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SLIDING MODE FAULT-TOLERANT CONSENSUS OF LEADER-FOLLOWING MULTI-AGENT SYSTEMS

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Abstract. *This paper investigates the problem of fault-tolerant consensus control for leader-following multi-agent systems subject to partial loss of actuator effectiveness and system uncertainty. The actuator faults are assumed to be time-varying and the system uncertainty is bounded by a more general form. Based on the relative states of neighbors, the consensus error dynamic is given. Then, a sliding mode surface based on consensus errors is proposed and the stability of the states on the sliding mode surface is proved by using Lyapunov theory. Furthermore, an adaptive sliding mode fault-tolerant consensus protocol is proposed to compensate the actuator faults under the online updating parameters. On the basis of the sliding mode control theory, the new proposed distributed adaptive sliding mode fault-tolerant controller ensures that the consensus of multi-agent systems can be reached as time goes to infinity. Finally, an example of leader-following multi-robot system is presented to demonstrate the effectiveness of the proposed controller.*

Keywords: Sliding mode, Fault-tolerant, Multi-agent systems, Consensus, Actuator faults

1. **Introduction.** In recent years, the consensus control problem of multi-agent systems has been paid more and more attention. Since multi-agent systems can accomplish complex tasks by mutual cooperation among individuals, they are widely used in many areas, including multi-satellite formation, sensor networks and multi-robot systems and so forth [1-4].

Sliding mode control is a robust control method, in which a switching surface is defined first, and a sliding mode controller is then designed to drive the system to reach and remain on the sliding mode surface. A lot of papers use the sliding mode theory to study the consistency of multi-agent systems. [5] proves that finite-time consensus tracking of multi-agent systems can be achieved on the terminal sliding mode surface. In [6], a distributed controller is developed by using terminal sliding mode and Chebyshev neural networks for second-order multi-agent systems in the presence of uncertain dynamics and bounded external disturbances. [7] proposes a new adaptive backstepping sliding mode control approach for leader-following multi-agent systems. [8] investigates the finite-time consensus problem of second-order multi-agent systems by integral sliding mode. In [9], sliding mode control method is applied to solving the tracking problem of nonlinear multi-agent systems under a time-varying topology.

However, the aforementioned studies on consensus of multi-agent systems using sliding mode control theory are based largely on the basis of an ideal actuator, while ignoring the unavoidable faults. Once the actuator faults occur in the operation of multi-agent systems, the control law which is designed on the condition of an ideal actuator cannot be completely executed. This may lead to the failure of the whole task and thus bring a lot of limitations on the application of such technologies. To be more practical, in this paper, we take the actuator faults into consideration and investigate the fault-tolerant consensus problem for the leader-following multi-agent systems.

When it comes to the fault-tolerant consensus problem for multi-agent systems, most of the existing researches use the adaptive methods to design a compensating control protocol with time-varying parameters. Moreover, the parameters are determined according to the adaptive law with projection operator. To name a few, in [10-13], a distributed adaptive protocol is designed by estimating the faults and updating the feedback gain online. However, only constant faults are considered. In [14], a constructive design method is proposed to achieve the tracking control of multi-agent systems with both the partial loss of actuator effectiveness and the actuator bias faults, but the intrinsic non-linear dynamics is not considered.

As for the sliding mode fault-tolerant control protocol, most of the existing fault-tolerant schemes are proposed on the basis of a single system. For example, [15] is concerned with the design of sliding mode control for uncertain systems with partial actuator degradation in presence of time delay. Based on adaptive sliding mode control, [16] proposes the robust fault-tolerant attitude control for anon-orbiting spacecraft with partial loss of actuator and external disturbance. In [17], a novel scheme is proposed to cope with the total failure of certain actuators, in which integral sliding mode method is incorporated with control allocation. [18] investigates a construction method of integral sliding surfaces by integrating the matrix factorization technique and adaptive laws.

