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PARAMETER ESTIMATION FOR BURR TYPE XII DISTRIBUTION WITH DIFFERENTIAL EVOLUTION AND QUASI-NEWTON APPROACHES BASED ON PROGRESSIVELY TYPE I INTERVAL-CENSORED SAMPLES

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Abstract. *Burr type XII distribution (BXIID) has earned more attention in the few past decades because of the flexibility of distribution shape for model fitting. However, no analytical closed formula solutions of the maximum likelihood estimates (MLEs) of BXIID parameters can be obtained based on progressively type I interval-censored (PTIIC) samples. In this manuscript, the differential evolution algorithm (DE) and quasi-Newton method (QN) are applied to searching the MLEs of BXIID parameters based on PTIIC samples. The performance of DE and QN is evaluated by means of Monte Carlo simulations. Simulation results show that the DE outperforms QN in terms of smaller bias and mean squared error (MSE) of the MLEs.*
Keywords: Burr type XII distribution, Maximum likelihood estimation, Differential evolution method, Progressively type I interval-censored scheme, Quasi-Newton method

1. **Introduction.** BXIID was initially introduced by Burr [1]. Tadikamalla [2] established the connection of BXIID to some popular lifetime distributions. Because of having two shape parameters, the BXIID is particularly useful to model unimodal-distributed lifetime data. Studies about using BXIID for reliability inference with different censoring schemes can be found in [3-9]. The probability density function and cumulative distribution function of BXIID are defined, respectively, by

$$f(x|c,k) = ckx^{c-1}(1+x^c)^{-k-1}, \tag{1}$$

and

$$F(x|c,k) = 1-(1+x^c)^{-k},\ x>0,\ c>0,\ k>0, \tag{2}$$

where c is inner parameter and k is outer shape parameter. Denote the two-parameter BXIID, which is defined by Equations (1) and (2), by BXIID(c,k).

Because most of today's lifetime products are highly reliable, it is difficult to collect product lifetimes from life tests in affordable time with given budget. Censored schemes have been widely used to conduct life tests for saving testing time and cost. The time censoring scheme, failure censoring scheme and type I interval-censored scheme are three popular censoring schemes in reliability applications. The time censoring scheme and failure censoring scheme are also named the type I censoring scheme and type II censoring scheme, respectively. The type I censoring scheme allows experimenters to terminate life test at a predetermined time, say t_*. The exact lifetime of surviving units longer than t_* is censored and cannot be observed. The type II censoring scheme allows experimenters to terminate life test once a fix number of failure times are collected, say $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(m)}$. The exact lifetime of surviving units longer than $x_{(m)}$ is censored and cannot be observed.

Experimenters usually favor type I censoring scheme due to the merit of constant experimental time. When implementing a type I censored test, the experimenter may only count the number of failed units in non-overlap time intervals at the endpoints of the time intervals. This censoring scheme is named type I interval-censored scheme. The type I interval-censored scheme is easy for operation, but experimenters cannot obtain the exact failure times of units for making reliability inferences. To avoid using only shortest lifetimes of test units for conducting reliability inference, experimenters could occasionally remove some surviving units during the life testing. Progressively censoring schemes can be then introduced into life tests instead of using traditional censoring schemes, see [6-8,10].

2. **Motivations and Organization.** Considering administrative convenience for running a censoring test with more extreme lifetimes in a constant experimental time, we would like to study reliable parameter estimation procedures for the BXIID with PTIIC samples. Because no close form of the MLEs of the BXIID parameters can be found based on PTIIC samples, computation procedures, for example the QN, genetic algorithm (GA) and DE, can be used to search the MLEs of BXIID parameters.

QN uses the gradient and the Hessian matrix of second derivatives of the function to search MLEs. The QN is very sensitive to the initial inputs of BXIID parameters and may fail if practitioners cannot accurately set up the initial inputs of model parameters or the likelihood function is complicated. Tsai et al. [11] used GA to obtain reliable MLEs of BXIID parameters. They found that the GA based MLEs have smaller bias and MSE than the QN based MLEs. The GA, introduced by Holland [12] in 1975, is an evolutionary algorithm that generates solutions for optimizing the target function based on the technique inspired by natural evolution. However, the GA could not perform well for optimizing real number function.

