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IMPULSIVE CONTROL AND EXPONENTIAL STABILITY FOR NONLINEAR TIME-VARYING SYSTEMS WITH TIME-VARYING DELAYS

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Abstract. *In this paper, the problem of exponential stability for nonlinear time-varying systems with time-varying delays via impulsive control is considered. By virtue of the formula for the variation of parameters and the Cauchy matrix, the exponential stability for such systems is discussed. In particular, the derived criteria can be used to stabilize the unstable nonlinear time-varying systems with time-varying delays by using the impulsive control. As an application, the generalized Nicholson's blowflies model with time-varying delays is given to show the feasibility of the obtained results.*
Keywords: Time-varying delays, Exponential stability, Time-varying systems, Impulses

1. **Introduction.** Recently, special attention has been devoted to the problem on the exponential stability of nonlinear autonomous systems with time-varying delays since such systems have been successfully applied into various signal processing problems such as optimization, image processing, associative memory and many other fields, see, [1-14] and the references therein. However, as we know, time-varying phenomena often occur in many realistic systems. In particular when we consider the long-time dynamical behavior of a system, the parameters of the systems are usually subjected to environmental disturbances and frequently vary with time. In this case, time-varying systems model can describe evolutionary processes of networks in detail. Thus, the study on the dynamical properties for time-varying systems with time-varying delays is very interesting and there are some excellent results on the time-varying systems, see [15-20] and the references therein.

On the other hand, impulsive systems are typically characterized by ordinary differential equations with instantaneous state jumps [21]. From the view of control theory, impulsive control strategies, as a useful control means, in the past several years, have been extensively used to stabilize some nonlinear unstable dynamical systems and chaotic systems [22-29]. The crucial principle of impulsive control is to alter the states of a system whenever some conditions are satisfied. In [23], Chen and Zheng have considered the exponential stability of nonlinear time-delay systems with delayed impulse effects by utilizing the Razumikhin-type theorem. In [30], with the help of the formula for the variation of parameters and the Cauchy matrix, the robust stability problem of an uncertain system with time-varying delay via impulsive control has been considered. In [31], Xu and Xu have also discussed the exponential stability in mean square moment of impulsive stochastic time-varying delay systems by utilizing the formula for the variation of parameters and the Cauchy matrix. In [32], by using an impulsive delay differential inequality, some criteria on the exponential stability and the impulsive controller design of nonlinear impulsive autonomous systems with time-varying delays have been obtained. However, the obtained results in [30-32] cannot be used to analyze the nonlinear impulsive time-varying systems with time-varying delays since the difficulty mainly comes from the existence of the time-varying phenomena. To the best of the authors' knowledge, there are few works on the exponential stability of nonlinear time-varying systems with time-varying delays via impulsive control. Thus, how to consider such problem becomes the main motivation of this paper.

In this paper, the problem on exponential stability of nonlinear time-varying systems with time-varying delays via impulsive control is analyzed. By using the formula for the variation of parameters and the Cauchy matrix, the exponential stability for such systems is considered. The novelty in this brief is stated as follows: 1) Compared with the technique proposed in [30-32], the difficulty mainly come from the existence of the non-autonomous phenomena can be overcome. Certainly, the methods proposed in [22-29] are also impracticable for our concerned problem. Besides, when our concerned problem is degraded into the autonomous case, our obtained results are the same as one given in [32]; 2) Due to the existence of the impulsive effect, the methods utilized in [15-20] cannot be employed to discuss the problem in this paper. However, when the impulses are removed, the problems in [15-20] can also be discussed by using the method in this paper; 3) Our results can weaken the conservatism of the ones given in [23] which is obtained by using the Razumikhin-type theorem. Finally, the generalized Nicholson's blowflies model with time-varying delays is provided to show the effectiveness of the derived result.

This paper is organized as follows. In Section 2, the problem statements and the preliminaries are given. In Section 3, main results are presented. Section 4 provides one example to check the effectiveness and feasibility of the theoretical results obtained. Section 5 makes a conclusion.