It is worth noting that although the aforementioned sliding mode fault-tolerant schemes have been developed, there are few works in multi-agent system. In this paper, a sliding mode fault-tolerant control protocol is proposed for multi-agent systems with partial loss of actuator effectiveness. In this paper, a sliding mode fault-tolerant control protocol is proposed for multi-agent systems with partial loss of actuator effectiveness. Firstly, the time-varying actuator fault model is given, and the error dynamic equation is obtained. Secondly, the existence of the sliding mode and the global stability of the system are analyzed by Lyapunov method. Finally, a distributed fault-tolerant controller is designed to guarantee that the sliding mode surface is reachable and the actuator faults are compensated. The simulation results show that this method can realize the fault-tolerant consensus of multi-agent systems and improve the robustness abilities to system uncertainty.

The novelties of this paper can be summarized as follows: (i) Both actuator faults and system uncertainty are investigated. The system uncertainty is bounded by a more general form such that the obtained results will be more common; (ii) Compared with some existing results in [10-13], actuator faults are assumed to be time-varying in this paper. Adaptive sliding mode fault-tolerant controller is designed to compensate the actuator faults under the online updating parameters; (iii) The proposed control protocol inherits the merits of sliding mode control strategy; therefore, the multi-agent systems are robust to the parametric uncertainty and disturbance.

Throughout this article, $\mathbf{R}^n$ stands for the n-dimensional Euclidean space. $\mathbf{R}^{m\times n}$ denotes the $m\times n$ matrix. The "T" denotes the matrix transposition. The matrix diag$\{\ldots\}$ is a diagonal block matrix. I and $\mathbf{0}$ represent, respectively, the identity matrix and zero matrix. $\|\cdot\|$ is 2-norm and $\otimes$ denotes the Kronecker product. $\mathbf{1}$ denotes the vector with all entries being one.

2. Problem Formulation and Preliminaries.

2.1. Graph theory. A graph is denoted by $\bar{\mathcal{G}} = \left(\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{A}}\right)$. $\bar{\mathcal{V}} = \{0, 1, 2, \ldots, n\}$ is a finite set of nodes in the leader-following multi-agent systems, where 0 represents the leader and $i = 1, 2, \ldots, n$ represent followers. $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}}\times\bar{\mathcal{V}}$ is a set of edges, and $\bar{\mathcal{A}} = (a_{ij}) \in \mathbf{R}^{(n+1)\times(n+1)}$

is the associated adjacency matrix. For $\bar{\mathcal{G}}$, if there is a path in $\bar{\mathcal{G}}$ from the leader to every follower, then $\bar{\mathcal{G}}$ is called a connected graph.

Denote $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ as the subgraph of $\bar{\mathcal{G}}$, which is formed by n followers. $\mathcal{A} = (a_{ij})$, $\forall i, j = 1, 2, \ldots, n$. An edge rooted at node j and ended at node i is denoted by (v_j, v_i), which means information can flow from node j to node i. If $(v_j, v_i) \in \mathcal{E}$, then $a_{ij} = 1$; otherwise $a_{ij} = 0$. Let $\mathcal{D} = \mathrm{diag}\{d_1, \ldots, d_n\}$ be the degree matrix of $\mathcal{G}$, whose diagonal elements are given by $d_i = \sum_{j=1, j\neq i}^{n} a_{ij}$ $(i = 1, 2, \ldots, n)$. Then the Laplacian matrix L of the weighted graph $\mathcal{G}$ is defined by $L = \mathcal{D} - \mathcal{A}$. Matrix $L = [l_{ij}] \in \mathbf{R}^{n\times n}$ is the Laplacian matrix with the elements l_{ij} defined as $l_{ij} = -a_{ij}$ $(i \neq j)$ and $l_{ij} = d_i$ $(i = j)$. From the definition of the Laplacian matrix, one has the following equality,

$$L \cdot \mathbf{1} = \mathbf{0}, \tag{1}$$

and (1) is an important property of Laplacian matrix. Define $B = \mathrm{diag}\{b_1, \ldots, b_n\}$ as the leader adjacency matrix associated with $\mathcal{G}$, where $b_i > 0$ if the ith agent has access to the leader agent, and $b_i = 0$, otherwise. Meanwhile, a neighboring set $\mathcal{N}_i = \{j = 1, 2, \ldots, n | (i, j) \in \mathcal{E}\}$ is defined, where agent i can receive information from its neighbors.

