ICIC Express Letters
Part B: Applications
Volume **9**, Number **1**, January **2018**

ICIC International ©2018 ISSN 2185-2766
pp. 11–16

MASTER-SLAVE SYNCHRONIZATION OF ZHANG AND LORENZ CHAOTIC SYSTEMS WITH UNCERTAIN PARAMETERS, AN ACTIVE NONLINEAR FEEDBACK CONTROLLER

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Received July 2017; accepted September 2017

Abstract. *In the present article, the synchronization and anti-synchronization task between Zhang chaotic system and Lorenz chaotic system is addressed. The parameters of both drive and response chaotic systems are considered known. Thus, an active nonlinear method is used for designing an appropriate feedback control law and parameters estimation law. The stability of dynamical errors and validity of the proposed method are verified by means of Lyapunov stability theorem. Furthermore, some numerical simulations are done to show the effectiveness of the proposed synchronization method.*
Keywords: Master-slave synchronization, Zhang chaotic system, Lorenz chaotic system, Active nonlinear feedback control

1. **Introduction.** Sensitivity to the initial conditions and non-predictable behavior of any chaotic system are prominent features of chaotic system, which makes them popular in investigations. Since Lorenz in [1] found his chaotic system, many potential applications arise from chaotic systems and also many chaotic systems have been investigated in physics, chemistry, electronics and secure communications. Lv chaotic system [2], Chen chaotic system [3], Chua chaotic system, Lorenz chaotic system [1] and Zhang chaotic system [4] are some of the studied chaotic systems.

The majority of synchronization approach can be classified into two main categories: master-slave synchronization methods and coupling methods. Master-slave methods are also called as leader-response systems, which are widely studied by the researchers. The ultimate goal of master-slave synchronization is to design a controller to force the motion trajectories of the slave state variables to track the paths of the salve system state variables. To this end, many synchronization methods are developed and studied in the last two decades. Active method [5,6], adaptive method [7-9], backstepping method [10,11], generalized method [12], phase method [13,14], sliding method [15-17] and projective method [18-22] are some of them. Among these methods, active method is a common control method, which plays an important role in many other synchronization methods. When the parameters of the system are considered known, the chaos synchronization between two chaotic systems can be easily done by means of active control method. In this paper, the synchronization problem of the Zhang chaotic system and Lv chaotic systems was firstly addressed. Chaos synchronization is carried out by introducing a new adaptive nonlinear feedback control law.

Motivated by the above discussions, synchronization and anti-synchronization between the Lorenz chaotic system and the Zhang chaotic system are addressed, in this paper. Section 2 gives some preliminaries and mathematical modeling. Then, the chaos synchronization between Zhang chaotic system, as the leader chaotic system and the Lv chaotic system as the response system is addressed in Section 2. An adaptive control law and a parameter estimation law are obtained based on the Lyapunov stability theorem and the adaptive control. Numerical simulations are presented in Section 3, in order to verify the effectiveness of the theoretical discussion given in Section 2. Finally, concluding remark is given in Section 4.

2. **Synchronization.** In this section, the synchronization problem between the Zhang chaotic system, as the master system and the Lorenz chaotic system as the slave system is addressed. Since the parameters of the systems are considered known, a nonlinear active feedback controller is designed to provide the master-slave synchronization problem.

Zhang chaotic system was recently introduced in [4], which is constructed based on the three-dimensional dynamical system with three state variables. Zhang chaotic system can be presented as follows:

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) - x_2 x_3 \\ \dot{x}_2 &= b x_1 - x_1^2 \\ \dot{x}_3 &= -c x_3 + x_2^2 \end{aligned} \tag{1}$$

where x_1, x_2 and x_3 represent the state variables of the system and a, b and c indicate the three positive constant values as, $a = 10$, $b = 30$ and $c = 6$. The phase portrait of the Zhang chaotic system (1) is shown in Figure 1, with initial system state variables as, $x_1 = 5$, $x_2 = 2$ and $x_3 = 30$. As can be seen the behavior of the Zhang chaotic system is chaotic. In addition, Lorenz in [1] has represented a chaotic system, which can be

