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AN ADAPTATION MODEL WITH VARIABLE WINDOW SIZE FOR FUZZY TIME SERIES

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Abstract. *In this paper, a novel time variant forecasting model is proposed for predicting fuzzy time series. In the training phase, with fuzzy c-means clustering method and genetic algorithm, a kind of information granule generation technique is applied to optimizing the partition of the universe. And the proposed method can also dynamically tune the window size of defuzzification process and adaptive expectations for fuzzy time series by computing the deviation between the forecasting value and the actual value at each time. Then, in the testing phase, fuzzy inference is used to deduce the fuzzy description in the future. Based on the heuristic rule, corresponding window size can also be determined. Numerical simulation results show that the proposed method can achieve high forecasting accuracy for fuzzy time series.*
Keywords: Fuzzy time series, Information granule, Time variant forecasting algorithm

1. **Introduction.** Time series theory is a useful tool for forecasting and analyzing many dynamic models which can be described by a series of data, such as economic system, temperature model and production process. In some applications the historical data of each time is usually of linguistic value, not the real number. To track this difficulty, fuzzy time series is proposed in [1]. Some scholars apply fuzzy time series to solving various forecasting problem, for example, enrollment forecasting and stock indices forecasting.

In existing researches, appropriate fuzzy relationships have been established to improve the forecasting capabilities of fuzzy time series. In [2,3], fuzzy time series based on fuzzy relationship is constructed to solve the forecasting of enrollment problem. In order to make fuzzy relationship reflect the inherent law of time series, in [4] neural network is used to generate fuzzy rules. Besides, some scholars also investigate how to determine the appropriate parameters of fuzzy time series. In [5] the interval length of fuzzy time series is computed by optimization technology. In [6], information granules are used to determine the partition of the universe of discourse. And, some adaptive expectation models are introduced to modify the results of fuzzy time series [7-9].

In previous works, the orders or the size of windows of fuzzy time series are usually fixed. In practical application, fixed window size could not reflect the dynamic feature of the objective model relevantly. In [10,11], adaptive time variant model, in which the window size of time series can be tuned automatically by heuristic rules, is designed. Besides, the variable window approach has been used for the prediction of interval time series in [12,13]. However, for each step of model in [10,11], only two kinds of orders can be chosen to predict value in the next state. It may lose some information and limit the width of the window size. To deal with these problems and improve the forecasting accuracy of fuzzy time series, in this paper, a time variant forecasting model is established for fuzzy time series, in which the window size can be chosen among multiple values. Furthermore, a kind of adaptive expectation strategy based on corresponding variant window size is also introduced to determine forecast results.

This paper is organized as follows. In Section 2, some preliminary knowledge and definitions of fuzzy time series are introduced. In Section 3, a forecasting method with variant window size for fuzzy time series is presented. In Section 4, the proposed method is applied to forecasting several empirical time series. Some conclusions are summarized in Section 5.

2. **Preliminaries of Fuzzy Time Series.** In this section, we will introduce the basic concepts and denotations which are used in the paper.

Suppose that U is the universe of discourse and it is an interval. In the modeling process, U is partitioned into several subintervals, denoted by u_l ($l = 1, \ldots, s$). A fuzzy set A on U is defined as: $A = \mu_A(u_1)/u_1 + \mu_A(u_2)/u_2 + \cdots + \mu_A(u_s)/u_s$, where $\mu_A(u_l)$ indicates the degree of membership of u_l on fuzzy set A.

Definition 2.1. [1,2] *Let $Y(t)$ ($t = \ldots, 0, 1, 2, \ldots$), a subset of $\mathbb{R}^1$, be the universe of discourse on which fuzzy sets $f_i(t)$ ($i = 1, 2, \ldots$) are defined and $F(t)$ is the collection of $f_i(t)$ ($i = 1, 2, \ldots$). Then $F(t)$ is called a fuzzy time series on $Y(t)$ ($t = \ldots, 0, 1, 2, \ldots$).*

Definition 2.2. [1] *Suppose $F(t)$ is caused by $F(t-1)$, $F(t-2)$, …, $F(t-m)$, ($m > 0$). This relation can be expressed as the fuzzy relational equation:*

$$F(t) = (F(t-1) \times \cdots \times F(t-1)) \circ R(t, t-m). \tag{1}$$

Then, Equation (1) is called the m-th order model of $F(t)$.