2.2. **System description.** In this paper, a group of second-order multi-agent system with one leader and n followers is considered. The dynamics of the leader can be described by

$$\dot{x}_0 = v_0, \ \dot{v}_0 = u_0 + f(x_0, v_0, t), \tag{2}$$

where $x_0 \in \mathbf{R}^m$ and $v_0 \in \mathbf{R}^m$ are the position and velocity states of the leader respectively. $u_0 \in \mathbf{R}^m$ is the pre-designed control input. $f(x_0, v_0, t)$ represents the leader's lumped system uncertainty induced by modelling errors and external disturbance. The dynamics of the ith $(i = 1, \ldots, n)$ follower is expressed by

$$\dot{x}_i = v_i, \ \dot{v}_i = u_i^F + f(x_i, v_i, t), \tag{3}$$

where $x_i \in \mathbf{R}^m$ and $v_i \in \mathbf{R}^m$ are the position and velocity states of the ith follower respectively. $u_i^F \in \mathbf{R}^m$ is the actual control input and the unknown vector function $f(x_i, v_i, t)$ represents the ith follower's lumped system uncertainty and external disturbance.

Assumption 2.1. *For the system uncertainties $f(x_0, v_0, t)$ and $f(x_i, v_i, t)$ $(i \in \{1, 2, \ldots, n\})$, there exist non-negative constants h_1 and h_2, such that*

$$\|f(x_i, v_i, t) - f(x_0, v_0, t)\| \leq h_1\|x_i - x_0\| + h_2\|v_i - v_0\|.$$

Remark 2.1. *It is noted that in this paper, the system uncertainty is simultaneously considered in the leader and follower nodes. However, neither $f(x_0, v_0, t)$ within the leader dynamics nor the $f(x_i, v_i, t)$ within the follower dynamics has been considered in* [14]*. Moreover, the condition in Assumption 2.1 is more general compared with* [7]*, which can be satisfied by many practical systems.*

2.3. **Actuator fault model.** In order to formulate the fault-tolerant control problem, the fault model is established first. Here, we consider actuator faults are time-varying. For node i $(i \in \{1, 2, \ldots, n\})$, let u_i represent the input signal of the ith follower and u_i^F represent the actuator fault which include loss of effectiveness in the ith follower node. A general actuator fault model is described as follows

$$u_i^F = (I_m - \theta_i(t))u_i, \tag{4}$$

where $\theta_i(t) = \mathrm{diag}\{\theta_{i1}(t), \ldots, \theta_{im}(t)\} \in \mathbf{R}^{m\times m}$ is the failure matrix of the actuator in the ith follower, and the time-varying function $\theta_{ip}(t)$ $(p \in \{1, 2, \ldots, m\})$ denotes loss effectiveness of the pth actuator channel which satisfies $0 \leq \theta_{ip}(t) < 1$.

Remark 2.2. *It should be pointed out that if* $\theta_{ip}(t) = 0$ ($i \in \{1, 2, \dots, N\}$, $p \in \{1, 2, \dots, m\}$), *which indicates that* $\theta_i(t) = 0$, *Equation (4) can be simplified as* $u_i^F(t) = u_i(t)$. *This is the case with ideal actuator, which has been studied in many researches, see* [5, 8] *for example; if* $0 < \theta_{ip}(t) < 1$, *Equation (4) indicates partial loss of effectiveness.*

The objective of the consensus problem is to design the fault-tolerant control protocol $u_i(t)$ such that the following conditions can be achieved

$$\begin{cases} \lim\limits_{t\to\infty} x_i - x_0 = 0, & \forall i \in 1, 2, \dots, n \\ \lim\limits_{t\to\infty} v_i - v_0 = 0, & \forall i \in 1, 2, \dots, n \end{cases} \tag{5}$$

To achieve the fault-tolerant control objective, we introduce the following standard assumption and recall the following lemma which are essential for this paper.

