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ADAPTIVE MODIFIED PROJECTIVE SYNCHRONIZATION AND PARAMETER ESTIMATION OF LORENZ CHAOTIC SYSTEM AND RÖSSLER CHAOTIC SYSTEM

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Abstract. *In this paper, the chaos synchronization problem between the Lorenz chaotic system as the leader system and the Rössler chaotic system as the follower system is addressed. During this paper, the parameters of the Lorenz chaotic system are considered unknown and are estimated by the parameter of the Rössler chaotic system. An adaptive control law and a parameter estimation law are presented based on the modified projective synchronization (MPS) and Lyapunov candidate theorem. Then, the validity of the proposed method is studied by Lyapunov stability theorem, analytically. After that, some numerical simulations are carried out to show the effectiveness of the proposed method. The numerical results verify the accuracy and convergence speed of the proposed method.*
Keywords: Rössler chaotic system, Lorenz chaotic system, Lyapunov stability theorem

1. **Introduction.** Chaos synchronization problem between two (hyper) chaotic systems have been widely investigated by the researchers, due to its vast applications in many scientific areas, including physics, chemistry, electronics and secure communications. Up to now, many synchronization methods have been designed and investigated to perform synchronization task between two identical/non-identical chaotic systems. Active method [1,2], adaptive method [3-5], backstepping method [6,7], generalized method [8], phase method [9,10], sliding method [11-13] and projective method [14-17] are some of them. Among the numerous synchronization schemes by researchers, projective synchronization has been extensively noticed due to its proportional system errors up to a scaling factor. The follower chaotic system state variables can track the motion trajectories of the leader chaotic system by a scaling factor. Projective synchronization includes many synchronization methods. When scaling factor is set to $+1$, complete synchronization (CS) achieves. On the other hand, when the scaling factor is set to -1, anti-synchronization (AS) would be achieved. When the scaling function is a diagonal matrix with different elements in its diagonals, modified projective synchronization (MPS) achieves [18-22]. Furthermore, when the diagonal elements of the scaling matrix are functions of time t, the modified function synchronization (MFS) achieves.

In this paper, the synchronization problem between a typical Lorenz chaotic system as the leader system and a Rössler chaotic system as the follower system is studied via MPS method. The main highlights of this paper are as follows.

- A new adaptive modified projective synchronization scheme is derived to achieve synchronization.
- Stability analysis of the proposed method is verified by means of Lyapunov stability theorem.
- Some numerical simulations are performed to validate the analytical discussions.

The rest of this paper is organized as follows. In Section 2, the synchronization problem between Lorenz chaotic system and Rössler chaotic system is addressed. An adaptive feedback controller and a parameter estimation strategy are introduced based on the Lyapunov stability theorem. After that, in Section 3, some numerical simulations are given to verify the theoretical discussions of the proposed method. Finally, some concluding remarks are given in Section 4.

2. **MPS Method.** In this section, the Lorenz chaotic system and the Rössler chaotic systems are described. Furthermore, and their chaotic behavior is shown by some figures.

$$\begin{cases} \dot{x}_1 = -ax_1 + ax_2 \\ \dot{x}_2 = bx_1 - x_2 - x_1x_3 \\ \dot{x}_3 = x_1x_2 - cx_3 \end{cases} \tag{1}$$

The phase portrait of the dynamical Lorenz system (1) is shown in Figure 1, with system parameters $a = 11$, $b = 27$ and $c = 2.7$. It is clear, from these figures that the behavior of the Lorenz system is chaotic.