Other competitive heuristic methods, for example the DE, can be better candidates to improve the estimation performance of QN and GA. DE uses evolutionary computation algorithm to optimize the target function by iteratively trying to improve candidate solution utilizing a specific measure of quality. Moreover, DE uses actual real number, and the ideas of mutation and crossover in DE are substantially different from that in the GA. In DE, the mutation and crossover operations create a new vector through using the difference between two or more vectors in the population such that the DE has higher opportunities to reach optimal solutions than GA in many instances. DE is a heuristic computation method and does not use gradient function to obtain optimal solutions. Hence, DE has good potential to replace the GA and QN methods to search reliable MLEs of the BXIID parameters based on PTIIC samples. Some recent studies to address the better strengths of using DE than GA for optimization can be found in [13-23]. Because of the aforementioned merits of DE, we are motivated to study the performance of using DE to obtain MLEs of the BXIID parameters based on PTIIC samples. Moreover, the performance comparison between using DE and QN to obtain reliable MLEs of BXIID parameters based on PTIIC samples is studied via simulations.

The rest of paper is organized as follows. Section 3 addresses the statistical model for the BXIID with PTIIC samples. Moreover, the principals to use QN and DE for searching the MLEs of BXIID parameters are also presented. An algorithm is provided in Section 4 to generate PTIIC samples, and the estimation performance of using DE and QN to search the MLEs of BXIID parameters is evaluated through using Monte Carlo simulations. In Section 5, some concluding remarks are given.

3. **Statistical Model and Differential Evolution Method.** A PTIIC sample can be collected as follows. A random sample of n units are drawn from BXIID(c, k) for life testing. The life test starts at $t_0 = 0$ and terminates at t_m. Count the number of failed units in each time interval at the scheduled times t_1, t_2, $\ldots$, t_m, respectively, and remove R_i surviving units at t_i. Let y_i denote the number of failed units in $(t_{i-1}, t_i]$ for $i = 1, 2, \ldots, m$. The likelihood function can be obtained (see [10]) by

$$L(c, k) \propto \prod_{i=1}^{m} [F(t_i|c, k) - F(t_{i-1}|c, k)]^{y_i} [1 - F(t_i|c, k)]^{R_i}, \tag{3}$$

where $F(t_0|c, k) = 0$, and the log-likelihood function can be presented by

$$\ell(c, k) \equiv \log[L(\theta)] = \sum_{i=1}^{m} \left[y_i \log \left[(1 + t_{i-1}^c)^{-k} - (1 + t_i^c)^{-k} \right] - kR_i \log(1 + t_i^c) \right]. \tag{4}$$

The MLEs $\hat{c}$ and $\hat{k}$ are the solutions to maximize $\ell(c, k)$ by

$$\left(\hat{c}, \hat{k} \right) = \arg \max_{c,k>0} \ell(c, k), \tag{5}$$

or $\hat{c}$ and $\hat{k}$ can be the solutions of the likelihood equations of $\partial \ell(c, k)/\partial c = 0$ and $\partial \ell(c, k)/\partial k = 0$. Because both the likelihood equations are complicated, no closed forms of maximizers for $\ell(c, k)$ in (4) can be obtained. Iterative numerical search methods are suggested to obtain the MLEs of c and k. The QN is the typical numerical computation method for statisticians or engineers to search the MLEs for maximizing the log-likelihood function in (4). Denote the QN based MLEs of c and k by $\hat{c}_M$ and $\hat{k}_M$, respectively. Two difficulties for obtaining the $\hat{c}_M$ and $\hat{k}_M$ are found.

(i) The QN uses the gradient and the Hessian matrix of second derivatives of the function to search MLEs such that the QN often fails to search the MLEs of c and k due to the terms $\log \left[(1 + t_{i-1}^c)^{-k} - (1 + t_i^c)^{-k} \right]$ and $\log(1 + t_i^c)$ could be divergent with undefined values. Moreover, it is risky that the QN could fall into the trap with undefined values of $\log \left[(1 + t_{i-1}^c)^{-k} - (1 + t_i^c)^{-k} \right]$ and $\log(1 + t_i^c)$ during the computation procedure.

(ii) The QN is sensitive to the initial inputs of c and k. However, the identification of initial inputs could be difficult in practical applications by using PTIIC samples.

To overcome these two difficulties with using QN to search the MLEs of BXIID(c, k) based on PTIIC samples, we consider DE as an alternative method to search the MLEs of c and k. The principal of DE is briefly given as follows.

1. Choose an initial population.
2. Determine the fitness of each individual.
3. Perform mutation.
4. Perform recombination.
5. Perform selection.
6. Determine the fitness of each individual.
7. The above process is repeated until a termination condition has been reached.

Common termination condition(s) can be one of the following conditions, or combinations of them.

1. A solution is reached to meet the specific criteria.

2. The fixed number of iterations is reached.
3. The allocated budget is reached.
4. The highest ranking solution's fitness is reached, or the solutions cannot be improved by successive iterations.