If $F(t-1) = A_{l1}$, …, $F(t-m) = A_{lm}$ and $F(t) = A_j$, then fuzzy relation $R(t, t-m)$ can be represented by $R(t, t-m) = (A_{l1} \wedge \cdots \wedge A_{lm}) \rightarrow A_j$, where A_{lk} ($k = 1, \ldots, m$) and A_j are fuzzy sets respectively and "$\rightarrow$" is the implication operator. In the following, implication operator is usually chosen as "min" operator.

From Definition 2.1 and Definition 2.2, the mathematical mechanism of fuzzy time series can be concluded as follows. Firstly, each state $F(t)$ of fuzzy time series is represented by a fuzzy set A_l. Then, first order fuzzy relation or higher order fuzzy relation is chosen to describe the feature of the time series. Further, defuzzifying technology is applied to obtaining the actual output of the time series.

3. **A Variable Window Size Adaptation Forecasting Model for Fuzzy Time Series.** We will construct a novel time variant model for forecasting fuzzy time series in this section. There are mainly three issues for this model to be handled, i.e., determining the window size, optimizing the partition of the universe and computing the forecasting value.

3.1. **Determination of the window size.** The main processes for tuning the window size are presented as follows.

Window size determination method.

Step a-1. Set the maximum value of window size W, where W is a positive integer.

Step a-2. Set $t = 1$ and initial window size $w = 1$.

Step a-3. Use window size $w = 1$ to calculate the value of the next time.

Step a-4. Let $t = k$. If $k \leq W$ then the alternative window sizes $w \in \{1, \ldots, k\}$ are respectively used to forecast the actual value of the next time; else window sizes $w \in \{1, \ldots, W\}$ are used to compute the actual value. The value which can achieve the best forecasting accuracy is set to be the optimum window size at time t. Then, $t = t + 1$.

Step a-5. Repeat Step a-4 until time t reaches the end of the fuzzy time series.

In this way, the optimum window size of the fuzzy time series at each time can be obtained. Compared to the fixed order fuzzy time series, proposed model can dynamically determine the appropriate window size by measuring forecasting accuracy. Hence, this model is more suitable for describing the dynamic variation feature of the objective model. Different from the methods in [10,11], there are more choices in the proposed model which can improve the forecasting capability of the proposed model.

3.2. **Defuzzification and modification.** Then, we introduce the defuzzifying and modification method. Suppose that a fuzzy relationship is represented as $A_l \rightarrow A_j$, where A_l denotes the current state of time t, A_j denotes the forecasting state of time $t+1$. The window size of time t is w. $\left[* A_j \right]$ is the corresponding u_j for which makes membership function $A_j(x)$ take the maximum value. $L\left[* A_j \right]$ and $U\left[* A_j \right]$ are respectively the lower bound and the upper bound of interval u_j. $M\left[* A_j \right]$ is the mid-value of u_j. E_t is the actual value of time t. D_w reflects the corresponding order of differences in the past w times. F_{t+1} is the forecasting value of time $t+1$, which has been adjusted by the forecasting errors of past time with weight parameters h_k $(k = 1, \ldots, w)$.