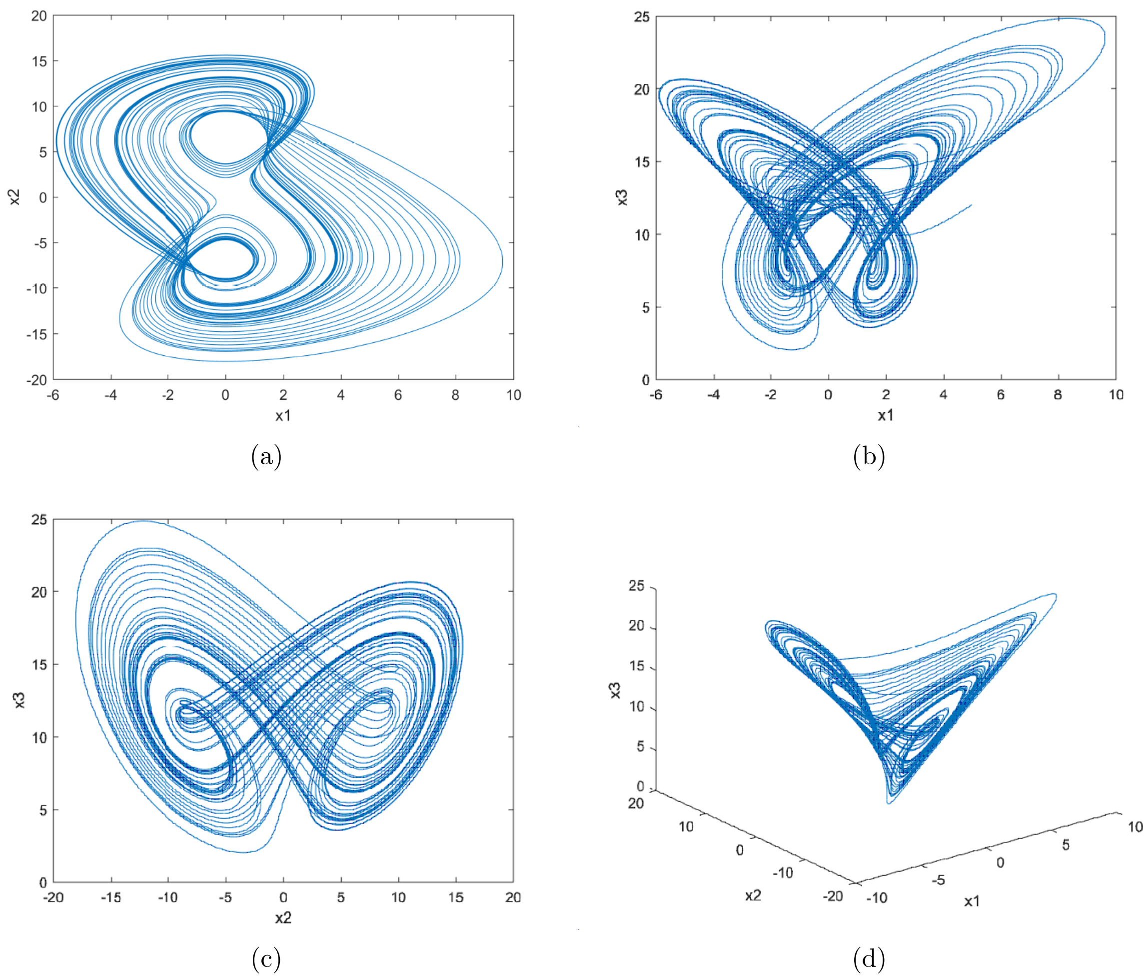

FIGURE 1. Time portrait of the Zhang chaotic system

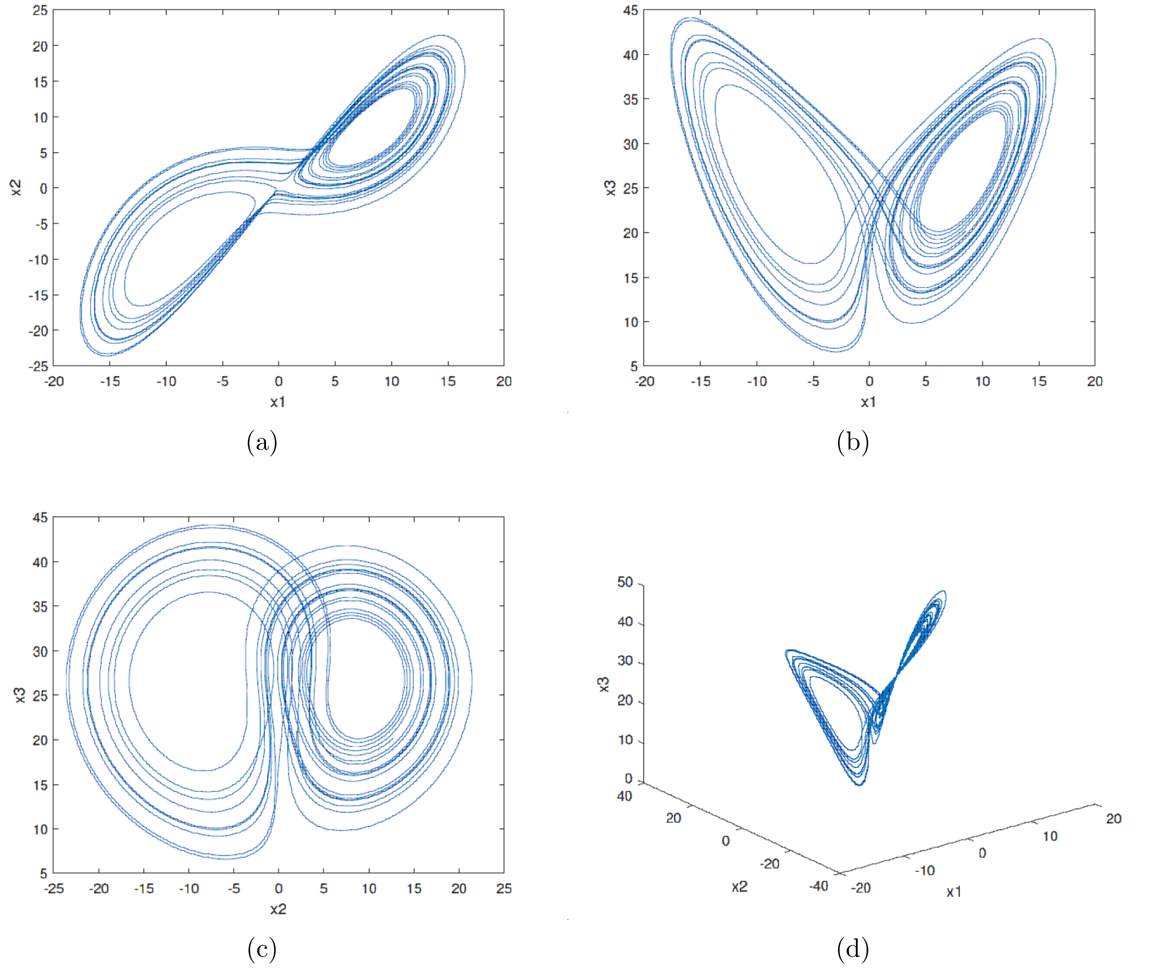

FIGURE 2. Time portrait of the Lorenz chaotic system

described as follows:

$$\begin{aligned} \dot{y}_1 &= -\alpha y_1 + \alpha y_2 \\ \dot{y}_2 &= \beta x_1 - y_2 - y_1 y_3 \\ \dot{y}_3 &= y_1 y_2 - \gamma y_3 \end{aligned} \tag{2}$$

where y_1, y_2 and y_3 are the three state variables of the system and α, β and γ are the parameters of the system. When $\alpha = 10$, $\beta = 28$ and $\gamma = 8/3$, the behavior of the Lorenz system (2) is chaotic. This chaotic behavior of the Lorenz system (2) is depicted in Figure 2, with initial system state variables as, $x_1 = 9$, $x_2 = 7$ and $x_3 = 1$.

Consider the Zhang chaotic system presented in (1) as the master chaotic system. Then the slave system can be defined based on the Lorenz chaotic system (2), as follows:

$$\begin{aligned} \dot{y}_1 &= -\alpha y_1 + \alpha y_2 + u_1 \\ \dot{y}_2 &= \beta x_1 - y_2 - y_1 y_3 + u_2 \\ \dot{y}_3 &= y_1 y_2 - \gamma y_3 + u_3 \end{aligned} \tag{3}$$

where u_1, u_2 and u_3 denote the feedback controller, which have to be designed.

