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ADAPTIVE NONSINGULAR FAST TERMINAL SLIDING-MODE FTC DESIGN FOR A CLASS OF NONLINEAR SYSTEMS WITH ACTUATOR FAULTS

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Abstract. *In this paper, an active fault-tolerant control (FTC) strategy is proposed for a class of nonlinear systems with actuator faults using the adaptive nonsingular fast terminal sliding-mode technique. Firstly, a second-order nonlinear systems mode is described. Then, a reconfigurable adaptive nonsingular fast terminal sliding-mode controller is designed to tackle the multiple constraints of actuator faults, external disturbances and model uncertainties. Next, the Lyapunov stability analysis shows that the designed fault tolerant controller can guarantee all state signals of the closed-loop systems convergent to zero in a finite time, and has a good fault tolerant capability. In the end, the proposed FTC method is applied to the attitude control of a spacecraft and simulation results demonstrate the effectiveness of the proposed FTC scheme.*
Keywords: Fault tolerant control, Nonlinear systems, Nonsingular fast terminal sliding-mode control, Adaptive control

1. **Introduction.** It is generally known that all kinds of faults caused by actuators, sensors, or other components, usually occur in many complex control systems. Once the faults occur, the faulty control systems may result in performance degradation even produce serious accidents [1,2]. Fault tolerant control (FTC), which usually consists of fault detection, fault isolation, fault estimation and accommodation components, has attracted considerable attention and has become of paramount importance.

In recent years, some FTC methods have been reported for the various linear or nonlinear systems. In [3], a new FTC method incorporating online control allocation (CA) has been developed to tackle actuator fault, unknown inertia moment and external disturbance. In [4], the longitudinal control problem is studied for the attitude systems of aircraft using the integral sliding mode control (SMC) allocation approach subject to actuator fault. In [5], the fault tolerant attitude tracking control is studied for a rigid spacecraft, and a finite time controller using SMC technology is proposed to accommodate four types of actuator failure. In [6], an improved integral-type sliding mode fault tolerant controller is proposed for compensating actuator failures without controller reconfiguration. Compared with the passive FTC methods described in [3-6], active FTC approach can compensate for faults either by selecting precomputed control laws or by synthesizing new control strategies online. In [7], an active FTC design method is developed for a nonlinear system based on non-singular terminal sliding mode control (NTSMC) and nonsingular fast terminal sliding mode control (NFTSMC), but the upper bound of unknown disturbance must be known in advance. In [8], an FTC approach is proposed for a class of

Takagi-Sugeno (T-S) fuzzy systems using the terminal sliding mode control technique. In [9], the variable structure reliable control (VSRC) issue is studied for a class of nonlinear systems; however, the uniformly ultimately boundedness of the closed loop systems can be only achieved. In [10], a robust controller is designed for the T-S fuzzy systems, but the tracking error of the plant cannot converge to zero within a finite time.

Based on the above descriptions, the active FTC problem is studied in this paper for a class of nonlinear systems with actuator faults using the adaptive nonsingular fast terminal sliding mode control (ANFTSMC) technique. The main contributions of this paper are stated as follows. (i) A novel ANFTSMC approach is proposed, which is robust against model uncertainties, external disturbances in presence of unknown actuator faults and avoid the singularity phenomena. (ii) The sliding mode surface designed in this paper can further improve the transient performance of the plant and ensure that the convergence rate is faster than the traditional SMC. (iii) The control approach designed in this paper is continuous and less chattering. The developed FTC method is independent of the upper bound of system uncertainties.

The rest of the paper is organized as follows. In Section 2, the second-order nonlinear systems mode with actuator fault is given. In Section 3, an active fault tolerant controller is designed by using adaptive nonsingular fast terminal sliding mode technique, and the closed-loop state stability of nonlinear systems under actuator fault are analyzed by using Lyapunov approach. In Section 4, a numerical example is shown to demonstrate the benefit of the developed FTC approach. Finally, the conclusion is given in Section 5.

2. **Problem Statement and Preliminaries.**

Consider a class of second-order nonlinear control systems, which can be described by the following

$$\dot{\mathrm{x}}_1 = \mathrm{x}_2 \tag{1}$$

$$\dot{\mathrm{x}}_2 = \mathrm{f}(\mathrm{x}) + G(\mathrm{x})\mathrm{u} + \mathrm{d} \tag{2}$$

where $\mathrm{x}_1 = [x_1, \ldots, x_n]^T \in R^n$, $\mathrm{x}_2 = [x_{n+1}, \ldots, x_{2n}]^T \in R^n$, and $\mathrm{x} = (\mathrm{x}_1, \mathrm{x}_2)^T$ represent the system states, $\mathrm{u} = [u_1, \ldots, u_m]^T \in R^m$ with $m \geq n$ are the control inputs, $\mathrm{d} = [d_1, \ldots, d_n]^T \in R^n$ represents the total uncertainties and disturbances in the model, and $\mathrm{f}(\mathrm{x}) \in R^n$ and $G(\mathrm{x}) \in R^{n\times m}$ are smooth functions with $\mathrm{f}(0) = 0$.