In this paper, we denote the DE based MLEs of c and k by $\hat{c}_D$ and $\hat{k}_D$, respectively.

4. **Simulations and Performance Comparison.** Because the DE uses actual real number and the ideas of the mutation and crossover operations in the DE create a new vector through using the difference between two or more vectors in the population, the DE has higher opportunities to reach optimal solutions than using GA in many instances. We extend the simulation scale of Tsai et al. [11] to study the performance of parameter estimation through using the DE and QN methods. In this paper, three removal schemes, removing items with constant probability at each check time point, removing items at the earliest stage of the life test and removing items at the end stage of the life test, are taken for the simulation study. We focus on the performance comparison through using the DE and QN for searching the MLEs of c and k based on the PTIIC samples with these three removal schemes.

The R package "*optim*" provides a general-purpose optimization based on the Nelder-Mead, quasi-Newton and conjugate-gradient algorithms. The optimization procedure of "*optim*" includes an option for box-constrained optimization and simulated annealing. In this paper, the R package "*optim*" with L-BFGS-B method is used to implement QN for searching the MLEs of BXIID(c, k). The R package "*DEoptim*", which was published in 2015, is used to implement the DE for searching the MLEs of BXIID(c, k). The package "*DEoptim*" provides global optimization of a real-valued function of a real-valued parameter vector. Algorithm I is used to generate PTIIC samples of $(y_1, y_2, \ldots, y_m)$ and $(R_1, R_2, \ldots, R_m)$.

Algorithm I. Generating PTIIC samples.

Step 1. Let $y_0 = R_0 = 0$. Set up inspection times of t_1, t_2, $\ldots$, and t_m, and withdraw probabilities of p_1, p_2, $\ldots$, and p_m, where $0 \leq p_j < 1$ for $j = 1, 2, \ldots, (m-1)$, and $p_m = 1$;

Step 2. Let $i = 0$ and $y_s = r_s = 0$;

Step 3. Let $i = i + 1$; Generate y_i from the binomial distribution that has sample size $(n - y_s - r_s)$ and the success probability

$$\delta_i = \frac{F(t_i|c, k) - F(t_{i-1}|c, k)}{1 - F(t_{i-1}|c, k)}; \tag{6}$$

Let $R_i = \left\lfloor p_i \times \left(n - \sum_{j=1}^{i-1}(y_j + R_j) - y_i\right)\right\rfloor$, where $\lfloor z \rfloor$ is the largest integer equal or smaller than z;

Step 4. Let $y_s = (y_s + y_i)$ and $r_s = (r_s + R_i)$;

Step 5. If $i < m$, go to Step 3; otherwise stop the algorithm.

To implement the DE, the probability of crossover 0.5, weighting factor 0.8 and the maximum iterations allowed 200 were used to implement the DE method. Because the page size limitation, we only display the simulation results for $m = 5$ in this paper. Let $(p_1, p_2, \ldots, p_5) = (0.05, 0.05, 0.05, 0.05, 1)$ denote Scheme I that removes surviving items with constant probability at each check time point, $(p_1, p_2, \ldots, p_5) = (0, 0, 0, 0.2, 1)$ denote Scheme II that removes surviving items at the end stage of the life test and $(p_1, p_2, \ldots, p_5) = (0.2, 0, 0, 0, 1)$ denote Scheme III that removes surviving items at the earliest stage of the life test. The parameter combinations $(c, k) = (3, 5)$ and $(2, 7)$ and the sample sizes $n = 30, 50, 100$ and 200 are used to conduct the simulation study. The performance comparison through using the DE and QN are evaluated based on the indices of bias and MSE with 10000 MLEs of $\left(\hat{c}_M, \hat{k}_M\right)$ and $\left(\hat{c}_D, \hat{k}_D\right)$.

The initial inputs of c and k are needed to implement the QN. However, practitioners could not have enough knowledge to set up the initial inputs of c and k. In the simulation study, the initial inputs $(c_0, k_0) = (1, 1)$, $(5, 5)$ and $(10, 10)$ are considered to implement the QN for searching the MLEs of c_M and k_M. Let $\hat{c}_M^I$ and $\hat{k}_M^I$ denote the QN based MLEs by using $c_0 = 1$ and $k_0 = 1$ as initial inputs, $\hat{c}_M^{II}$ and $\hat{k}_M^{II}$ denote the QN based MLEs by using $c_0 = 5$ and $k_0 = 5$ as initial inputs and $\hat{c}_M^{III}$ and $\hat{k}_M^{III}$ denote the QN based MLEs by using $c_0 = 10$ and $k_0 = 10$ as initial inputs. All simulation results are reported in Tables 1-6. In summary, we obtain the following results.