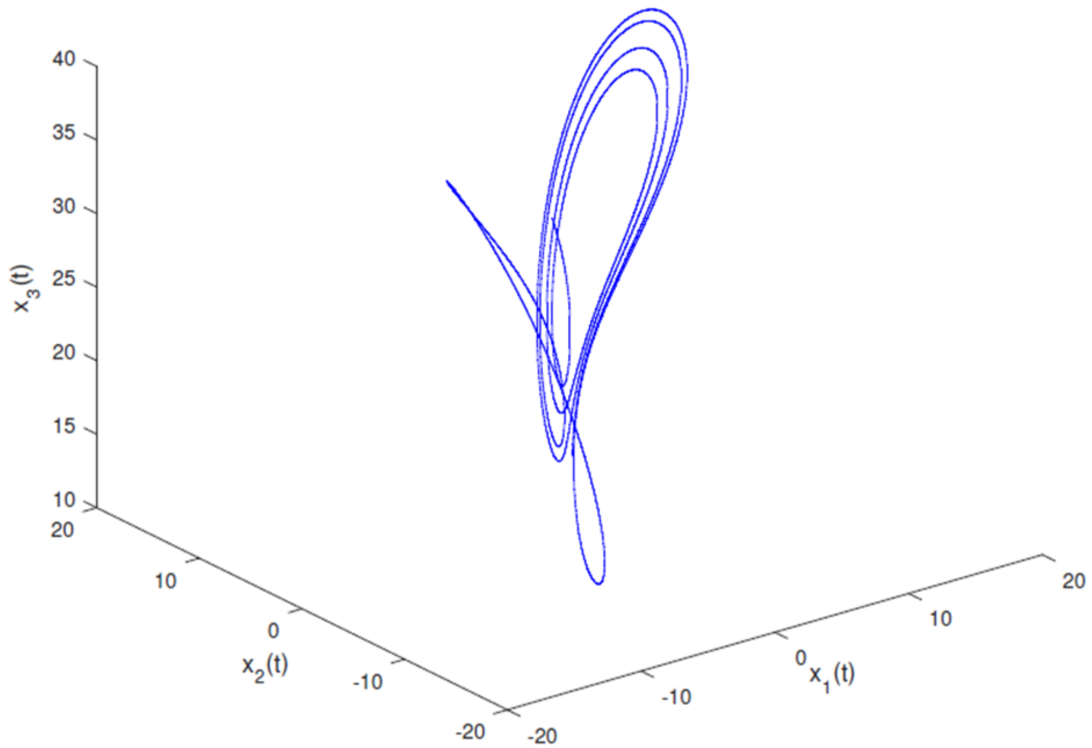


FIGURE 1. Time portrait of the Lorenz chaotic system

Rössler in [24] proposed a chaotic attractor that can be presented by a three simple nonlinear integer-based differential equations that depends on the three positive coefficient parameters as follows:

$$\begin{aligned} \dot{y}_1 &= -(y_2 + y_3) \\ \dot{y}_2 &= y_1 + ay_2 \\ \dot{y}_3 &= b + y_3(y_1 - c) \end{aligned} \tag{2}$$

where $\dot{y}_1$, $\dot{y}_2$ and $\dot{y}_3$ are the time derivatives of the state variables y_1, y_2 and y_3 of the chaotic system, respectively. The parameters of the system are denoted by a, b and c. The dynamical Rössler system (1) shows chaotic behavior for a wide variety amount of initial state values.

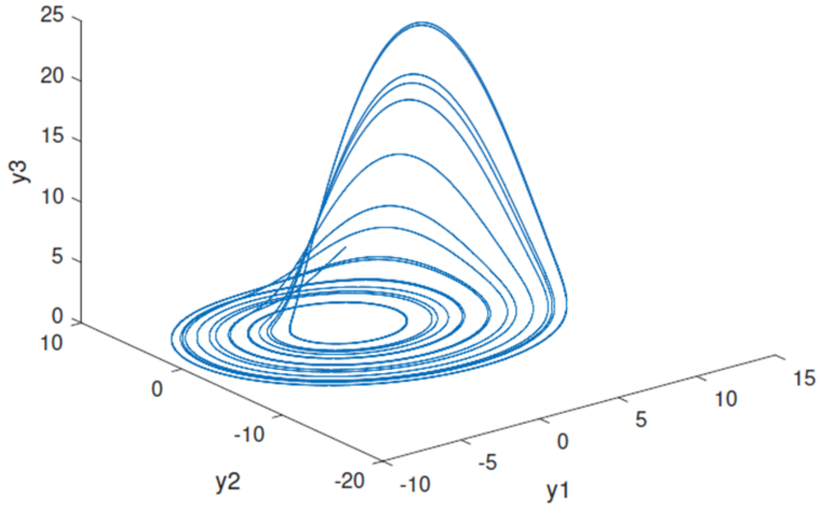


FIGURE 2. Time portrait of the Rössler chaotic system

The phase portrait of the dynamical Rössler system (2) is shown in Figure 2, with system parameters $a = 0.2$, $b = 0.2$ and $c = 5.7$. It is clear, from these figures that the behavior of the Rössler system (2) is chaotic.