For the purpose of FTC design, the actuator loss of effectiveness fault is considered in this paper. For the dynamical systems (1) and (2) with the existence of actuator fault, the overall actuators can be divided into two groups: 1) health, H and 2) fault, F. It is assumed that only the actuators in group F are allowed to fail while all of the actuators in group H must be in healthy case during operation. Then, systems (1) and (2) can be rewritten as

$$\dot{\mathrm{x}}_1 = \mathrm{x}_2 \tag{3}$$

$$\dot{\mathrm{x}}_2 = \mathrm{f}(\mathrm{x}) + G_H(\mathrm{x})\mathrm{u}_H + G_F(\mathrm{x})\mathrm{u}_F + \mathrm{d} \tag{4}$$

where $G(\mathrm{x}) = (G_H(\mathrm{x})|G_F(\mathrm{x}))$ and $\mathrm{u} = (\mathrm{u}_H|\mathrm{u}_F)$.

In the rest of this paper, it is assumed that $\mathrm{u}_H \in R^k$, $\mathrm{u}_F \in R^{m-k}$, and $m \geq k \geq n$. In order to succeed in FTC design, the following assumptions are given.

Assumption 2.1. [7] *For all system state* $\mathrm{x} \in R^{2n}$*, the matrix* $G_H(\mathrm{x}) \in R^{n\times k}$ *has full row rank, namely,* $rank(G_H(\mathrm{x})) = n$.

Note that Assumption 2.1 means that there are sufficient healthy actuators to perform the FTC task. Moreover, the control input in the group F is diagnosed as

$$\mathrm{u}_F = \hat{\mathrm{u}}_F + \tilde{\mathrm{u}}_F \tag{5}$$

where $\hat{\mathrm{u}}_F$ and $\tilde{\mathrm{u}}_F$ denote the estimated control value and estimated error of the faulty actuator u_F, respectively.

Therefore, the systems (3) and (4) can be rewritten as:

$$\dot{\mathrm{x}}_1 = \mathrm{x}_2 \tag{6}$$

$$\dot{\mathrm{x}}_2 = \mathrm{f}(\mathrm{x}) + G_H(\mathrm{x})\mathrm{u}_H + G_F(\mathrm{x})\left(\hat{\mathrm{u}}_F + \tilde{\mathrm{u}}_F\right) + \mathrm{d} \tag{7}$$

Based on systems (6) and (7), an appropriate FTC law u_H will be organized, so that all states of the closed-loop systems converge to the origin in a finite amount of time, even when the actuators in F are detected and diagnosed as experiencing faults by FDD mechanism.

The following assumptions and lemma are mainly used for the controller design and stability analysis processes of the subsequent sections.

Assumption 2.2. *The desired output trajectory is continuous and bounded; moreover, there exists a known compact set* Ψ *such that*

$$[\mathrm{x}_1, \dot{\mathrm{x}}_1, \ddot{\mathrm{x}}_1] \in \Psi_d \tag{8}$$

Assumption 2.3. [7] *The total system uncertainties and faults satisfy the following condition:*

$$\|G_F(\mathrm{x})\tilde{\mathrm{u}}_F + d\| \leq \eta(\mathrm{x}, t) \tag{9}$$

where $\eta(\mathrm{x}, t)$ *is a nonnegative function.*

Lemma 2.1. *The extended Lyapunov description of finite-time stability with faster finite time convergence is given as* [11]*:*

$$\dot{V}(x) + \lambda_1 V(x) + \lambda_2 V^r(x) \leq 0, \ \forall\ t \geq t_0, \ V(x_{t_0}) \geq 0 \tag{10}$$

Then, for any given t_0*, the convergence time is given as:*

$$T_r \leq t_0 + \frac{1}{\lambda_1(1-r)} \ln \frac{\lambda_1 V^{1-r}(x_{t_0}) + \lambda_2}{\lambda_2} \tag{11}$$

where $\lambda_1 > 0$*,* $\lambda_2 > 0$ *and* $0 < r < 1$*.*

3. **Fault Tolerant Controller Design.** In this section, a novel sliding mode surface of ANFTSMC is firstly designed for the systems (6) and (7) to avoid singularity problem [13]:

$$S = \mathrm{x}_2 + l_1 \mathrm{x}_1 + l_2 \bar{S}(\mathrm{x}_1) \tag{12}$$

where $S = [S_1, \ldots, S_n]^T \in R^n$, $l_1 = \mathrm{diag}\{l_{11}, \ldots, l_{1n}\} \in R^{n\times n}$, $l_2 = \mathrm{diag}\{l_{21}, \ldots, l_{2n}\} \in R^{n\times n}$ and $l_{1i} > 0$, $l_{2i} > 0$ $(i = 1, \ldots, n)$ are positive scalar. $\bar{S}(\mathrm{x}) = \left[\bar{S}_1(\mathrm{x}), \ldots, \bar{S}_n(\mathrm{x})\right]^T \in R^n$ are defined as

$$\bar{S}(\mathrm{x}) = \begin{cases} h_1 \mathrm{x} + h_2 \mathrm{sign}(\mathrm{x})\mathrm{x}^2, & \text{if } \bar{S}_i \neq 0,\ |\mathrm{x}| < \epsilon; \\ \mathrm{x}^{\frac{a}{b}}, & \text{otherwise.} \end{cases} \tag{13}$$

$S_i = \mathrm{x}_{2i} + l_{1i}\mathrm{x}_{1i} + l_{2i}\mathrm{x}_{1i}^{\frac{a}{b}}$ is the sliding variable, $a > 0$, $b > 0$ are positive odd integers, and $0 < \frac{a}{b} < 1$. $h_1 = \left(2 - \frac{a}{b}\right)\epsilon^{\frac{a}{b}-1}$, $h_2 = \left(\frac{a}{b} - 1\right)\epsilon^{\frac{a}{b}-2}$, and $\epsilon > 0$ is a small constant.

Then, the time derivative of sliding variables along the trajectory of systems (6) and (7) is

$$\dot{S} = \dot{\mathrm{x}}_2 + l_1 \dot{\mathrm{x}}_1 + l_2 D_{\mathrm{x}_1} \dot{\mathrm{x}}_1 \tag{14}$$

where D_{x_1} is defined as follows

$$D_{\mathrm{x}_1} = \begin{cases} h_1 I_n + 2h_2 \mathrm{diag}(\mathrm{sign}(\mathrm{x}_{1i})\mathrm{x}_{1i}), & \text{if } \bar{S}_i \neq 0,\ |\mathrm{x}_{1i}| < \epsilon; \\ \dfrac{a}{b}\mathrm{diag}\left(\mathrm{x}_{1i}^{\frac{a}{b}-1}\right), & \text{otherwise.} \end{cases} \tag{15}$$

I_n is n-order unit matrix.

The reaching condition is selected as the following form:

$$\dot{S} = -\vartheta_1 S - \vartheta_2 \mathrm{sign}^{\frac{a}{b}}(S) \tag{16}$$

where $\vartheta_1 = \mathrm{diag}\{\vartheta_{11}, \ldots, \vartheta_{1n}\}$, $\vartheta_2 = \mathrm{diag}\{\vartheta_{21}, \ldots, \vartheta_{2n}\}$ are two diagonal matrix and $\vartheta_{1i} > 0$, $\vartheta_{2i} > 0$ are designed parameters. $\mathrm{sign}^{\frac{a}{b}}(\cdot)$ are defined as

$$\mathrm{sign}^{\frac{a}{b}}(S) = \left[|S_1|\mathrm{sign}^{\frac{a}{b}}(S_1), \cdots, |S_n|\mathrm{sign}^{\frac{a}{b}}(S_n)\right]^T \tag{17}$$

Assumption 3.1. *There exists an unknown constant $\chi \geq 0$, which makes the following formula hold:*

$$\|l_1\dot{\mathrm{x}}_1 + l_2 D_{\mathrm{x}_1}\dot{\mathrm{x}}_1\| \leq \chi\|\dot{\mathrm{x}}_1\| \tag{18}$$

Remark 3.1. *By considering Assumption 2.2 and Formula (15), it is easy to know that $\|D_{\mathrm{x}_1}\| \leq \chi_1$ is satisfied, so that $\|l_1\dot{\mathrm{x}}_1 + l_2 D_{\mathrm{x}_1}\dot{\mathrm{x}}_1\| \leq \chi\|\dot{\mathrm{x}}_1\|$ is satisfied.*

Theorem 3.1. *Consider the nonlinear systems (6) and (7). If the system is controlled by the proposed fault tolerant controllers (19) and (20)*

$$\mathrm{u}_H = -G_H^T\left(G_H G_H^T\right)^{-1} \cdot \sigma \tag{19}$$

$$\sigma = \mathrm{f}(\mathrm{x}) + G_F\hat{\mathrm{u}}_F + \frac{S}{\|S\|}\left(\hat{\chi}\,\|\dot{\mathrm{x}}_1\| + \hat{\eta}\right) + \vartheta_1 S + \vartheta_2\mathrm{sign}^{\frac{a}{b}}(S) \tag{20}$$

where

$$\dot{\hat{\eta}} = -\tau_1^2\hat{\eta} + \frac{1}{\kappa_1}\|S\|, \quad \dot{\tau}_1 = -\frac{1}{p_1}\tau_1; \quad \dot{\hat{\chi}} = -\tau_2^2\hat{\chi} + \frac{1}{\kappa_2}\|S\|\|\dot{\mathrm{x}}_1\|, \quad \dot{\tau}_2 = -\frac{1}{p_2}\tau_2$$

$\tau_i > 0$, $\kappa_i > 0$, $p_i > 0$ $(i = 1, 2)$ denotes the adaptation gain, then the stability of the system and the convergence of the tracking error to zero can be guaranteed in a finite time.