Then the synchronization error between the master Zhang chaotic system (1) and the slave Lorenz chaotic system (2) can be defined as follows:

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3 \end{aligned} \tag{4}$$

From Equations (1), (3) and (4), the dynamics of system errors can be obtained as follows:

$$\begin{aligned}\dot{e}_1 &= \dot{y}_1 - \dot{x}_1 = -\alpha y_1 + \alpha y_2 + u_1 - a(x_2 - x_1) + x_2 x_3 \\ \dot{e}_2 &= \dot{y}_2 - \dot{x}_2 = \beta x_1 - y_2 - y_1 y_3 + u_2 - b x_1 + x_1^2 \\ \dot{e}_3 &= \dot{y}_3 - \dot{x}_3 = y_1 y_2 - \gamma y_3 + u_3 + c x_3 - x_2^2\end{aligned} \tag{5}$$

Definition 2.1. *For the master Zhang chaotic system (1) and the slave Lorenz chaotic system (3), it is said that the master-slave synchronization would be obtained if an appropriate feedback control law and a parameter estimation law are derived. Then the chaos synchronization would occur and the synchronization errors would be zero as time tends to infinity, meanly,*

$$\lim_{t\to\infty} |y_i - x_i| = 0 \quad \forall i = 1, 2, 3 \tag{6}$$

We need to obtain an appropriate feedback controller based on the nonlinear active control method. In the following theorem, an active feedback controller and a system parameter estimation law are given to provide the master-slave synchronization.

Theorem 2.1. *The master Zhang chaotic system presented in (1) with the state variables x_1, x_2 and x_3 and the system parameters a, b and c would be synchronized with the slave Lorenz chaotic system (3), and considering the active system state errors presented in (4), with the feedback controller defined as follows:*

$$\begin{aligned}u_1 &= +\alpha x_1 - \alpha y_2 + a(x_2 - x_1) - x_2 x_3 \\ u_2 &= -\beta x_1 + x_2 + y_1 y_3 + b x_1 - x_1^2 \\ u_3 &= -y_1 y_2 + \gamma x_3 + (\gamma - c) x_3 + x_2^2\end{aligned} \tag{7}$$

Substituting the control law in (7) in error dynamics (5), one can obtain:

$$\begin{aligned}\dot{e}_1 &= -\alpha e_1 \\ \dot{e}_2 &= -e_2 \\ \dot{e}_3 &= -\gamma e_3\end{aligned} \tag{8}$$

Proof: Let the Lyapunov stability theorem as follows:

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{9}$$

which is positive definite. Then, the derivative of V along the time domain would be obtained as follows:

$$\dot{V} = e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 \tag{10}$$

With considering the dynamical errors (7), the dynamical system (10) will be:

$$\dot{V} = -\alpha e_1^2 - e_2^2 - \gamma e_3^2$$

which is negative definite. Then the anticipated synchronization between Zhang chaotic system (1) as the master system and the Lorenz chaotic system (3), as the slave system would be achieved. By the other word, the synchronization errors converge to zero as $t \to \infty$. In the following section, some numerical results, related to the proposed synchronization scheme are given to verify the validity of the proposed method.

3. **Numerical Simulations.** Assume the parameters of the Zhang chaotic system, as the master system as: $a = 10$, $b = 30$ and $c = 6$, and the initial values for the master Zhang chaotic system (1) are taken as, $x_1(0) = 12$, $x_2(0) = 4$, and $x_3(0) = 7$. In additiion, the initial values of the slave Lorenz chaotic system (3) are selected as: $y_1(0) = 2$, $y_2(0) = 15$ and $y_3(0) = 0$.