Processes of defuzzifying and adjusting method:

if $w = 1$ then $F_{t+1} = E_t + h_1 \cdot \left(M\left[* A_j \right] - E_t \right)$;
if $w = 2$ then $D_w = ||E_t - E_{t-1}| - |E_{t-1} - E_{t-2}||$;
if $w > 2$ then $D_w = \left| |E_t - E_{t-1}| - \left(\sum_{k=1}^{w-2} \left| E_{t-k} - E_{t-(k+1)} \right| \right) + |E_{t-w+1} - E_{t-w}| \right|$
$R = 0$, $S = 0$;
for $r = 1/6, 1/4, 1/2, 1, 2, 3$, $X_{rt+1} = E_t + r \cdot D_w$, $Y_{rt+1} = E_t - r \cdot D_w$;
  if $X_{rt+1} \geq L\left[* A_j \right]$ and $X_{rt+1} \leq U\left[* A_j \right]$, then $R = R + X_{rt+1}$, $S = S + 1$;
  if $Y_{rt+1} \geq L\left[* A_j \right]$ and $Y_{rt+1} \leq U\left[* A_j \right]$, then $R = R + Y_{rt+1}$, $S = S + 1$;
end

$$\begin{aligned} F_{t+1} = \ & E_t + h_1 \cdot \left(\left(R + M\left[* A_j \right] \right) / (S+1) - E_t \right) \\ & + \sum_{k=2}^{w} h_k \cdot \left(E_{t+2-k} - E_{t+1-k} \right), \ h_k \in (-1, 1). \end{aligned} \tag{2}$$

In Expression (2), h_k $(k = 1, \ldots, w)$ are optimized by genetic algorithm.

From above method, we find that the forecasting value of fuzzy time series is determined by the subinterval, i.e., the partition points of the universe and the window size.

3.3. **Algorithms of proposed method.** In the following, we will give the detailed procedures for forecasting fuzzy time series from two aspects: training phase and testing phase. And information granular technology is applied to optimizing the division of the universe for improving the forecasting capability in the training phase.

Computing the forecasting value in the training phase.

Step b-1. Initialize some modeling parameters, including the maximum value of window size W, the partition number of subinterval for the universe s.

Step b-2. Use fuzzy c-means clustering method to determine the center of clusters, which are denoted by v_l $(l = 1, \ldots, s)$, where $a < v_1 < v_2 < \cdots < v_s < b$.

Step b-3. Determine partition points of the universe. For a given universe $U = [a, b]$, the subintervals are respectively denoted by $\left[\hat{x}_{l-1}^R, \hat{x}_l^R \right]$, where $\hat{x}_l^R$ $(l = 1, \ldots, s-1)$ are the partition points and $\hat{x}_0^R = a$, $\hat{x}_s^R = b$. Let D_l $(l = 1, \ldots, s)$ be the l-th cluster and v_l be the cluster center of D_l. By the principle of justifiable granularity [14], partition points are computed by the equation: $\hat{x}_l^R = \left(p_l^R + p_{l+1}^L \right)/2$, where p_l^R and p_{l+1}^L can be obtained by solving the following optimization problems:

$$p_l^R = \arg\max_{b_l} \left\{ Cov(b_l) \cdot Sp(b_l) \right\}, \quad p_{l+1}^L = \arg\max_{a_{l+1}} \left\{ Cov(a_{l+1}) \cdot Sp(a_{l+1}) \right\},$$

and

$$Cov(b_l) = \sum_{x_i \in (v_l, b_l] \cap D_l} \frac{b_l - x_i}{b_l - v_l}, \quad Sp(b_l) = 1 - \frac{1}{2} \cdot \frac{b_l - v_l}{\max\{D_l\} - v_l},$$

$$Cov(a_{l+1}) = \sum_{x_i \in [a_{l+1}, v_{l+1}) \cap D_{l+1}} \frac{x_i - a_{l+1}}{v_{l+1} - a_{l+1}}, \quad Sp(a_{l+1}) = 1 - \frac{1}{2} \cdot \frac{v_{l+1} - a_{l+1}}{v_{l+1} - \min\{D_{l+1}\}}.$$

Step b-4. Define fuzzy membership functions. For each interval $\left[\hat{x}_{l-1}^R, \hat{x}_l^R\right]$, we can define a fuzzy set A_l on it.

Step b-5. Use Steps a-1 to a-5 to deduce the window size at each time t.

Step b-6. Use Expression (2) to compute the forecasting value of the fuzzy time series.