Proof: Define the error variables as $\tilde{\eta} = \hat{\eta} - \eta$, $\tilde{\chi} = \hat{\chi} - \chi$. Consider a Lyapunov function candidate

$$V = \frac{1}{2}S^T S + \frac{\kappa_1}{2}\tilde{\eta}^T\tilde{\eta} + \frac{\kappa_2}{2}\tilde{\chi}^T\tilde{\chi} + \frac{\kappa_1 p_1}{2}\eta^2\tau_1^2 + \frac{\kappa_2 p_2}{2}\chi^2\tau_2^2 \tag{21}$$

Taking the derivative of V along the systems (6) and (7), we obtain

$$\begin{aligned}
\dot{V} &= S^T\dot{S} + \kappa_1\tilde{\eta}^T\dot{\hat{\eta}} + \kappa_2\tilde{\chi}^T\dot{\hat{\chi}} + \kappa_1 p_1\eta^2\tau_1\dot{\tau}_1 + \kappa_2 p_2\chi^2\tau_2\dot{\tau}_2 \\
&= S^T\left[\dot{\mathrm{x}}_2 + l_1\dot{\mathrm{x}}_1 + l_2 D_{\mathrm{x}_1}\dot{\mathrm{x}}_1\right] + \kappa_1\tilde{\eta}^T\dot{\hat{\eta}} + \kappa_2\tilde{\chi}^T\dot{\hat{\chi}} + \kappa_1 p_1\eta^2\tau_1\dot{\tau}_1 + \kappa_2 p_2\chi^2\tau_2\dot{\tau}_2 \\
&= S^T\left[\mathrm{f}(\mathrm{x}) + G_H\mathrm{u}_H + G_F\left(\hat{\mathrm{u}}_F + \tilde{\mathrm{u}}_F\right) + d + l_1\dot{\mathrm{x}}_1 + l_2 D_{\mathrm{x}_1}\dot{\mathrm{x}}_1\right] - \kappa_1\tau_1^2\tilde{\eta}^T\hat{\eta} + \tilde{\eta}^T\|S\| \\
&\quad - \kappa_2\tau_2^2\tilde{\chi}^T\hat{\chi} + \tilde{\chi}^T\|S\|\|\dot{\mathrm{x}}_1\| - \kappa_1\eta^2\tau_1^2 - \kappa_2\chi^2\tau_2^2
\end{aligned} \tag{22}$$

Substituting fault tolerant controllers (19) and (20) into (22), we have

$$\begin{aligned}
\dot{V} &= S^T\left[\left(G_F\tilde{\mathrm{u}}_F + d - \frac{S}{\|S\|}\hat{\eta}\right) + \left(l_1\dot{\mathrm{x}}_1 + l_2 D_{\mathrm{x}_1}\dot{\mathrm{x}}_1 - \frac{S}{\|S\|}\hat{\chi}\|\dot{\mathrm{x}}_1\|\right) - \vartheta_1 S - \vartheta_2\mathrm{sign}^{\frac{a}{b}}(S)\right] \\
&\quad - \kappa_1\tau_1^2\tilde{\eta}^T\hat{\eta} + \tilde{\eta}^T\|S\| - \kappa_2\tau_2^2\tilde{\chi}^T\hat{\chi} + \tilde{\chi}^T\|S\|\|\dot{\mathrm{x}}_1\| - \kappa_1\eta^2\tau_1^2 - \kappa_2\chi^2\tau_2^2 \\
&= \left[S^T\left(G_F\tilde{\mathrm{u}}_F + d - \frac{S}{\|S\|}\hat{\eta}\right) + \tilde{\eta}^T\|S\|\right] + \left[S^T\left(l_1\dot{\mathrm{x}}_1 + l_2 D_{\mathrm{x}_1}\dot{\mathrm{x}}_1 - \frac{S}{\|S\|}\hat{\chi}\|\dot{x}_1\|\right)\right. \\
&\quad \left. + \tilde{\chi}^T\|S\|\|\dot{\mathrm{x}}_1\|\right] - \vartheta_1 S^T S - \vartheta_2 S^T\mathrm{sign}^{\frac{a}{b}}(S) - \kappa_1\tau_1^2\tilde{\eta}^T\hat{\eta} - \kappa_2\tau_2^2\tilde{\chi}^T\hat{\chi} - \kappa_1\eta^2\tau_1^2 - \kappa_2\chi^2\tau_2^2 \\
&= \left[S^T\left(G_F\tilde{\mathrm{u}}_F + d\right) - \eta\|S\|\right] + \left(l_1 S^T\dot{\mathrm{x}}_1 + l_2 S^T D_{\mathrm{x}_1}\dot{\mathrm{x}}_1 - \|S\|\chi\|\dot{\mathrm{x}}_1\|\right) - \vartheta_1 S^T S \\
&\quad - \vartheta_2 S^T\mathrm{sign}^{\frac{a}{b}}(S) - \kappa_1\tau_1^2\tilde{\eta}^T\hat{\eta} - \kappa_2\tau_2^2\tilde{\chi}^T\hat{\chi} - \kappa_1\eta^2\tau_1^2 - \kappa_2\chi^2\tau_2^2 \\
&\leq \|S\|\left(\|G_F\tilde{\mathrm{u}}_F + d\| - \eta\right) + \|S\|\left(\|l_1\dot{\mathrm{x}}_1 + l_2 D_{\mathrm{x}_1}\dot{\mathrm{x}}_1\| - \chi\|\dot{\mathrm{x}}_1\|\right) - \vartheta_1 S^T S \\
&\quad - \vartheta_2 S^T\mathrm{sign}^{\frac{a}{b}}(S) - \kappa_1\tau_1^2\tilde{\eta}^T\hat{\eta} - \kappa_2\tau_2^2\tilde{\chi}^T\hat{\chi}
\end{aligned}$$

