycardias (heart rate above 100 beats per minute) and bradycardias (heart rate below 60 beats per minute). The tachycardias are sub-divided into supraventricular and ventricular. The bradycardias are also sub-divided into sinus bradycardia, sino-atrial block, and atrioventricular block. VF is a tacharrhythmia characterized by a high ventricular rate (180 $\sim$ 250 beats per minute) with a regular rhythm. SB is one of the bradycardias, where the number of the first waves in the sinus node is less than the normal one.

3.3. **VdP model of heart dynamics.** The general heartbeat dynamics is generated by the coupling of Van del Pol (VdP) oscillators of a different heart region signal. The normal cardiac rhythm is primarily generated by the SA node, which is considered as the normal pacemaker [4]. Besides, the AV node is another pacemaker. Each one of these presents an actuation potential that is fundamental to the heart dynamics, but not necessarily the most expressive to compose the ECG signal [4]. Moreover, the third oscillator that represents the pulse propagation through the ventricles, which physiologically represents the His-Purkinje complex, is composed by the His bundle and the Purkinje fibers [4]. Gois and Savi [4] proposed a mathematical model to describe heart rhythms considering three modified Van der Pol oscillators with delays:

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -a_{SA}x_2(x_1 - w_{SA1})(x_1 - w_{SA2}) - x_1(x_1 + d_{SA})(x_1 + e_{SA}) \\
&\quad + \rho_{SA}\sin(\omega_{SA}t) + k_{SA-AV}\left(x_1 - x_3^{\tau_{SA-AV}}\right) + k_{SA-HP}\left(x_1 - x_5^{\tau_{SA-HP}}\right) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -a_{AV}x_4(x_3 - w_{AV1})(x_3 - w_{AV2}) - x_3(x_3 + d_{AV})(x_3 + e_{AV}) \\
&\quad + \rho_{AV}\sin(\omega_{AV}t) + k_{AV-SA}\left(x_3 - x_1^{\tau_{AV-SA}}\right) + k_{AV-HP}\left(x_3 - x_5^{\tau_{AV-HP}}\right) \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= -a_{HP}x_6(x_5 - w_{HP1})(x_5 - w_{HP2}) - x_5(x_5 + d_{HP})(x_5 + e_{HP}) \\
&\quad + \rho_{HP}\sin(\omega_{HP}t) + k_{HP-SA}\left(x_5 - x_1^{\tau_{HP-SA}}\right) + k_{HP-AV}\left(x_5 - x_3^{\tau_{HP-AV}}\right),
\end{aligned}
$$

where $x_i^\tau = x_i(t - \tau)$, τ represents a time delay, and $\sin(\omega t)$ is an external forcing.

The ECG signal is built from the composition of these internal states as

$$ECG = \alpha_0 + \alpha_1 x_1 + \alpha_3 x_3 + \alpha_5 x_5.$$

We focus on three coupling parameters from the His-Purkinje (HP) complex to the sinoatrial (SA) node, from the SA node to the atrioventicular (AV) node, and from the AV node to the HP complex. The VF occurs if the coupling from SA node to AV node is cut. The SB appears if the coupling from AV node to HP complex is disconnected. The three coupled VdP oscillators can be rewritten by a linear time-varying MIMO system with exogenous signals and coupling terms with the coupling parameters. From the results in Gois and Savi [4], the coupling parameters are given as $k_{SA-HP} \neq 0$, $k_{AV-SA} \neq 0$, $k_{HP-AV} \neq 0$, and the other parameters are zeros. The following parameters in the VdP equations are selected as in [4]:

$$
\begin{aligned}
&a_{SA} = 3,\ w_{SA_1} = 0.2,\ w_{SA_2} = -1.9,\ d_{SA} = 3; \\
&e_{SA} = 4.9,\ a_{AV} = 3,\ w_{AV_1} = 0.1,\ w_{AV_2} = -0.1; \\
&d_{AV} = 3,\ e_{AV} = 3,\ a_{HP} = 5,\ w_{HP_1} = 1,\ w_{HP_2} = -1; \\
&d_{HP} = 3,\ e_{HP} = 7; \\
&\tau_{SA-AV} = 0.8,\ \tau_{AS-HP} = 0.1,\ \text{vanishing all others.}
\end{aligned}
$$