Assumption 2.2. *Suppose that* $\bar{\mathcal{G}}$ *is strongly connected, and the subgraph* $\mathcal{G}$ *is undirected.*

Lemma 2.1. [19] *If* $\bar{\mathcal{G}}$ *is connected, then the matrix* $L+B$ *associated with* $\bar{\mathcal{G}}$ *is symmetric and positive definite.*

3. Fault-Tolerant Consensus Control Protocol Design and Analysis.

In this section, we first present the consensus error dynamic system based on state information shared by the neighboring agents. Then, by adopting sliding mode technique, the states of consensus error system are proven to be asymptotically stable on the sliding mode surface. Furthermore, the proposed fault-tolerant consensus control protocol for each follower guarantees the reaching and sliding conditions and the actuator faults can be compensated. Therefore, the fault-tolerant consensus problem can be solved.

3.1. Error dynamic system.

The position consensus error and velocity consensus error of the ith follower are defined as follows

$$\begin{aligned} e_{xi} &= \sum_{j\in\mathcal{N}_i} a_{ij}(x_i - x_j) + b_i(x_i - x_0), \\ e_{vi} &= \sum_{j\in\mathcal{N}_i} a_{ij}(v_i - v_j) + b_i(v_i - v_0), \end{aligned} \tag{6}$$

where a_{ij} is the element of the weighted adjacency matrix A and b_i is the element of the leader adjacency matrix B.

Denote $\bar{x}_i = x_i - x_0$, $\bar{v}_i = v_i - v_0$, $\bar{x} = \left[\bar{x}_1^T, \dots, \bar{x}_n^T\right]^T$, $\bar{v} = \left[\bar{v}_1^T, \dots, \bar{v}_n^T\right]^T$, $e_x = \left[e_{x1}^T, \dots, e_{xn}^T\right]^T$, $e_v = \left[e_{v1}^T, \dots, e_{vn}^T\right]^T$, then Equation (6) can be reexpressed in the following compact form

$$\begin{aligned} e_x &= ((L+B)\otimes I_m)\cdot \bar{x}, \\ e_v &= ((L+B)\otimes I_m)\cdot \bar{v}. \end{aligned} \tag{7}$$

Thus, the fault-tolerant consensus problem is equivalent to the stability of the following overall system by taking the time derivative of (7)

$$\begin{aligned} \dot{e}_x &= e_v, \\ \dot{e}_v &= ((L+B)\otimes I_m)(u - \theta u - \mathbf{1}\otimes u_0 + F - \mathbf{1}\otimes f_0), \end{aligned} \tag{8}$$

where $u = \left[u_1^T, \dots, u_n^T\right]^T$, $F = \left[f_1^T, \dots, f_n^T\right]^T$, $\theta = \operatorname{diag}\{\theta_1, \dots, \theta_n\}$.

3.2. **Sliding mode surface.** The distributed sliding mode surface for the ith follower is defined as

$$s_i = e_{vi} + K e_{xi}, \tag{9}$$

which can be expressed in a compact form as

$$s = e_v + (I_n \otimes K) e_x, \tag{10}$$

where $s = \left[s_1^T, \ldots, s_n^T\right]^T$, and K is the positive gain matrix to be designed. The asymptotic stability of the overall system (8) is given in the following theorem.