$$\begin{aligned}
&\le -\vartheta_1 S^T S - \vartheta_2 S^T \mathrm{sign}^{\frac{a}{b}}(S) - \frac{\kappa_1 \tau_1^2}{2}\left(\tilde{\eta}^T\tilde{\eta} + \eta^T\eta\right) - \frac{\kappa_2\tau_2^2}{2}\left(\tilde{\chi}^T\tilde{\chi} + \chi^T\chi\right) \\
&\le -2\min(\vartheta_{1i})\left(\frac{1}{2}S^T S\right) - \min(\vartheta_{2i})2^{\left(\frac{a}{b}+1\right)/2}\left(\frac{1}{2}S^T S\right)^{\left(\frac{a}{b}+1\right)/2} \\
&\le -2\left(\min(\vartheta_{1i}) - \frac{\Omega}{V}\right)V - (2)^{\left(\frac{a}{b}+1\right)/2}\left(\min(\vartheta_{2i}) - \left(\frac{\Omega}{V}\right)^{\left(\frac{a}{b}+1\right)/2}\right)V^{\left(\frac{a}{b}+1\right)/2} \le 0
\end{aligned} \tag{23}$$

where $\Omega = \frac{\kappa_1}{2}\tilde{\eta}^T\tilde{\eta} + \frac{\kappa_2}{2}\tilde{\chi}^T\tilde{\chi} + \frac{\kappa_1 p_1}{2}\eta^2\tau_1^2 + \frac{\kappa_2 p_2}{2}\chi^2\tau_2^2$, and $\frac{\Omega}{V} < 1$, $\left(\frac{\Omega}{V}\right)^{\left(\frac{a}{b}+1\right)/2} < 1$. Therefore, the closed-loop system is stable in the Lyapunov sense.

According to Lemma 2.1 and (23), the convergence time satisfies the following inequality:

$$T_S \le t_0 + \frac{2}{\gamma_1\left(1 - \frac{a}{b}\right)}\ln\frac{\gamma_1 V^{\left(1-\frac{a}{b}\right)/2}(S_{t_0}) + \gamma_2}{\gamma_2} \tag{24}$$

where t_0 is the initial time and $V(S_{t_0})$ is the initial value, and $\gamma_1 = 2\left(\min(\vartheta_{1i}) - \frac{\Omega}{V}\right)$, $\gamma_2 = 2^{\left(\frac{a}{b}+1\right)/2}\left(\min(\vartheta_{2i}) - \left(\frac{\Omega}{V}\right)^{\left(\frac{a}{b}+1\right)/2}\right)$.

Remark 3.2. *In order to eliminate chattering in the proposed fault tolerant controllers (19) and (20), the sign function sign(x) can be replaced by the saturation function sat(x), sat(x) is defined as follows:*

$$sat(x,\varsigma) = \begin{cases} 1, & x > \varsigma; \\ \dfrac{x}{\varsigma}, & |x| \le \varsigma; \\ -1, & x < -\varsigma. \end{cases} \tag{25}$$

where $\varsigma > 0$ is a small positive constant. Meanwhile, we apply the linear function $\frac{S}{\|S\|+p}$ to replacing the nonlinear function $\frac{S}{\|S\|}$ in the controller (20), where $p > 0$ is a small positive constant, thus can help to eliminate chattering.