Based on the above process, we can use the training data to obtain the optimized length of each subinterval and the window size at each time.

Then we will investigate how to compute the forecasting value in the testing phase. In this phase, the division of the universe and the corresponding membership function remain the same as them in the training phase. Assume that fuzzy relationship of time $t-1$ and time t is $A_l \to A_j$ and the window size at time t is w. The fuzzy relationship between time t and time $t+1$ is unknown, i.e., $A_j \to \#$, which means that the fuzzy set of time $t+1$ is unknown. In order to get it, we respectively choose A_j and $A_l \to A_j$ as the fuzzy input and fuzzy rule. By fuzzy inference method, we can deduce the fuzzy output, denoted by A^*. Approximately, we assume that A^* is the fuzzy state of time $t+1$. And the window size of time $t+1$ is equal to the window size of time t.

Suppose that there are M data in testing phase. The fuzzy set and window size at time t are known. However, they are unknown from time $t+1$ to time $t+M$. The basic procedures for computing the forecasting value in the testing phase are listed below.

Computing the forecasting value in the testing phase.

Step c-1. Let $i = 1$.

Step c-2. Establish fuzzy relationship of time $t+i-2$ to time $t+i-1$.

Step c-3. For above fuzzy relationship and fuzzy set of time $t+i-1$, use fuzzy inference to deduce the fuzzy set of time $t+i$.

Step c-4. Use Expression (2) to compute the forecasting value F_{t+i}, where the window size of time $t+i$ equals the one of time $t+i-1$.

Step c-5. $i = i+1$. If $i > M$, then stop; else go to Step c-2.

4. **Numerical Simulation.** In this section, we will utilize the proposed method to predict some time series including the enrollments at the University of Alabama and the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX), where the mean square error (MSE), the root mean square error (RMSE) and the average forecasting error rate (AFER) are used to evaluate the performance of proposed method, i.e.,

$$\text{MSE} = \sum_{i=1}^{n} \left(|E_i - F_i|^2 / n \right), \quad \text{RMSE} = \sqrt{\sum_{i=1}^{n} \left(|E_i - F_i|^2 / n \right)},$$

$$\text{AFER} = \frac{1}{n} \sum_{i=1}^{n} \left(|E_i - F_i| / E_i \right),$$

where E_i denotes the actual value, F_i denotes the forecasted value and n is the number of data in fuzzy time series.

Here, we choose the enrollments at the University of Alabama as an example to introduce how to use the proposed method to forecast fuzzy time series. For the parameters in GA, the crossover rate is 67%, and the crossover point is chosen according to a uniform probability distribution. The mutation rate is 0.05. In each iteration 20 chromosomes and 10 new chromosomes are selected to produce next generation.

Example 4.1. *Forecasting of enrollments at the University of Alabama.*

The universe of the fuzzy time series is $U = [13000, 20000]$. The maximum value of window size is $W = 5$. By information granular technology, seven subintervals can be obtained, i.e., $u_1 = [13000, 14614]$, $u_2 = [14614, 15429]$, $u_3 = [15429, 15803]$, $u_4 = [15803, 16703]$, $u_5 = [16703, 17075]$, $u_6 = [17075, 18653]$ and $u_7 = [18653, 20000]$.

Fuzzy sets based on the discrete universe $\{u_1, \ldots, u_7\}$ can be defined as follows:

$$A_1 = \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7},\ A_2 = \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7},$$
$$A_3 = \frac{0}{u_1} + \frac{0.5}{u_2} + \frac{1}{u_3} + \frac{0.5}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7},\ A_4 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0.5}{u_3} + \frac{1}{u_4} + \frac{0.5}{u_5} + \frac{0}{u_6} + \frac{0}{u_7},$$
$$A_5 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0.5}{u_4} + \frac{1}{u_5} + \frac{0.5}{u_6} + \frac{0}{u_7},\ A_6 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0.5}{u_5} + \frac{1}{u_6} + \frac{0.5}{u_7},$$
$$A_7 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0.5}{u_6} + \frac{1}{u_7}.$$

The fuzzy set and window size at each time are shown in Table 1. Comparisons among other methods under the same intervals are shown in Table 2. It can be seen that the proposed method can achieve higher forecasting accuracy in the training phase.