Theorem 3.1. *Suppose that Assumptions 2.1 and 2.2 hold. Then with the sliding mode surface function given by (9), if the sliding mode surface $s = \dot{s} = 0$ is reached, then the tracking error $\bar{x}$ converges to zero and the states of system (8) are asymptotically stable.*

Proof: If the sliding mode surface $s = 0$ is reached, then we can obtain

$$\dot{e}_x = -(I_n \otimes K) e_x. \tag{11}$$

Choosing the Lyapunov function as

$$V = \frac{1}{2} e_x^T e_x. \tag{12}$$

Taking the derivative of (12) and substituting (11) into the derivative, we can obtain

$$\dot{V} = -e_x^T (I_n \otimes K) e_x \leq 0,$$

which implies that the consensus error e_x converges to zero. Note that

$$e_x = ((L + B) \otimes I_m) \cdot \bar{x},$$

hence the tracking error $\bar{x}$ converges to zero and the states of system (8) are asymptotically stable. The proof is completed.

3.3. **Fault-tolerant consensus control protocol.** Now we are ready to present the control protocol. The fault-tolerant control protocol is proposed to guarantee the consensus of the second order leader-following multi-agent systems with the actuator faults. Based on the consensus errors $e_{xi} \in \mathbf{R}^m$ and $e_{vi} \in \mathbf{R}^m$, we design the fault-tolerant control protocol for the ith follower as

$$u_i = \left(\sum_{j \in \mathcal{N}_i} a_{ij} + b_i \right)^{-1} (b_i u_0 - K e_{vi}) - \eta s_i - \gamma_i \mathrm{sgn}(s_i), \tag{13}$$

with

$$\gamma_i = \zeta_i + \phi_i, \ \zeta_i = -\varphi_i + \hat{\omega}_i \varphi_i, \tag{14}$$

and

$$\phi_i = \left(\sum_{j \in N^i} a_{ij} + b_i \right)^{-1} (h_1 \|e_{xi}\| + h_2 \|e_{vi}\|) + c_2, \tag{15}$$

where $\hat{\omega}_i$ is the estimation of ω_i and $\omega_i = \frac{1}{1 - \|\theta_i\|}$, $\dot{\hat{\omega}}_i = \sigma \varphi_i \|s_i\|$, $\varphi_i = \left\| \left(\sum_{j \in N^i} a_{ij} + b_i \right)^{-1} (b_i u_0 - K e_{vi}) - \eta s_i \right\| + \phi_i + c_1$. σ, η, c_1, c_2 are positive constants.

The fault-tolerant control protocol (13) with (14) and (15) can be rewritten in the following compact form

$$u = \left((L + B)^{-1} \otimes I_m\right) (\mathbf{b} \otimes u_0 - K e_v) - \eta s - \gamma \mathrm{sgn}(s), \tag{16}$$

where $\gamma = \mathrm{diag}\{\gamma_1, \ldots, \gamma_n\}$, $\mathbf{b} = [b_1, \ldots, b_n]^T$.

Theorem 3.2. *Under Assumptions 2.1 and 2.2, there exists the fault-tolerant control protocol (13) with parameters defined in (14) and (15), which can force the states of systems to reach a sliding mode surface and then maintain on the sliding surface. Under the control protocol, the consensus of multi-agent systems (2) and (3) can be obtained.*

Proof: Define the estimation error of ω_i as $\tilde{\omega}_i = \omega_i - \hat{\omega}_i$. Consider the following Lyapunov function

$$V = \frac{1}{2} s^T ((L+B)\otimes I_m)^{-1} s + \sum_{i=1}^{n} \frac{(1-\|\theta_i\|)\tilde{\omega}_i^2}{2\sigma}. \tag{17}$$

Taking the derivative of (17), one can obtain

$$\begin{aligned}\dot{V} &= s^T((L+B)\otimes I_m)^{-1}\big\{Ke_v + ((L+B)\otimes I_m)(u - \theta u - \mathbf{1}\otimes u_0 + F - \mathbf{1}\otimes f_0)\big\} \\ &\quad - \sum_{i=1}^{n} \frac{(1-\|\theta_i\|)\tilde{\omega}_i\dot{\hat{\omega}}_i}{\sigma}.\end{aligned} \tag{18}$$