2. **Problem Statement and Preliminaries.** Let $R^+_{t_0} = [t_0, +\infty)$, $N = \{1, 2, 3, \ldots\}$, and E denotes an identity matrix. For any $x \in R^n$, let $\|x\|$ be the Euclid vector norm, and represent the induced matrix norm and the matrix measure (a logarithmic matrix norm), respectively, by

$$\|A\| = \sup_{x \neq 0} \frac{\|Ax\|}{\|x\|}, \quad \mu_A(t) = \lim_{\varepsilon \to 0^+} \frac{\|E + \varepsilon A\| - 1}{\varepsilon}.$$

The norm and the measure of vector and matrix used in this brief are given as follows:

$$\|x\| = \sum_{j=1}^{n} |x_j|, \quad \|A\| = \max_{1 \le j \le n} \sum_{i=1}^{n} |a_{ij}|, \quad \mu_A = \max_{1 \le j \le n} \left\{ a_{jj} + \sum_{i=1, i \neq j}^{n} |a_{ij}| \right\},$$

and $x(t^+) = \lim_{s \to 0^+} x(t + s)$, $x(t^-) = \lim_{s \to 0^-} x(t + s)$, $D^+ x(s) = \limsup_{t_n \downarrow s^+} \frac{x(t_n) - x(s)}{t_n - s}$.

Consider the nonlinear time-varying systems with time-varying delays:

$$\begin{cases} \dot{x}(t) = A(t)x(t) + F(t, x(t), x(t - h_1(t)), x(t - h_2(t)), \ldots, x(t - h_m(t))), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

on $t \ge t_0$, where $x \in R^n$ is the state vector, $y \in R^m$ is the output vector, $A(\cdot) : [t_0, +\infty) \to R^{n \times n}$, and $C \in R^{m \times n}$. The time varying delay $h_i(t)$ is continuous and satisfies $0 \le h_i(t) \le h$ $(i = 1, 2, \ldots, m)$, $F : [t_0, +\infty) \times \overbrace{R^n \times R^n \times \cdots \times R^n}^{m+1} \to R^n$ is continuous function satisfying

$$\|F(t, x, \bar{y}_1, \bar{y}_2, \ldots, \bar{y}_m)\| \leq l_0(t)\|x\| + \sum_{i=1}^{m} l_i(t)\|\bar{y}\|, \tag{2}$$

where $l_i(t)$ $(i = 0, 1, 2, \ldots, m)$ are some nonnegative continuous functions.

Remark 2.1. *When $A(t)$ and $l_i(t)$ $(i = 0, 1, 2, \ldots, m)$ are independent on t, systems (1) have been discussed in [32]. However, when considering systems (1) in this paper, the method provided in [32] is not impracticable.*

An impulsive control law of systems (1) can be given in form of the following control sequences $\{t_k, B_k Cx(t_k^-)\}$ [21]:

$$\begin{cases} 0 \leq t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots, \ \lim_{k\to+\infty} t_k = +\infty, \\ \Delta x(t_k^+) = x(t_k^+) - x(t_k^-) = B_k Cx(t_k^-), \ k \in N. \end{cases} \tag{3}$$

Then, we can derive the following nonlinear impulsive time-varying systems with time-varying delays:

$$\begin{cases} \dot{x}(t) = A(t)x(t) + F(t, x(t), x(t - h_1(t)), x(t - h_2(t)), \ldots, x(t - h_m(t))), \\ \qquad t \neq t_k, \ \ t \geq t_0; \\ \Delta x(t_k) = x(t_k^+) - x(t_k^-) = C_k x(t_k^-), \ \ k \in N, \\ x(t) = \varphi(t), \ t \in [t_0 - h, t_0], \end{cases} \tag{4}$$

where $C_k = B_k C$ $(k = 1, 2, \ldots)$, and $\varphi :\ [t_0 - h, t_0] \to R^n$ is continuous. It is always assumed that $x(t)$ is right continuous at $t = t_k$, i.e., $x(t_k) = x(t_k^+)$. Thus, the solution of systems (4) is the piecewise right-hand continuous functions with discontinuities of the first kind only at $t = t_k$, $k \in N$.