Remark 3.3. *In practice, due to unfavorable effects such as system uncertainties, actuator faults, external disturbances and bias on state measurements, it is not possible to achieve the objective $S = 0$. In the presence of above unfavorable effects, system state finally converges into small neighborhood of origin, and the neighborhood is small enough with large enough parameters ϑ_{1i} and ϑ_{2i} $(i = 1, \ldots, n)$. Simultaneously, the larger parameters ϑ_{1i} and ϑ_{2i} $(i = 1, \ldots, n)$ provide the faster convergence rate.*

Remark 3.4. *Through analysis of the sliding mode surface designed in* [8-10]*, it is noted that the main results obtained in* [8-10] *cannot avoid singularity phenomenon, which can be resolved by the designed sliding mode surface (12) in this paper.*

Remark 3.5. *Compared with power reaching law selected in* [7,10]*, the reaching law (16) selected in this paper is essentially a combination of the traditional power reaching law and the exponential reaching law, which is an improvement to the traditional power reaching law. It can make the state variables of plant reach the designed sliding surface (12) in a shorter time.*

4. **Simulation Results.** The dynamic model of a spacecraft attitude systems described in [9] has the same form as (1) and (2) with $n = 3$, $m = 4$, in which $\mathrm{x} = (\mathrm{x}_1, \mathrm{x}_2)^T$, $\mathrm{x}_1 = (x_1, x_2, x_3)^T = (\phi, \theta, \psi)^T$, $\mathrm{x}_2 = (x_4, x_5, x_6)^T = \left(\dot{\phi}, \dot{\theta}, \dot{\psi}\right)^T$, $\mathrm{u} = [u_1, u_2, u_3, u_4]^T$ and $\mathrm{d} = [d_1, d_2, d_3]^T$, where ϕ, θ and ψ are Eulers angles for the x, y, and z axes, respectively.

u denote actuators, providing the force torques by reaction wheels or thrusters in four directions, and f(x) and G(x) are computed as follows:

$$G = \begin{bmatrix} 0.67 & 0.67 & 0.67 & 0.67 \\ 0.69 & -0.69 & -0.69 & 0.69 \\ 0.28 & 0.28 & -0.28 & -0.28 \end{bmatrix}$$

$$\begin{aligned}
f_1(x) &= \omega_0 x_6 cx_3 cx_2 - \omega_0 x_5 sx_3 sx_2 + \frac{I_y - I_z}{I_x}\Bigg[x_5x_6 + \omega_0 x_5 cx_1 sx_3 sx_2 + \omega_0 x_5 cx_3 sx_1 \\
&\quad + \omega_0 x_6 cx_3 cx_1 + \frac{1}{2}\omega_0^2 s(2x_3)c^2x_1 sx_2 + \frac{1}{2}\omega_0^2 c^2x_3 s(2x_1) - \omega_0 x_6 sx_3 sx_2 sx_1 \\
&\quad - \frac{1}{2}\omega_0^2 s^2x_2 s^2x_3 s(2x_1) - \frac{1}{2}\omega_0^2 s(2x_3) sx_2 s^2x_1 - \frac{3}{2}\omega_0^2 c^2x_2 s(2x_1)\Bigg] \\
f_2(x) &= \omega_0 x_6 sx_3 cx_1 + \omega_0 x_4 cx_3 sx_1 + \omega_0 x_6 cx_3 sx_2 sx_1 + \omega_0 x_5 sx_3 cx_2 sx_1 + \omega_0 x_4 sx_3 sx_2 cx_1 \\
&\quad + \frac{I_z - I_x}{I_y}\Bigg[x_4x_6 + \omega_0 x_4 cx_1 sx_3 sx_2 + \omega_0 x_4 cx_3 sx_1 - \omega_0 x_6 sx_3 cx_2 \\
&\quad - \frac{1}{2}\omega_0^2 s(2x_2)s^2x_3 cx_1 - \frac{1}{2}\omega_0^2 cx_2 sx_1 s(2x_3) + \frac{3}{2}\omega_0^2 s(2x_2)cx_1\Bigg] \\
f_3(x) &= \omega_0 x_4 sx_1 sx_3 sx_2 - \omega_0 x_6 cx_1 cx_3 sx_2 - \omega_0 x_5 cx_1 sx_3 cx_2 + \omega_0 x_6 sx_3 sx_1 - \omega_0 x_4 cx_3 cx_1 \\
&\quad + \frac{I_x - I_y}{I_z}\Bigg[x_4x_5 + \omega_0 x_4 cx_3 cx_1 - \omega_0 x_4 sx_3 sx_2 sx_1 - \omega_0 x_5 sx_3 cx_2 \\
&\quad - \frac{1}{2}\omega_0^2 s(2x_3)cx_2 cx_1 + \frac{1}{2}\omega_0^2 s^2x_3 sx_1 s(2x_2) - \frac{3}{2}\omega_0^2 s(2x_2)sx_1\Bigg]
\end{aligned}$$

where I_x, I_y, and I_z are the moments of inertia with respect to the three body axes, ω_0 denotes the constant orbital rate, and c and s denote the cos and sin functions, respectively.