Consider the Lorenz chaotic system (1) as the leader chaotic system. Then its follower chaotic system can be described based on the Rössler chaotic system (2) as follows:

$$\begin{aligned} \dot{y}_1 &= -(y_2 + y_3) + u_1 \\ \dot{y}_2 &= y_1 + (a + \Delta a)y_2 + u_2 \\ \dot{y}_3 &= (b + \Delta b) + y_3(y_1 - (c + \Delta c)) + u_3 \end{aligned} \tag{3}$$

where Δa, Δb and Δc indicate the amount of disparity of the unknown system parameters a, b and c in the leader chaotic system (1), respectively. u_1, u_2 and u_3 denote the feedback controllers, which have to be designed. Let the leader and follower system errors be:

$$\begin{aligned} e_1 &= y_1 - \sigma_1 x_1 \\ e_2 &= y_2 - \sigma_2 x_2 \\ e_3 &= y_3 - \sigma_3 x_3 \end{aligned} \tag{4}$$

where σ_1, σ_2 and σ_3 are the modified projective scaling factors, which align the synchronization between the leader chaotic system (1) and the follower system (2). Then the dynamical representation of system errors (4) can be obtained as follows:

$$\begin{aligned} \dot{e}_1 &= \dot{y}_1 - \sigma_1 \dot{x}_1 \\ \dot{e}_2 &= \dot{y}_2 - \sigma_2 \dot{x}_2 \\ \dot{e}_3 &= \dot{y}_3 - \sigma_3 \dot{x}_3 \end{aligned} \tag{5}$$

The ultimate goal of chaos synchronization is to design a control law and necessary parameter estimation law, in such way to force the state variables of the follower chaotic system (3) to track the motion trajectories of the leader chaotic system (1). In other word, the synchronization errors (3) converge to zero, meanly,

$$\lim_{t\to\infty} e_i = \lim_{t\to\infty} (y_i - \sigma_i x_i) = 0$$

In the following theorem a new feedback controller and a parameter estimation law are given to provide the leader and follower synchronization practically.

Theorem 2.1. *The Lorenz chaotic system (1) with unknown state variables x_1, x_2 and x_3 and unknown system parameters a, b, c is globally and exponentially synchronized by* *considering the modified projective synchronization errors defined in (4) and the controller and parameter estimation laws are defined as follows:*

$$\begin{aligned} u_1(t) &= y_2 + y_3 + \sigma_1(a + \Delta a)(x_2 - x_1) - k_1 e_1 \\ u_2(t) &= -y_1 - (a + \Delta a)y_2 + \sigma_2((b + \Delta b)x_1 - x_2 - x_1 x_3) - k_2 e_2 \\ u_3(t) &= -(b + \Delta b) - y_3(y_1 - (c + \Delta c)) + \sigma_3(x_1 x_2 - (c + \Delta c)) - k_3 e_3 \end{aligned} \tag{6}$$

and,

$$\begin{aligned} \dot{\Delta a} &= -e_1 \sigma_1 \Delta a(x_2 - x_1) \\ \dot{\Delta b} &= -e_2 \sigma_2 \Delta b x_1 \\ \dot{\Delta c} &= +e_3 \sigma_3 \Delta c x_3 \end{aligned} \tag{7}$$

where Δa, Δb and Δc are the disparity amount of the system parameters.

Proof: System errors stability is a sufficient condition for synchronization task of leader Lorenz choatic system (1) and the follower Rössler chaotic system (3). To this end, a typical Lyapunov candidate function is provided based on system errors and parmaeter estimation errors. Let us consider the Lyapunov candidate function as follows:

$$V(t) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + (\Delta a)^2 + (\Delta b)^2 + (\Delta c)^2\right) \tag{8}$$

It is clear that V is positive definite. The time derivative of the Lyapunov candidate function (8) can be described as follows:

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + \Delta a \Delta' a + \Delta b \Delta' b + \Delta c \Delta' c \tag{9}$$

With substituting the dynamical error system (5) and the proposed controller (6) and the parameter estimation (7), the dynamical Equation (9) can be simplified as follows:

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - \phi_1(\Delta a)^2 - \phi_2(\Delta b)^2 - \phi_3(\Delta c)^2 \tag{10}$$

When k_i $(i = 1, 2, 3)$ and ϕ_i $(i = 1, 2, 3)$ are positive constants, the dynamical representation of the Lyapunov candidate function (10) is negative definite. This means that the anticipated synchronization between the leader Lorenz chaotic system (1) and the follower Rössler chaotic system (3) will be achieved, based on the Lyapunov stability theorem. So the theorem is proved, namely, $\lim |E_s(t)| \to 0$ as time tends to infinity.