Lemma 2.1. *Let $\Phi(t, t_0)$ be the Cauchy solution of the following systems:*

$$\begin{cases} \dot{x}(t) = A(t)x(t), \ \ t \neq t_k, \ \ t \geq t_0; \\ \Delta x(t_k) = C_k x(t_k^-), \ \ k \in N. \end{cases} \tag{5}$$

If a constant is given with $r \geq \|E + C_k\|$ for any $k \in N$, then we have the following results:

(1) when $0 < r < 1$, and $\rho = \sup_{k\in N}\{t_k - t_{k-1}\} < +\infty$,

$$\|\Phi(t, t_0)\| \leq \frac{1}{r} \exp\left\{\int_{t_0}^{t} \left(\mu_A(s) + \frac{\ln r}{\rho}\right) ds\right\}, \ \ t \geq t_0;$$

(2) when $r \geq 1$ and $\theta = \inf_{k\in N}\{t_k - t_{k-1}\} > 0$,

$$\|\Phi(t, t_0)\| \leq r \exp\left\{\int_{t_0}^{t} \left(\mu_A(s) + \frac{\ln r}{\theta}\right) ds\right\}, \ \ t \geq t_0. \tag{6}$$

Proof: For any $x_0 \in R^n$, let $x(t) = x(t, t_0, x_0)$ be a solution of systems (4) with its initial value (t_0, x_0). For any $t \in [t_k, t_{k+1})$, we have

$$\|x(t)\| \leq \|x(t_k)\| \exp\left\{\int_{t_k}^{t} \mu_A(s) ds\right\},$$

and

$$\|x(t)\| \leq \|E + C_k\| \|x(t_k^-)\|, \ \ k \in N.$$

Furthermore, it follows

$$\|x(t)\| \leq \left(\Pi_{t_0 < t_k \leq t} \|E + C_k\|\right) \|x(t_0)\| \exp\left\{\int_{t_0}^{t} \mu_A(s) ds\right\},$$

for any $t \geq t_0$.

Case i): when $0 < r < 1$, we have

$$\|x(t)\| \le r^{\frac{t-t_0}{\rho}} \|x(t_0)\| \exp\left\{ \int_{t_0}^{t} \mu_A(s) ds \right\}$$
$$\le \frac{1}{r} \|x(t_0)\| \exp\left\{ \int_{t_0}^{t} \left(\mu_A(s) + \frac{\ln r}{\rho} \right) ds \right\}.$$

Since $x(t) = \Phi(t, t_0)x(t_0)$, it yields

$$\|\Phi(t, t_0)\| \le \frac{1}{r} \exp\left\{ \int_{t_0}^{t} \left(\mu_A(s) + \frac{\ln r}{\rho} \right) ds \right\},$$

for any $t \ge t_0$.

Case ii): Similarly, for $r \ge 1$, it is obtained that

$$\|\Phi(t, t_0)\| \le r \exp\left\{ \int_{t_0}^{t} \left(\mu_A(s) + \frac{\ln r}{\theta} \right) ds \right\},$$

for any $t \ge t_0$.

Lemma 2.2. [33]: *Assume that $p(t)$ and $q(t)$ are continuous on $[t_0, +\infty)$ with $p(t) - q(t) \ge \varrho > 0$, and there exists a positive constant M such that $0 \le q(t) \le M$ for all $t \ge t_0$, then $\lambda = \inf_{t \ge t_0}\{\lambda(t) > 0 : \lambda(t) - p(t) + q(t)e^{\lambda(t)h} = 0\} > 0$.*

3. Main Results.

Theorem 3.1. *Let $\rho = \sup_{k \in N}\{t_k - t_{k-1}\} < +\infty$. If there exists a constant $0 < \gamma < 1$ such that*

$$\|E + B_k C\| \le \gamma, \quad \text{and} \quad \frac{\ln \gamma}{\rho} + \mu_A(t) + \frac{\sum_{i=0}^{m} l_i(t)}{\gamma} < 0, \quad k \in N, \tag{7}$$

then the solution of systems (4) is exponentially stable, and the exponential convergence rate of systems (4) is larger than or equal to λ, where $\lambda > 0$ is defined as

$$\lambda = \inf_{t \ge t_0} \left\{ \lambda(t) > 0 : \lambda(t) + \mu_A(t) + \frac{\ln \gamma}{\rho} + \frac{\sum_{i=0}^{m} l_i(t) e^{\lambda(t)h}}{\gamma} = 0 \right\} > 0. \tag{8}$$

Proof: The solution $x(t)$ of systems (4) can be given by

$$\begin{aligned} x(t) = \Phi(t, t_0)x(t_0) + \int_{t_0}^{t} &\Phi(t, s)F(s, x(s), x(s - h_1(s)), x(s - h_2(s)), \\ &\dots, x(s - h_m(s)))ds, \quad t \ge t_0, \end{aligned} \tag{9}$$

where $\Phi(t, t_0)$ is the Cauchy matrix of the impulsive systems (5).