The effectiveness of the proposed control method for synchronization of the Zhang chaotic system (1) and the Lorenz chaotic system (3) with known master and slave system

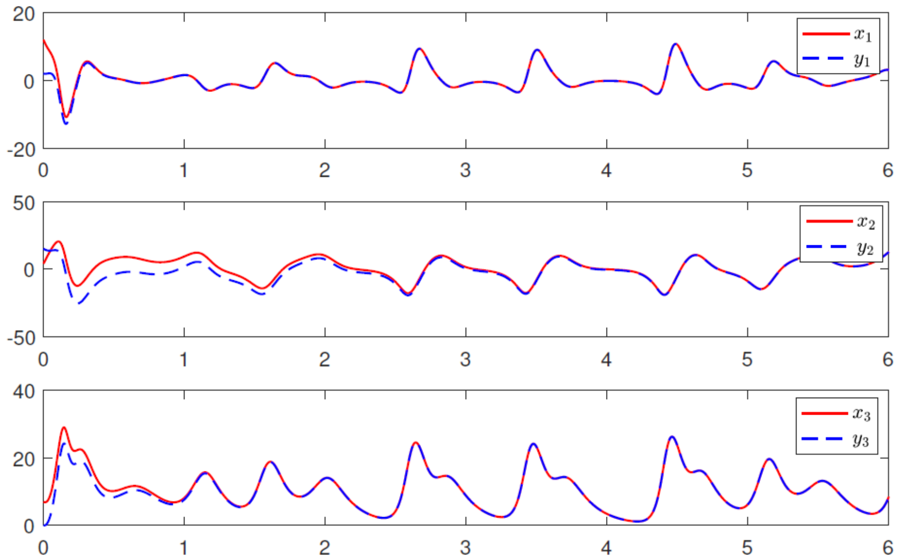

FIGURE 3. Motion trajectories of the state variables of the Zhang and Lorenz state variables

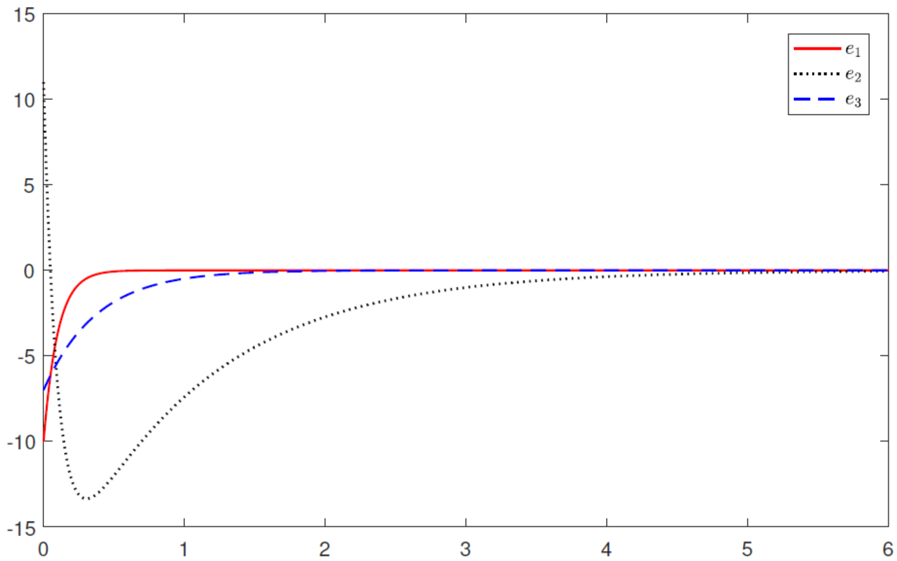

FIGURE 4. Synchronization errors between Zhang chaotic system (1) and Lorenz chaotic system (2)

parameters is shown in Figures 3 and 4. Figure 3 shows that the state variables of the system (1) converge to zero. In addition, Figure 4 exhibits that the errors between master and slave system state variables and its estimation values converge to zero.

4. **Conclusion.** In this paper, a nonlinear active control method for synchronization of Zhang chaotic system as the master system and the Lorenz chaotic system as the slave system is derived. The parameters of the drive chaotic system are considered known. Thus, an appropriate active feedback control is designed based on the Lyapunov stability theorem to force the motion trajectories of the slave Lorenz chaotic system to track the trajectories of the master Zhang chaotic system. Then, numerical simulations are carried out to verify the effectiveness of the proposed method. As can be seen from the simulated

results, the anticipated master-slave synchronization is achieved and the synchronization errors of the system state variables also converge to zero as time goes to the infinity.

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