1. The DE based MLE beats the QN based MLE with smaller bias and MSE.
2. The bias and MSE are decreased as n increases for both the DE and QN.
3. The QN based MLEs are unreliable and probably result in huge MSE in many simulation cases.
4. To implement the DE, a PTIIC sample of size thirty for $m = 5$ is enough to obtain reliable MLEs of the BXIID(c, k).
5. No consistent conclusion regarding which removal scheme can produce most accurate MLEs. The impact of removal scheme depends on the sample size.

Overall, the log-likelihood function for the BXIID with PTIIC samples is complicated, and the QN often fails and cannot obtain reliable MLEs of the BXIID parameters. The DE is more efficient than the QN to obtain reliable MLEs of the BXIID parameters.

Table 1. Bias and MSEs of the MLEs for $(c, k) = (3, 5)$ with Scheme I

	Bias				MSE			
	$n = 30$	$n = 50$	$n = 100$	$n = 200$	$n = 30$	$n = 50$	$n = 100$	$n = 200$
$\hat{c}_D$	2.590	2.421	1.671	0.748	17.934	17.553	12.353	4.885
$\hat{c}_M^I$	6.172	5.477	3.055	0.794	222.480	179.765	86.279	15.300
$\hat{c}_M^{II}$	4.538	4.654	2.901	0.788	224.180	180.661	86.475	15.313
$\hat{c}_M^{III}$	4.832	4.862	2.966	0.800	226.580	182.582	87.112	15.442
$\hat{k}_D$	1.610	0.920	0.347	0.134	10.115	5.549	1.678	0.492
$\hat{k}_M^I$	9.643	5.140	1.198	0.179	257.840	135.852	28.093	1.894
$\hat{k}_M^{II}$	9.351	4.986	1.167	0.177	242.990	128.007	26.509	1.809
$\hat{k}_M^{III}$	11.679	6.211	1.415	0.190	373.400	196.855	40.420	2.548

5. **Conclusions.** In this paper, the DE and QN are used to obtain the MLEs of BXIID parameters based on PTIIC samples. An extensive simulation study is conducted to verify the estimation performance using these two computation procedures. The estimation performance is evaluated in terms of the bias and MSE. Simulation results show that the DE outperforms QN with smaller bias and MSEs.

The QN is sensitive to the initial inputs of parameters. Practitioners often lack enough knowledge to set accurate initial inputs of parameters. The selection of initial parameter inputs becomes a critical issue for using QN to obtain reliable MLEs, and this issue could be problematic for users to implement QN for numerical computation. DE is a heuristic computation method and does not use gradient function to search the MLEs of BXIID parameters. Based on the simulation results, we suggest that a PTIIC sample of size thirty with five failure times is enough to implement DE for obtaining reliable MLEs of the BXIID parameters. We also find that the impact of removal scheme depends on sample size, and no consistent conclusion regarding which removal scheme can produce

TABLE 2. Bias and MSEs of the MLEs for $(c, k) = (2, 7)$ with Scheme I

	Bias				MSE			
	$n = 30$	$n = 50$	$n = 100$	$n = 200$	$n = 30$	$n = 50$	$n = 100$	$n = 200$
$\hat{c}_D$	3.207	3.262	3.157	2.724	19.318	20.884	21.960	20.591
$\hat{c}_M^I$	4.575	4.828	4.703	3.963	102.050	108.766	104.352	87.357
$\hat{c}_M^{II}$	1.089	1.887	2.727	3.110	98.629	105.930	102.467	86.595
$\hat{c}_M^{III}$	1.381	2.171	2.996	3.306	99.560	107.211	104.361	88.456
$\hat{k}_D$	1.901	1.509	0.977	0.510	8.127	6.862	4.624	2.317
$\hat{k}_M^I$	18.248	15.419	10.524	4.812	445.690	379.229	260.211	117.530
$\hat{k}_M^{II}$	17.654	14.913	10.177	4.656	417.920	355.595	243.988	110.216
$\hat{k}_M^{III}$	22.382	18.937	12.939	5.900	663.770	564.822	387.602	174.956