It implies from Case i) in Lemma 2.1 and inequality (2) that

$$\begin{aligned} \|x(t)\| &\le \|\Phi(t, t_0)\| \|x(t_0)\| \\ &\quad + \int_{t_0}^{t} \|\Phi(t, s)\| \|F(s, x(s), x(s - h_1(s)), x(s - h_2(s)), \dots, x(s - h_m(s)))\| ds \\ &\le \frac{1}{\gamma} \exp\left\{ \int_{t_0}^{t} \left(\mu_A(s) + \frac{\ln \gamma}{\rho} \right) ds \right\} \\ &\quad + \frac{1}{\gamma} \int_{t_0}^{t} \exp\left\{ \int_{s}^{t} \left(\mu_A(u) + \frac{\ln \gamma}{\rho} \right) du \right\} \sum_{i=0}^{m} l_i(s) \|x(s - h_i(s))\| ds, \end{aligned} \tag{10}$$

for any $t \ge t_0$, where $h_0(t) = 0$. Define $\|x\|_h = \sup_{\theta \in [t_0 - h, t_0]} \|\phi(\theta)\|$.

Define

$$Q(t) = \begin{cases} \dfrac{1}{\gamma}\exp\left\{\displaystyle\int_{t_0}^{t}\left(\mu_A(s) + \frac{\ln\gamma}{\rho}\right)ds\right\} + \dfrac{1}{\gamma}\displaystyle\int_{t_0}^{t}\exp\left\{\int_{s}^{t}\left(\mu_A(u) + \frac{\ln\gamma}{\rho}\right)du\right\} \\ \times\displaystyle\sum_{i=0}^{m} l_i(s)\|x(s-h_i(s))\|ds, \quad t \geq t_0, \\ \|x\|_h, \quad t \in [t_0 - h, t_0]. \end{cases} \tag{11}$$

Obviously, it is seen that $\|x(t)\| \leq Q(t)$, for any $t \in [t_0 - h, +\infty)$.

Thus, from (11), we have

$$\begin{aligned} D^+Q(t) = & \left(\mu_A(t) + \frac{\ln\gamma}{\rho}\right)Q(t) + \frac{1}{\gamma}\sum_{i=0}^{m} l_i(t)\|x(t-h_i(t))\| \\ \leq & \left(\mu_A(t) + \frac{\ln\gamma}{\rho}\right)Q(t) + \frac{1}{\gamma}\sum_{i=0}^{m} l_i(t)Q(t-h_i(t)), \end{aligned} \tag{12}$$

for any $t \geq t_0$.

From Lemma 2.2 and inequality (6), λ can uniquely be determined in (7). Now, let

$$\Gamma(t) = \chi\|x\|_h e^{-\lambda(t-t_0)}, \quad t \geq t_0, \tag{13}$$

where $\chi > 1$. It is easily seen that for any $t \in [t_0 - h, t_0]$, we have $\Gamma(t) > Q(t)$. Now, we only prove that

$$Q(t) < \Gamma(t), \tag{14}$$

for any $t \geq t_0$. Due to the fact that $Q(t)$ is continuous on the interval $[t_0, +\infty)$, for the sake of contradiction, assume that there exists t^* such that

$$Q(t^*) = \Gamma(t^*), \quad \text{and} \quad Q(t) < \Gamma(t), \tag{15}$$

for any $t \in [t_0, t^*)$. Thus,

$$D^+Q(t^*) \geq \Gamma'(t^*). \tag{16}$$

From (14) and $Q(t) < \Gamma(t)$ for any $t \in [t_0 - h, t_0]$, we can obtain

$$Q(t^* - h_i(t^*)) < \Gamma(t^* - h_i(t^*)), \tag{17}$$

for any $i = 1, 2, \ldots, m$.