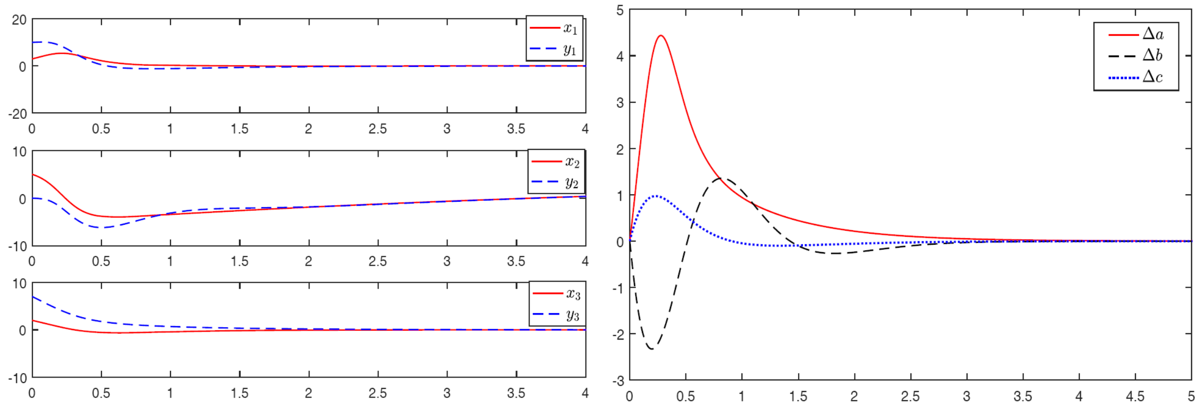
3. **Numerical Simulations.** The ultimate goal of numerical simulation is to verify the effectiveness of the proposed method for synchronization of chaotic systems. Therefore, some numerical simulations related to the synchronization of two chaotis systems: Lorenz chaotic system as the leader system and the Rössler chaotic system as the follower system are given.

The proposed program uses the fourth-order Runge-Kutta integration method with a fixed time-step size and a tolerance of $1.0E^{-6}$. The program benefits the adaptive-projective feedback control presented in (6), the dynamical system for parameter estimation in (7) and the dynamical error system presented in (4) for simulation purpose.

Let the attractor parameteres be $a = 0.2$, $b = 0.2$, and $c = 5.7$, and the initial values of the estimated follower attractor are set as: $\Delta a = 0.4$, $\Delta b = 0.1$ and $\Delta c = 0.3$.

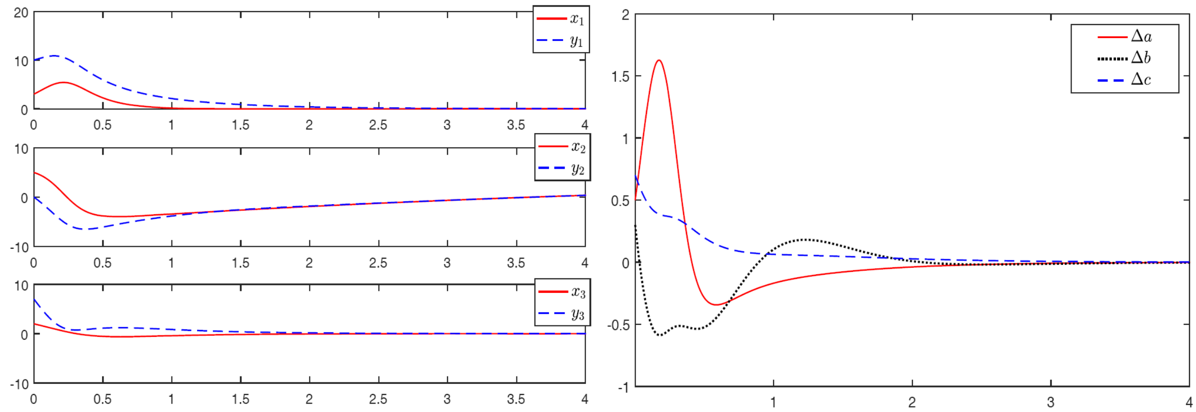
The initial state values for the leader attractor are: $x_1(0) = 2$, $x_2(0) = 1$ and $x_3(0) = 5$. Let us take the follower initial state values as: $y_1(0) = 5$, $y_2(0) = -2$ and $y_3(0) = 3$. So the initial differences between leader and follower state attractors are $(e_1(0), e_2(0), e_3(0)) = (5 - 2\delta_1, -2 - \delta_2, 3 - 5\delta_3)$, where α is projective constant factor.