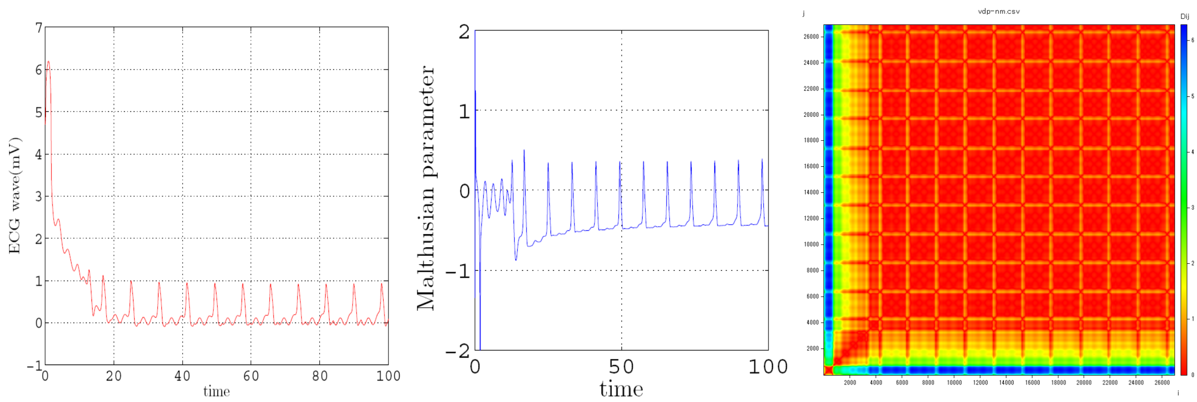
We have performed the simulation in the case of the normal ECG (N), the SB, and the VF [12]. Since we could not get the cardiac rhythms for the coupling parameters given in Gois and Savi [4], we searched the coupling parameters via our MATLAB/Simulink model. The normal ECG is generated when the coupling parameters $k_{SA-HP} = 0$, $k_{AV-SA} = 5$, $k_{HP-AV} = 20$. The SB is generated when the coupling parameters $k_{SA-HP} = 0$, $k_{AV-SA} = 50$, $k_{HP-AV} = 0$. The VF is generated when the coupling parameters $k_{SA-HP} = 0$, $k_{AV-SA} = 0$, $k_{HP-AV} = 20$. The parameters in the ECG signal were selected for each wave form as follows:

- N: $\alpha_0 = 4.7$, $\alpha_1 = 0.25$, $\alpha_3 = 0.05$, $\alpha_5 = 0.4$
- SB: $\alpha_0 = 4.5$, $\alpha_1 = 0.1$, $\alpha_3 = 0.2$, $\alpha_5 = 0.01$
- VF: $\alpha_0 = 4.5$, $\alpha_1 = 0.25$, $\alpha_3 = 5.0$, $\alpha_5 = 5.0$

In this paper, we generate the ECG signals using these parameters as the output of the mathematical model.

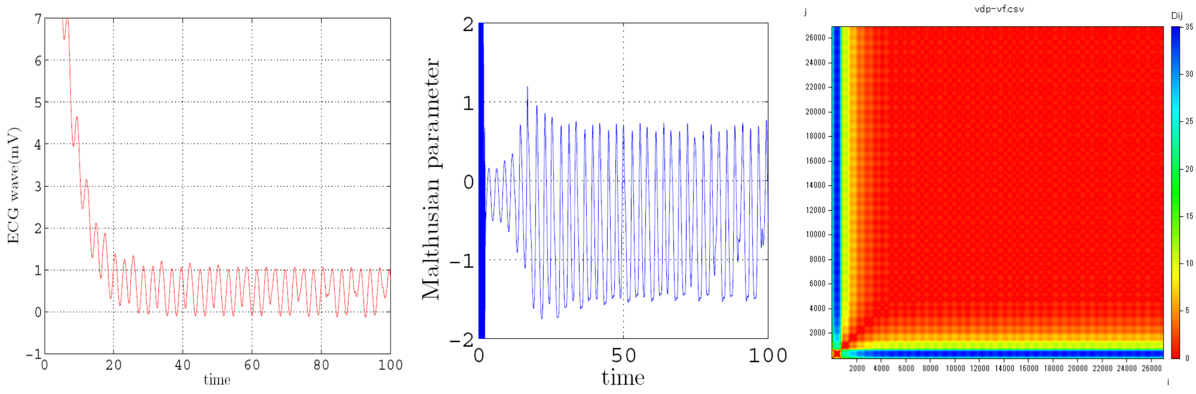
3.4. **Database: Physionet.** Cardiac electric signals on an intracellular level may be recorded with a microelectrode, which is inserted inside a cardiac muscle cell. The ECG is a measure of the extra-cellular electric behavior of the cardiac muscle tissue. The propagation wave-front of the cardiac electrical signal through the body presents a very complicated shape. PhysioBank databases [11] is a large and growing archive of well-characterized digital recordings of physiologic signals and related data for use by the biomedical research community [5]. We select a normal beat (data # 115), an SB beat (data # 232), and a VF beat (data # 219) to estimate the Malthusian parameter. The ECG beats include artifacts caused by an electrode misconnection, a both limb and a precordial, and an improper placement [13]. In this paper, we adopt the second-order band-pass filter with a lower cutoff frequency of 0.05 [Hz] and an upper cutoff frequency of 3 [Hz]. Moreover, the ECG signals are normalized to the maximum amplitude of 1.