Besides, suppose that the enrollments from 1971 to 1989 are known, and we need to predict the enrollments from 1990 to 1992. When the number of the subintervals is 7, comparisons among different methods in the testing phase are shown in Table 3. The simulation results demonstrate the superiority of the proposed method.

Table 1. The fuzzy set and window size for each time

Year	Fuzzy Set	Window Size	Year	Fuzzy Set	Window Size
1971	A_1	1	1982	A_3	1
1972	A_1	1	1983	A_3	4
1973	A_1	2	1984	A_2	4
1974	A_2	2	1985	A_2	4
1975	A_3	1	1986	A_4	4
1976	A_2	4	1987	A_5	5
1977	A_3	2	1988	A_6	1
1978	A_4	4	1989	A_7	2
1979	A_5	1	1990	A_7	5
1980	A_5	4	1991	A_7	2
1981	A_4	2	1992	A_7	1

Table 2. Comparisons among other methods about enrollments forecasting

Model	[21]	[6]	[20]	[16]	[10]	[15]	[4]	Proposed method
MSE	324900	242930	183723	95306	85895	62976	32849	10575

Table 3. Comparisons among different methods in the testing phase

	Year				
Model	1990	1991	1992	RMSE	AFER (%)
Actual	19328	19337	18876	–	–
[17]	18685	19138	19176	425.46	1.98
[18]	18500	19500	19500	605.95	2.81
[10]	18970	19306	19315	327.54	1.45
Proposed method	19314	19339	19116	138.80	0.45

Further, to illustrate the forecasting capability of the proposed method in the testing phase, and to verify the validity of the method for some more complex empirical data sets, we apply it to handling TAIEX data in the next example.

Example 4.2. *TAIEX forecasting.*

Firstly, TAIEX data from 2000/11/02 to 2000/12/30 are considered. The number of subintervals is $s = 15$. The maximum window size is 4. The proposed method runs 10 times to get the average RMSE. Comparisons with other methods are shown in Table 4. From the simulation results, we find that the proposed method possesses higher forecasting capability.

Table 4. Comparisons of TAIEX from 2000/11/02 to 2000/12/30

Model	RMSE	AFER (%)
[18]	159.40	2.36
[19]	41.32	0.58
[10]	28.37	0.38
Proposed method	24.51	0.26

Then, we apply the proposed method to predicting TAIEX data from 1999 to 2004. In each year, the historical data of TAIEX from January to October are chosen as the training set, and the historical data in November and December are chosen as the testing set. The number of subintervals is chosen as $s = 14$. The maximum window size is 5. The proposed method runs 20 times to get the average RMSE. Comparison results are shown in Table 5. Simulation results show that the proposed method can also possess high forecasting capability in the testing phase.

Table 5. Comparisons results of TAIEX from 1999 to 2004

Method	Year						Average RMSE
	1999	2000	2001	2002	2003	2004	
[22]	116.64	123.62	123.85	71.98	58.06	57.73	91.98
[23]	103	154	120	77	54	85	98.83
[24]	109	152	130	84	56	116	107.83
[25]	104.99	124.52	114.66	64.79	53.63	52.96	85.93
Proposed method	99.32	123.38	114.73	67.12	52.38	53.51	85.07

5. **Conclusions.** In this paper, a novel time variant forecasting method for fuzzy time series is obtained. Interval fuzzy sets are established through solving a series data driven optimization problems based on the information granule theory. The window size of the fuzzy time series is not a constant here. At each time, the window size of defuzzification and adaptation can be dynamically tuned by computing the error between the actual value and the forecasting value. Some numerical examples show that the proposed method can achieve higher forecasting accuracy than other methods in the training phase and in the testing phase. In the future, the proposed method can be applied to solving the forecasting problem of fuzzy time series with multiple factors.

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