The original condition and disturbance model are chosen as $\mathrm{x}(0) = (-0.6, 0.3, -0.02, -0.1, -0.1, -0.07)^T$ and $\mathrm{d} = 0.05 \cdot (\sin(t), \cos(2t), \sin(3t))^T\mathrm{N{\cdot}m}$, respectively. Suppose that the actuator u_2 fails to work at $t = 4$s. In order to eliminate the chattering phenomena in the proposed FTC scheme, the saturation function (25) can replace the sign function. Other parameters are designed as follows: $\varsigma = p = 0.002$, $l_1 = 0.5I_3$, $l_2 = 0.5I_3$, $m = 3$, $n = 5$, $\vartheta_1 = 10I_3$, $\vartheta_2 = I_3$, $\epsilon = 0.0001$.

In the simulation, the fault diagnosis approach developed in [9] can be used to obtain the estimated value of unknown actuator fault. It was shown that the observer described by Equations (10) and (11) in [9] can estimate the output value of the faulty actuator. The corresponding simulation comparisons are given in this section between the FTC method proposed in this paper and the FTC result proposed in [7]. Figures 1 and 2 show the system state output response curves, which indicate that the proposed ANFTSMC-based fault tolerant controller has a good fault tolerant capability under actuator fault case, compared to the NFTSMC proposed in [7], and ANFTSMC in this paper results in the faster state convergence rate. Figures 3 and 4 show the three sliding variables and four control inputs responses curves, respectively. In Figure 3, the FTC controller developed in this paper can make the state variables reach the sliding surface in a finite time, and the time is shorter than NFTSMC in [7]. In Figure 4, the ANFTSMC-based FTC approach provides smooth control inputs compared with the result obtained in [7]. From the simulation results, it can demonstrate the benefit of the FTC scheme proposed in this study.

5. Conclusions.

In this paper, the FTC problem for a class of nonlinear control systems is studied based on a novel ANFTSMC method. The finite time convergence of all