The constants (gains) values are all taken as $k_i = 2$ $(\forall i = 1, \ldots, 3)$ and also $\phi_i = 1.5$ $(\forall i = 1, \ldots, 3)$. Some simulation results extracted from this section are shown in Figures 3 to 5. All simulations are carried out from $t = 0$s to $t = 10$s. The MPS between the



(a) State variables trajectories (b) Parameter estimation errors

FIGURE 3. MPS of the Lorenz chaotic system and the Rössler chaotic system with projective scaling factor $\boldsymbol{\delta}_1 = (1, 1, 1)$



(a) State variables trajectories (b) Parameter estimation errors

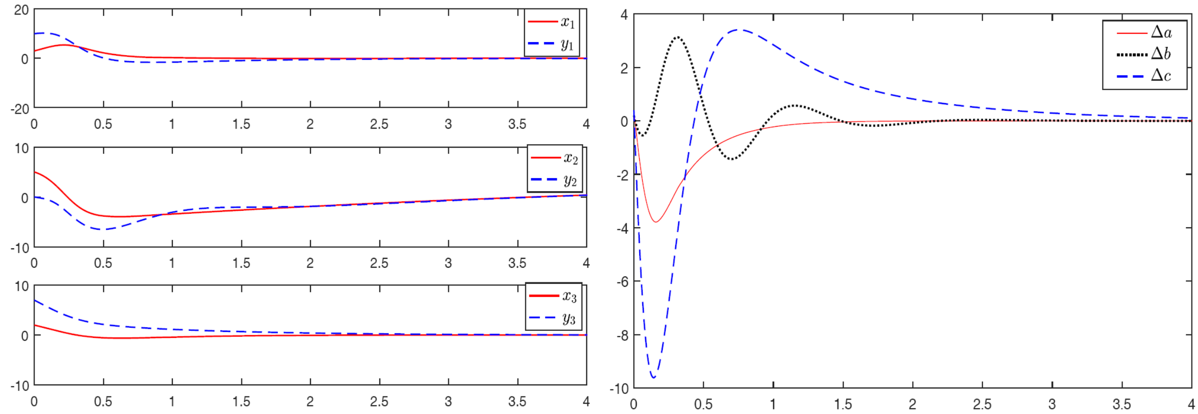
FIGURE 4. MPS of the Lorenz chaotic system and the Rössler chaotic system with projective scaling factor $\boldsymbol{\delta}_2 = (1 + \varepsilon_1 t, 1 + \varepsilon_2 t, 1 + \varepsilon_3 t)$

Lorenz chaotic system and the Rössler chaotic system is performed by three different scaling factors $\boldsymbol{\delta}_1$, $\boldsymbol{\delta}_2$ and $\boldsymbol{\delta}_3$ as follows:

$$\begin{aligned} \boldsymbol{\delta}_1 &= (1, 1, 1) \\ \boldsymbol{\delta}_2 &= (1 + \varepsilon_1 t, 1 + \varepsilon_2 t, 1 + \varepsilon_3 t) \\ \boldsymbol{\delta}_3 &= (1 + 0.01 \sin(\varepsilon_1 t), 1 + 0.03 \sin(\varepsilon_2 t), 1 + 0.2 \sin(\varepsilon_3 t)) \end{aligned}$$

where $\varepsilon_1 = 0.001$, $\varepsilon_2 = 0.0002$ and $\varepsilon_3 = 0.001$. Figure 3(a) shows the behavior of the state variables of the Lorenz chaotic system (1) and Rössler chaotic system (3) along the time with projective scaling factor as $\boldsymbol{\delta}_1 = (\sigma_1, \sigma_2, \sigma_3) = (1, 1, 1)$. In addition, the errors estimation of the system parameters is depicted in Figure 3(b). It is clear that the disparity amount of the system parameters Δa, Δb and Δc converges to the zero. Furthermore, in a similar manner, the behavior of the state variables of the Lorenz chaotic system (1) and Rössler chaotic system (3) and also the disparity amount of system paramteters are shown in Figures 4 and 5, for scaling factors $\boldsymbol{\delta}_2$ and $\boldsymbol{\delta}_3$, respectively.

4. **Conclusions.** In this paper, an adaptive modified projective synchronization (MPS) method for synchronization of Lorenz chaotic system as the drive system and the Rössler chaotic system as the response system is studied. The parameters of the drive chaotic system are considered unknown. An appropriated feedback control law and a parameter



(a) State variables trajectories

(b) Parameter estimation errors

FIGURE 5. MPS of the Lorenz chaotic system and the Rössler chaotic system with projective scaling factor $\boldsymbol{\delta}_3 = (1+0.01\sin(\varepsilon_1 t), 1+0.03\sin(\varepsilon_2 t), 1+0.2\sin(\varepsilon_3 t))$

estimation law are derived based on the Lyapunov stability theorem and the adaptive control theorem. Then, numerical simulations are carried out to verify the effectiveness method. As it can be seen from the simulated results, the anticipated drive-response synchronization is achieved and the synchronization errors of the system parameters and also errors form the disparity amount of system parameters tend to zero as time goes to the infinity.

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