From (11), (12), (14) and (16), it yields

$$\begin{aligned} & D^+Q(t^*) - \Gamma'(t^*) \\ \leq & \left(\mu_A(t^*) + \frac{\ln\gamma}{\rho}\right)Q(t^*) + \frac{1}{\gamma}\sum_{i=0}^{m} l_i(t^*)Q\left(t^* - h_i(t^*)\right) + \chi\lambda\|x\|_h e^{-\lambda(t^*-t_0)} \\ < & \left(\mu_A(t^*) + \frac{\ln\gamma}{\rho}\right)\Gamma(t^*) + \frac{1}{\gamma}\sum_{i=0}^{m} l_i(t^*)\Gamma(t^* - h_i(t^*)) + \chi\lambda\|x\|_h e^{-\lambda(t^*-t_0)} \\ \leq & \left[\lambda^* + \mu_A(t^*) + \frac{\ln\gamma}{\rho} + \frac{1}{\gamma}\sum_{i=0}^{m} l_i(t^*)e^{\lambda^* h}\right]\chi\|x\|_h e^{-\lambda(t^*-t_0)} \\ = & 0, \end{aligned}$$

which contradicts (15). Hence, (13) holds for any $t \in [t_0, +\infty)$. Letting $\chi \to 1$ in (13), we have $Q(t) \leq \|x\|_h e^{-\lambda(t-t_0)}$, for any $t \in [t_0, +\infty)$, which implies that systems (4) are exponentially stable.

Remark 3.1. *By virtue of Lemma 2.1 and Lemma 2.2, the exponential stability of systems (4) can be well discussed. When $A(t) = A$ and $l_i(t) = l_i$ $(i = 1, 2, \ldots, m)$, systems (4) are degraded into systems (4) in [32]. By using Theorem 3.1, the exponential stability of systems (4) in [32] can be derived. If the impulsive effects are removed, systems (4) have been extensively studied with the help of the Razuminkhin-type theorem in [16], the generalized Halanay Lemma in [17] and the construction of the comparison techniques in [18-20]. However, it is pointed out that there is much great difference on the method between our concerned problem and the autonomous systems. Hence, the techniques proposed in [16-20,32] cannot be utilized to consider our concerned problem in this paper.*

Remark 3.2. *Based on the method proposed in Theorem 3.1, it is easily obtained that the exponential stability of systems (4) can be guaranteed under the following condition:*

$$\mu_A(t) + \sum_{i=0}^{m} l_i(t) < 0. \tag{18}$$

Similarly, we can obtain the following result when case $\gamma \geq 1$. The detailed proof can be seen in Theorem 3.1, which is omitted for simplicity, here.

Theorem 3.2. *Let $\varrho = \inf_{k \in N}\{t_k - t_{k-1}\} < +\infty$. If there exists a constant $\gamma \geq 1$ such that*

$$\|E + B_k C\| \leq \gamma, \quad k \in N, \quad \text{and} \quad \frac{\ln \gamma}{\varrho} + \mu_A(t) + \gamma \sum_{i=0}^{m} l_i(t) < 0,$$

then the solution of systems (4) is exponentially stable, and the exponential convergence rate of systems (4) is larger than or equal to λ, where $\lambda > 0$ is defined as

$$\lambda = \inf_{t \geq t_0} \left\{ \lambda(t) > 0: \ \lambda(t) + \mu_A(t) + \frac{\ln \gamma}{\varrho} + \gamma \frac{\sum_{i=0}^{m} l_i(t) e^{\lambda(t) h}}{\gamma} = 0 \right\} > 0.$$

Remark 3.3. *In [23], Chen and Zheng have considered the exponential stability for systems (4) by using the Razumikhin-type theorem. When $\gamma \in (0, 1)$, in contrast to (iii) of Corollary 1 in [23], it is easily seen that Theorem 3.1 is much less conservative. Besides, the results under the case $\gamma \geq 1$ cannot be given in [23].*

4. **Example.**

In this section, we consider the following nonlinear time-varying Nicholson-type systems with time-varying delay [34]:

$$\begin{cases} \dot{x}_1(t) = -\alpha_1(t) x_1(t) + \beta_1(t) x_2(t) + c_1(t) x_1(t - h_1(t)) e^{-x_1(t - h_1(t))}, \\ \dot{x}_2(t) = -\alpha_2(t) x_2(t) + \beta_2 x_1(t) + c_2(t) x_2(t - h_1(t)) e^{-x_2(t - h_1(t))}, \\ y_1(t) = c_1 x_1(t) + c_2 x_2(t), \\ y_2(t) = c_3 x_1(t) + c_4 x_2(t), \quad t \geq t_0, \end{cases} \tag{19}$$

with its initial conditions:

$$x_{t_0} = \varphi = \text{col}[\varphi_1, \ \varphi_2] \in C_+, \quad \text{and} \quad \varphi(t_0) > 0,$$

where $C_+ = C([t_0 - h, t_0], R_+) \times C([t_0 - h, t_0], R_+)$.