Substituting (16) into (18), one has

$$\begin{aligned}\dot{V} &= s^T((L+B)\otimes I_m)^{-1}\{-(B\cdot\mathbf{1})\otimes u_0 + \mathbf{b}\otimes u_0 + ((L+B)\otimes I_m)(-\theta u - \eta s \\ &\quad - \gamma \operatorname{sgn}(s) + F - \mathbf{1}\otimes f_0)\} - \sum_{i=1}^{n} \frac{(1-\|\theta_i\|)\tilde{\omega}_i\dot{\hat{\omega}}_i}{\sigma} \\ &= -s^T(\theta u + \eta s + \gamma\operatorname{sgn}(s) - F + \mathbf{1}\otimes f_0) - \sum_{i=1}^{n} \frac{(1-\|\theta_i\|)\tilde{\omega}_i\dot{\hat{\omega}}_i}{\sigma}.\end{aligned} \tag{19}$$

Using $s^T\operatorname{sgn}(s) = \|s\|$, thus (19) is changed into

$$\begin{aligned}\dot{V} &= s^T(F - \mathbf{1}\otimes f_0) - \sum_{i=1}^{n} \phi_i\|s_i\| - s^T(\theta u + \eta s) + \sum_{i=1}^{n}(\varphi_i - \hat{\omega}_i\varphi_i)\|s_i\| \\ &\quad - \sum_{i=1}^{n} \frac{(1-\|\theta_i\|)\tilde{\omega}_i\dot{\hat{\omega}}_i}{\sigma}.\end{aligned} \tag{20}$$

From Assumption 2.2, one can obtain

$$\begin{aligned}\|F - \mathbf{1}\otimes f_0\| &\le h_1\|(\|x_1 - x_0\|, \ldots, \|x_n - x_0\|)\| + h_2\|(\|v_1 - v_0\|, \ldots, \|v_n - v_0\|)\| \\ &\le h_1\|\bar{x}\| + h_2\|\bar{v}\| \le \|(L+B)^{-1}\|(h_1\|e_x\| + h_2\|e_v\|) \le \sum_{i=1}^{n}(\phi_i - c_2).\end{aligned} \tag{21}$$

Hence,

$$s^T(F - \mathbf{1}\otimes f_0) - \sum_{i=1}^{n}\phi_i\|s_i\| \le \sum_{i=1}^{n}(\phi_i - c_2)\|s_i\| - \sum_{i=1}^{n}\phi_i\|s_i\| \le -\sum_{i=1}^{n} c_2\|s_i\|. \tag{22}$$

Therefore, the following inequality is obtained

$$\begin{aligned}\dot{V} &\le \sum_{i=1}^{n}\|\theta_i\|\,\|s_i^T\|\left(\left\|\sum_{j\in N^i}(a_{ij}+b_i)^{-1}(b_i u_0 - Ke_{vi}) - \eta s_i\right\| + \phi_i + \zeta_i\right) \\ &\quad + \sum_{i=1}^{n}(1-\hat{\omega}_i)\varphi_i\,\|s_i^T\| - \sum_{i=1}^{n}\frac{(1-\|\theta_i\|)\tilde{\omega}_i\dot{\hat{\omega}}_i}{\sigma} - \sum_{i=1}^{n}(\eta\|s_i\| + c_2)\|s_i\| \\ &\le \sum_{i=1}^{n}(1-\hat{\omega}_i)\varphi_i\,\|s_i^T\| + \sum_{i=1}^{n}\|\theta_i\|\,\|s_i^T\|(-c_1 + \hat{\omega}_i\varphi_i) - \sum_{i=1}^{n}\frac{(1-\|\theta_i\|)\tilde{\omega}_i\dot{\hat{\omega}}_i}{\sigma}\end{aligned}$$