4. **Results.** We calculate two indicators for heart beats: the Malthusian parameter and the recurrence plot using Sunday Chaos Time [10]. Figures 1-3 are the ECG signals generated by the mathematical model, the Malthusian parameter estimates, and the recurrence plots. Figure 1 is the result of the normal ECG, Figure 2 is of the VF, and Figure 3 is of the SB. Note that the ECG signal generated by the mathematical model includes a rise time. Figures 4-6 are the ECG signals generated by the mathematical model, the Malthusian parameter estimates, and the recurrence plots. Figure 4 is the result of the normal ECG, Figure 5 is of the VF, and Figure 6 is of the SB.



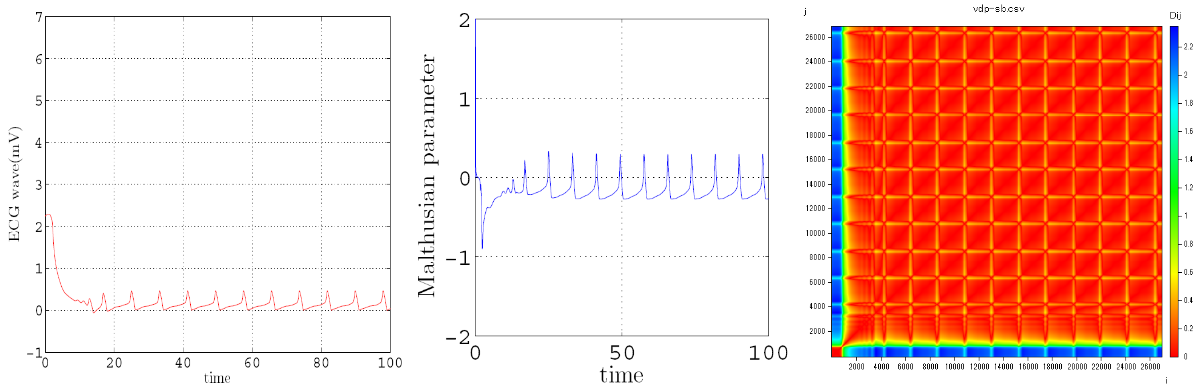
ECG signal | Malthusian parameter estimate | Recurrence plot

FIGURE 1. Normal ECG signal obtained by the mathematical model and the indicators



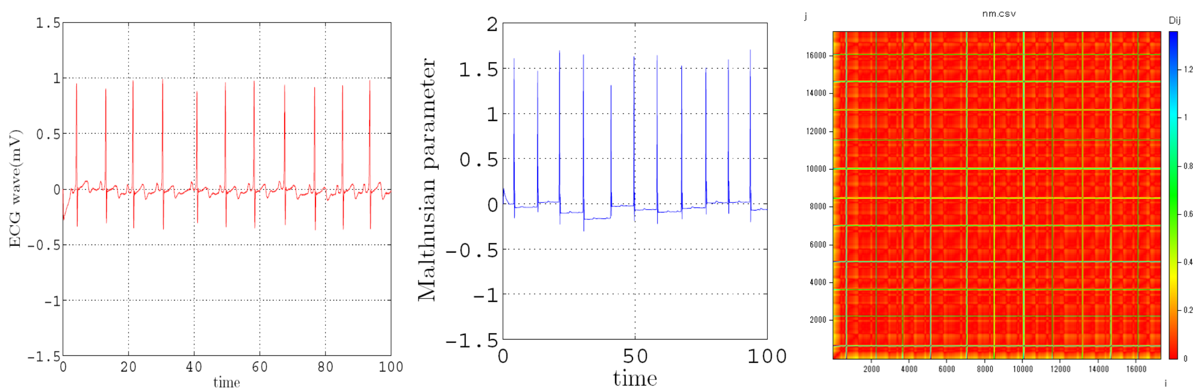
ECG signal | Malthusian parameter estimate | Recurrence plot