Suppose that $\alpha_1(t) = -4.9 + e^{-0.5t}$, $\alpha_2(t) = -4.35 + 2e^{-0.5t}$, $\beta_1(t) = 2 + 2e^{-0.5t}$, $\beta_2(t) = 2 + e^{-0.5t}$, $c_1(t) = 3 + 2e^{-0.5t}$, $c_2(t) = 1.5 + 2e^{-0.5t}$, and $h_1(t) = |\sin(t)|$. The matrix $C = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$.

Letting $x(t) = \mathrm{col}[x_1(t) \quad x_2(t)]$, $y(t) = \mathrm{col}[y_1(t) \quad y_2(t)]$, $A(t) = \begin{bmatrix} -\alpha_1(t) & \beta_1(t) \\ \beta_2(t) & -\alpha_2(t) \end{bmatrix}$, and

$$\begin{aligned} & F(t, x(t), x(t-h_1(t)), x(t-h_2(t)), \ldots, x(t-h_m(t))) \\ & = \mathrm{col}\Big[c_1(t)x_1(t-h_1(t))e^{-x_1(t-h_1(t))} \quad c_2(t)x_2(t-h_1(t))e^{-x_2(t-h_1(t))}\Big], \end{aligned}$$

thus, systems (19) can be written into the following form:

$$\begin{cases} \dot{x}(t) = A(t)x(t) + F(t, x(t), x(t-h_1(t)), x(t-h_2(t)), \ldots, x(t-h_m(t))), \\ y(t) = Cx(t), \quad t \geq t_0. \end{cases} \tag{20}$$

Assume that $x(t, t_0, \varphi)$ is the solution of systems (19), from Theorem 5.2.1 in [35], it is concluded that $x(t, t_0, \varphi) \geq 0$. By simple derivation, we have

$$\begin{aligned} \|E + \varepsilon A(t)\| &= \left\| \begin{bmatrix} 1 - \varepsilon\alpha_1(t) & \varepsilon\beta_1(t) \\ \varepsilon\beta_2(t) & 1 - \varepsilon\alpha_2(t) \end{bmatrix} \right\| \\ &= \max\left\{1 - 2.9\varepsilon + 3\varepsilon e^{-0.5t}, 1 - 2.35\varepsilon + 3\varepsilon e^{-0.5t}\right\} \\ &= 1 - \varepsilon \min\left\{2.9 - 3e^{-0.5t}, 2.35 - 3e^{-0.5t}\right\}, \end{aligned}$$

which implies that

$$\mu_A(t) = \lim_{\varepsilon \to 0^+} \frac{\|E + \varepsilon A(t)\| - 1}{\varepsilon} = -2.35 + 3e^{-0.5t},$$

and

$$\begin{aligned} & \|F(t, x(t), x(t-h_1(t)), x(t-h_2(t)), \ldots, x(t-h_m(t)))\| \\ \leq & \max_{i=1,2}\{c_i(t)\}\|x(t-h_1(t))\| = 3 + 2e^{-0.5t}. \end{aligned}$$

Obviously, in view of (17) in Remark 3.2, the solution of systems (18) is not exponentially stable. However, when introducing the impulsive controllers (3) of systems (18) with matrices

$$B_k = \begin{bmatrix} -0.5 & 0 \\ 1 & -0.5 \end{bmatrix}, \quad k \in N,$$

and $\|E + B_k C\|_2 = 0.5$ $(k \in N)$. By choosing $\gamma = 0.5$ and the impulsive control instant $\rho = \sup_{k \in N}\{t_k - t_{k-1}\} = 0.005$, it is concluded that (6) holds. Therefore, it follows from Theorem 3.1 that the exponential stability of systems (18) via the impulsive control can be guaranteed.

5. **Conclusion.** In this paper, the exponential stability for nonlinear time-varying systems with time-varying delays via impulsive control has been discussed. By using the formula for the variation of parameters and the Cauchy matrix, the exponential stability for such systems has been considered. The obtained stability criteria can be used to stabilize the unstable nonlinear time-varying systems with time-varying delays by employing the impulsive control, which is demonstrated by a generalized Nicholson's blowflies model.

Based on this paper, future work includes the discussion about the impulsive control and exponential stability for stochastic time-varying complex dynamical networks with time-varying delay.

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