$$
\begin{aligned}
&- \sum_{i=1}^{n} (\eta \|s_i\| + c_2)\|s_i\| \\
&\le -\sum_{i=1}^{n} c_1\|\theta_i\| \left\|s_i^T\right\| + \sum_{i=1}^{n}(1-\hat{\omega}_i + \|\theta_i\|\hat{\omega}_i)\varphi_i \left\|s_i^T\right\| - \sum_{i=1}^{n} \frac{(1-\|\theta_i\|)\tilde{\omega}_i\dot{\hat{\omega}}_i}{\sigma} \qquad (23)\\
&\quad - \sum_{i=1}^{n} (\eta \|s_i\| + c_2)\|s_i\| \\
&= -\sum_{i=1}^{n} c_1\|\theta_i\| \left\|s_i^T\right\| + \sum_{i=1}^{n}(1-\|\theta_i\|)\tilde{\omega}_i\varphi_i \left\|s_i^T\right\| - \sum_{i=1}^{n} \frac{(1-\|\theta_i\|)\tilde{\omega}_i\dot{\hat{\omega}}_i}{\sigma} \\
&\quad - \sum_{i=1}^{n} (\eta \|s_i\| + c_2)\|s_i\| \\
&= -\sum_{i=1}^{n} (c_1\|\theta_i\| + \eta\|s_i\| + c_2)\left\|s_i^T\right\| \\
&\le 0.
\end{aligned}
$$

Therefore, we can obtain the states of system (7) can reach the sliding mode surface $s = 0$ and maintain on it. From Theorem 3.1 and Theorem 3.2, we can know the fault-tolerant consensus of multi-agent systems (2) and (3) can be obtained with the proposed control protocol (13). This completes the proof.

4. **Simulation Results.** In this section, an example is provided to validate the effectiveness of the theoretical results. We will consider a multi-robot system which consists of one leader indexed by 0, and four followers indexed by i ($i = 1, 2, 3, 4$). The communication topology is described in Figure 1.

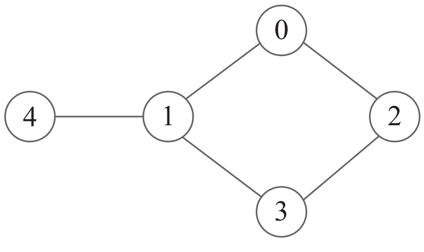

Figure 1. Communication topology of the leader-following system

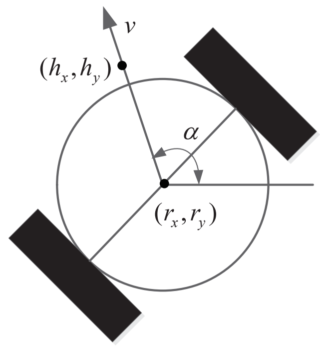

Figure 2. Non-holonomic differentially driven wheeled mobile robot

We consider the AmigoBots given in [2]. The scheme of the non-holonomic differentially driven wheeled mobile robot is shown in Figure 2, where (r_x, r_y) represents the position of the robot, and α denotes the orientation. Based on Figure 2, the kinematic equations for the ith robot are given as

$$\dot{r}_{xi} = v_i \cos(\alpha_i),\ \dot{r}_{yi} = v_i \sin(\alpha_i),\ \dot{\alpha}_i = \beta_i,$$

where v_i is the linear velocity of the robot i, (r_{xi}, r_{yi}) represents the position of the ith robot, and α_i denotes the orientation of the ith robot. β represents the angular rate. To simplify the control problem, we consider the problem of coordinating the hand positions of the robots rather than coordinating their center positions. The hand position can be described as

$$\begin{bmatrix} h_{xi} \\ h_{yi} \end{bmatrix} = \begin{bmatrix} r_{xi} \\ r_{yi} \end{bmatrix} + L_i \begin{bmatrix} \cos(\alpha_i) \\ \sin(\alpha_i) \end{bmatrix},$$