FIGURE 2. VF ECG signal obtained by the mathematical model and the indicators



ECG signal | Malthusian parameter estimate | Recurrence plot

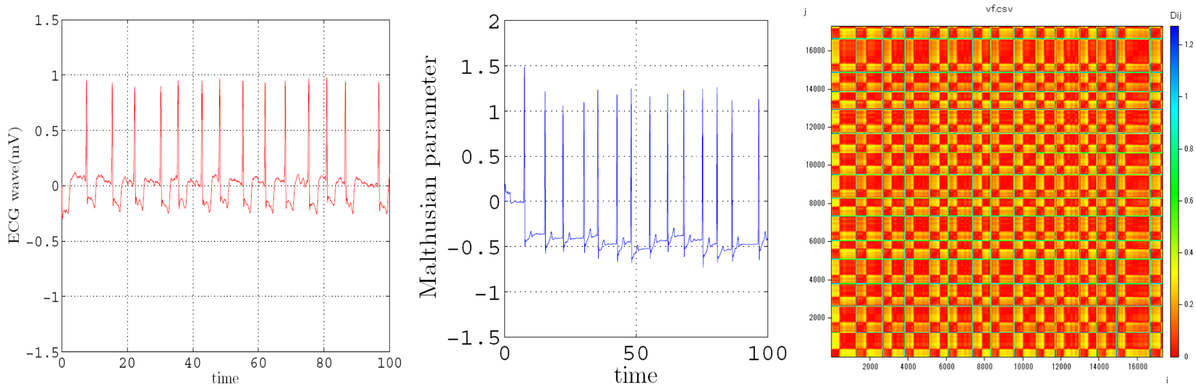
FIGURE 3. SB ECG signal obtained by the mathematical model and the indicators



ECG signal | Malthusian parameter estimate | Recurrence plot

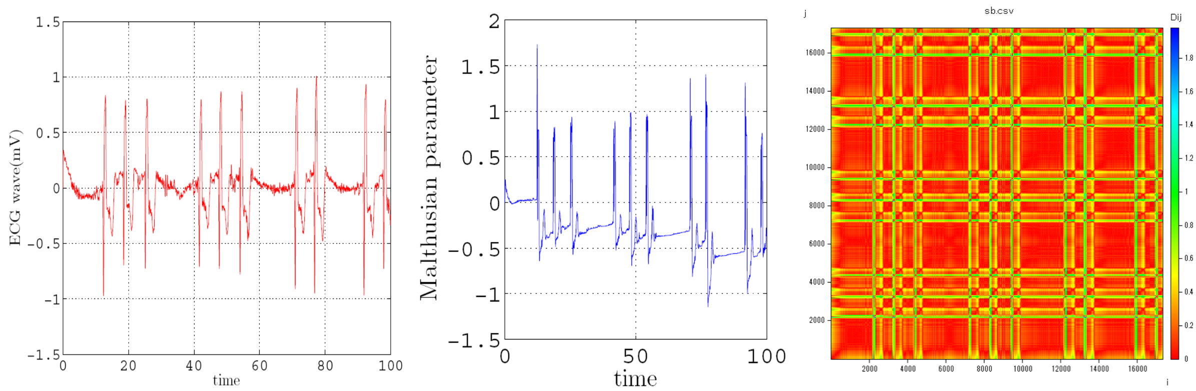
FIGURE 4. Normal ECG signal from the database and the indicators

The Malthusian parameter estimate emphasizes a peak signal and a different baseline for each ECG signal. As a result of the comparison of the Malthusian parameters, those of the mathematical model are different from those of the real data. This means that the mathematical model is not enough to approximate the heart rhythms. In the real data case, the Malthusian parameter can be used to distinguish among the N, the SB, and the



ECG signal | Malthusian parameter estimate | Recurrence plot

FIGURE 5. VF ECG signal from the database and the indicators



ECG signal | Malthusian parameter estimate | Recurrence plot

FIGURE 6. SB ECG signal from the database and the indicators

VF paying attention to the baseline and the interval of a peak. As another result of the recurrence plots, we can distinguish among the N, the VF, and the SB in the real data, but cannot find the difference between the N and the SB in the mathematical model.

5. Conclusions.

We proposed the decay/growth rate estimation method using the Malthusian parameter and applied it to the normal and abnormal ECG signals in PhysioBank databases. We compared the mathematical simulation data with the real data in the PhysioBank databases using the Malthusian parameter estimates and the recurrence plots. In comparing the simulation data with the real data, we may add the importance of the prefilter to remove artifacts.

Acknowledgment. The work was supported by the Grant-in-Aid for Scientific Research ((C)16K06421), Japan Society for the Promotion of Science.

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