TABLE 3. Bias and MSEs of the MLEs for $(c, k) = (3, 5)$ with Scheme II

	Bias				MSE			
	$n = 30$	$n = 50$	$n = 100$	$n = 200$	$n = 30$	$n = 50$	$n = 100$	$n = 200$
$\hat{c}_D$	2.638	2.426	1.615	0.701	18.058	17.411	11.694	4.520
$\hat{c}_M^I$	6.573	5.667	2.954	0.755	230.490	182.717	80.215	13.476
$\hat{c}_M^{II}$	4.830	4.751	2.768	0.745	232.180	183.690	80.402	13.486
$\hat{c}_M^{III}$	4.927	4.802	2.778	0.753	231.680	183.393	80.345	13.563
$\hat{k}_D$	1.610	0.920	0.346	0.134	10.114	5.548	1.678	0.492
$\hat{k}_M^I$	9.642	5.139	1.198	0.179	257.840	135.816	28.093	1.894
$\hat{k}_M^{II}$	9.350	4.985	1.167	0.177	242.990	128.007	26.509	1.810
$\hat{k}_M^{III}$	11.678	6.210	1.415	0.190	373.400	196.585	40.419	2.548

TABLE 4. Bias and MSEs of the MLEs for $(c, k) = (2, 7)$ with Scheme II

	Bias				MSE			
	$n = 30$	$n = 50$	$n = 100$	$n = 200$	$n = 30$	$n = 50$	$n = 100$	$n = 200$
$\hat{c}_D$	3.263	3.291	3.179	2.708	19.701	21.061	22.045	20.311
$\hat{c}_M^I$	4.581	4.889	4.905	4.055	98.571	107.388	107.271	88.142
$\hat{c}_M^{II}$	1.043	1.878	2.839	3.123	95.046	104.432	105.216	87.226
$\hat{c}_M^{III}$	1.240	2.046	2.954	3.179	94.394	103.869	104.852	87.165
$\hat{k}_D$	1.902	1.509	0.977	0.510	8.125	6.862	4.624	2.318
$\hat{k}_M^I$	18.248	15.419	10.523	4.812	445.690	379.230	260.211	117.530
$\hat{k}_M^{II}$	17.654	14.913	10.176	4.655	417.920	355.596	243.989	110.216
$\hat{k}_M^{III}$	22.382	18.937	12.938	5.900	663.770	564.823	387.602	174.956

most accurate MLEs. These findings can be valuable guidelines for practitioners to use DE for obtaining the MLEs of BXIID parameters based on PTIIC samples

Other computation methods, for example ant algorithm, evolution strategies and particle swarm optimization, could also be helpful to obtain reliable MLEs of the model

TABLE 5. Bias and MSEs of the MLEs for $(c, k) = (3, 5)$ with Scheme III

	Bias				MSE			
	$n = 30$	$n = 50$	$n = 100$	$n = 200$	$n = 30$	$n = 50$	$n = 100$	$n = 200$
$\hat{c}_D$	2.592	2.488	1.898	0.972	17.314	17.674	13.992	6.516
$\hat{c}_M^I$	6.046	5.601	3.666	1.212	231.520	195.827	112.186	27.697
$\hat{c}_M^{II}$	4.751	5.081	3.669	1.222	233.220	196.564	112.364	27.720
$\hat{c}_M^{III}$	5.641	5.795	4.000	1.259	244.430	205.536	116.505	28.163
$\hat{k}_D$	1.607	0.919	0.348	0.135	10.115	5.547	1.676	0.492
$\hat{k}_M^I$	9.643	5.140	1.198	0.179	257.840	135.817	28.093	1.894
$\hat{k}_M^{II}$	9.351	4.986	1.167	0.177	242.990	128.008	26.510	1.809
$\hat{k}_M^{III}$	11.679	6.211	1.416	0.190	373.400	196.585	40.420	2.548

TABLE 6. Bias and MSEs of the MLEs for $(c, k) = (2, 7)$ with Scheme III

	Bias				MSE			
	$n = 30$	$n = 50$	$n = 100$	$n = 200$	$n = 30$	$n = 50$	$n = 100$	$n = 200$
$\hat{c}_D$	3.201	3.241	3.203	2.899	18.942	20.288	21.615	21.301
$\hat{c}_M^I$	4.317	4.518	4.582	4.043	93.126	99.732	104.227	92.858
$\hat{c}_M^{II}$	0.971	1.784	2.882	3.443	89.977	97.274	102.862	92.580
$\hat{c}_M^{III}$	1.514	2.417	3.623	4.085	95.124	104.033	111.706	100.605
$\hat{k}_D$	1.900	1.508	0.976	0.510	8.130	6.864	4.625	2.317
$\hat{k}_M^I$	18.248	15.419	10.524	4.812	445.690	379.229	260.211	117.530
$\hat{k}_M^{II}$	17.654	14.914	10.177	4.656	417.920	355.595	243.988	110.217
$\hat{k}_M^{III}$	22.382	18.937	12.939	5.900	663.770	564.822	387.602	174.957

parameters. Moreover, optimal parameter design for implementing computation methods is also a critical issue. All these topics will be studied in the future.

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