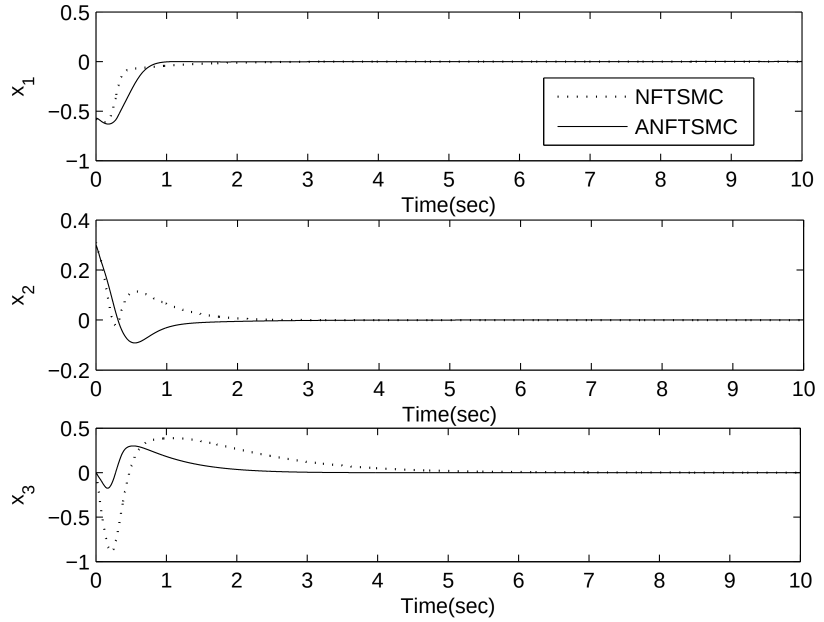

FIGURE 1. The system states output response curves of x_1, x_2, x_3

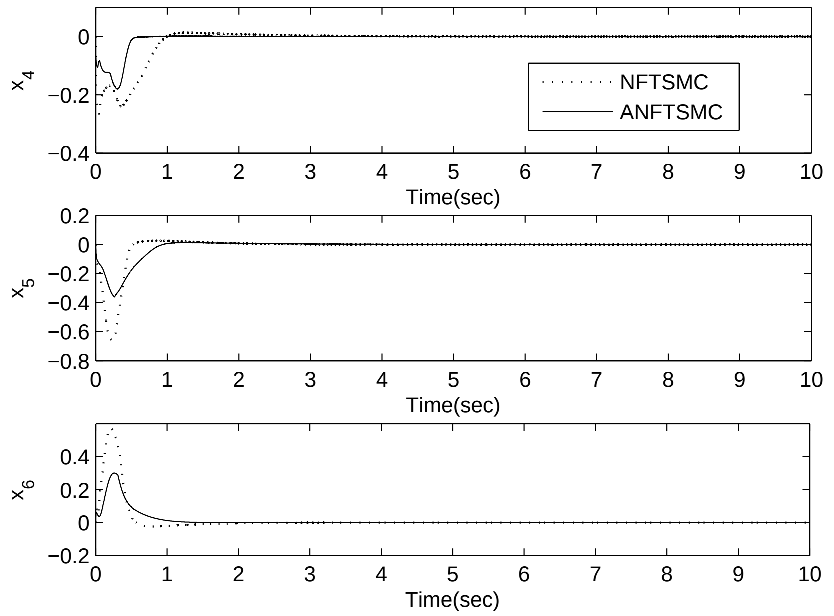

FIGURE 2. The system states output response curves of x_4, x_5, x_6

closed-loop signals has been proved using Lyapunov approach. The design novel sliding mode surface can avoid potential singularity phenomena that exist in traditional faster terminal sliding mode surface. Finally, simulation example on spacecraft attitude systems demonstrates the superiority of our FTC approach. In our future study, we will try to design a finite time fault tolerant controller against actuator saturation for the faulty nonlinear control systems.

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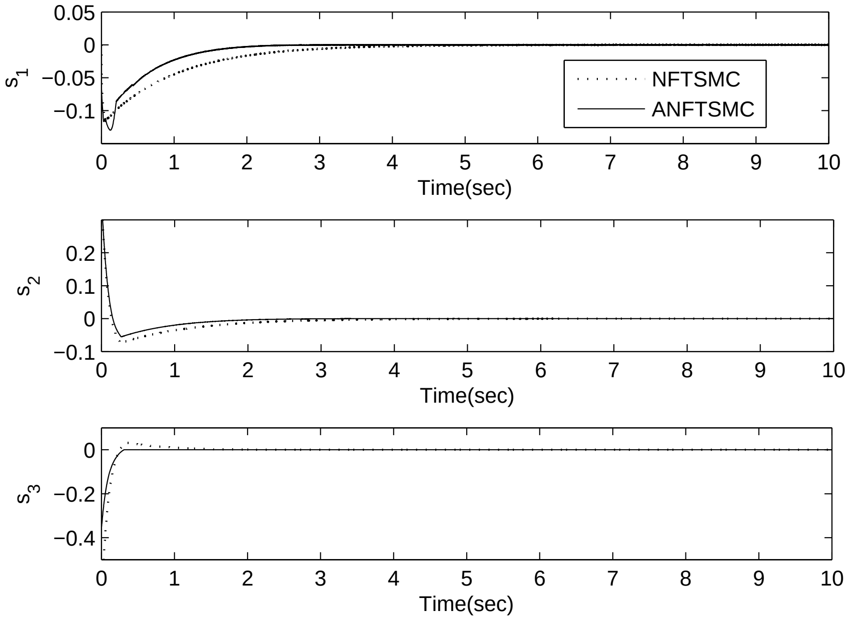

FIGURE 3. The three sliding variables output response curves of s_1, s_2, s_3

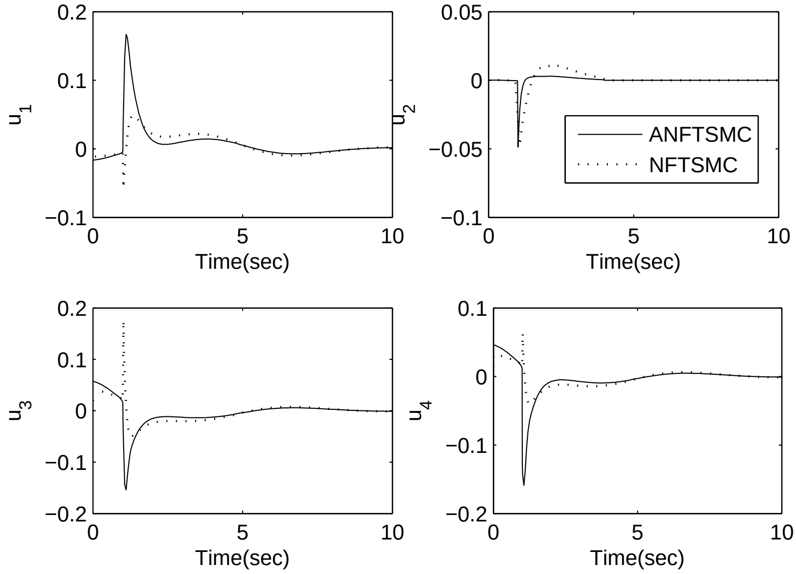

FIGURE 4. The four control inputs response curves of u_1, u_2, u_3, u_4

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