and by differentiating above equation twice with respect to time, one has

$$\begin{bmatrix} \ddot{h}_{xi} \\ \ddot{h}_{yi} \end{bmatrix} = \begin{bmatrix} \cos(\alpha_i) & -L_i \sin(\alpha_i) \\ \sin(\alpha_i) & L_i \cos(\alpha_i) \end{bmatrix} \begin{bmatrix} \dot{v}_i \\ \dot{\beta}_i \end{bmatrix} + \begin{bmatrix} \cos(\alpha_i) & -L_i \sin(\alpha_i) \\ \sin(\alpha_i) & L_i \cos(\alpha_i) \end{bmatrix}.$$

Let

$$\begin{bmatrix} \dot{v}_i \\ \dot{\beta}_i \end{bmatrix} = \begin{bmatrix} \cos(\alpha_i) & \sin(\alpha_i) \\ -\dfrac{1}{L_i} \sin(\alpha_i) & \dfrac{1}{L_i} \cos(\alpha_i) \end{bmatrix} \begin{bmatrix} u_{xi} \\ u_{yi} \end{bmatrix},$$

and

$$\begin{bmatrix} f_{1i} \\ f_{2i} \end{bmatrix} = \begin{bmatrix} -\sin(\alpha_i) v_i \beta_i & -L_i \cos(\alpha_i) \beta_i^2 \\ \cos(\alpha_i) v_i \beta_i & -L_i \sin(\alpha_i) \beta_i^2 \end{bmatrix}.$$

Hence, the dynamic equation of a differentially driven wheeled mobile robot can be written as

$$\begin{bmatrix} \ddot{h}_{xi} \\ \ddot{h}_{yi} \end{bmatrix} = \begin{bmatrix} u_{xi} \\ u_{yi} \end{bmatrix} + \begin{bmatrix} f_{i1} \\ f_{i2} \end{bmatrix}. \tag{24}$$

Thus, the model (24) has been taken into the form of (3) and the fault-tolerant control protocol (13) can be used by slight modification.

We assume that robot 2 and robot 3 are subject to the loss of effectiveness fault with $\theta_{21} = 0.3 + 0.1\sin(\pi t)$, $\theta_{22} = 0.2 + 0.2\cos(2t)$, $\theta_{31} = 0.5 + 0.2\sin(\pi t)$ and $\theta_{32} = 0.4 + 0.2\cos(2t)$. The initial conditions are chosen as $[h_{x0}, h_{y0}]^T = [0, 0]^T$, $[h_{x1}, h_{y1}]^T = [-4, -2]^T$, $[h_{x2}, h_{y2}]^T = [-0.5, -5.5]^T$, $[h_{x3}, h_{y3}]^T = [-2.5, -7]^T$ and $[h_{x4}, h_{y4}]^T = [-5.9, -2.5]^T$. The input of the leader is designed as $u_0 = 1$. The feedback gain and parameters are chosen as $K = 2$, $\eta = 0.5$, $\sigma = 0.362$, $c_1 = 0.137$ and $c_2 = 0.32$.

Under the proposed fault-tolerant consensus protocol, the simulation results of the position tracking error are shown in Figure 3, and the velocity tracking error trajectories are shown in Figure 4. It can be seen that the followers can track the leader asymptotically. In order to make a comparison, we carry out the normal consensus protocol proposed in [7] with the same actuator faults and obtain the tracking performances as shown in Figure 5 and Figure 6, respectively. It is shown that by using the normal consensus protocol given in [7], the followers 2 and 3 with loss effectiveness fault will not be able to track the leader. Thus, the fault-tolerant control scheme proposed in our paper ensures that the followers track the leader under system uncertainty and partial loss of actuator effectiveness.

5. **Conclusion.** In this work, a sliding mode fault-tolerant control protocol is proposed for leader-following multi-agent systems with partial loss of actuator effectiveness and system uncertainty. The adaptive sliding mode control algorithm has been applied to designing the distributed controller, which ensures that the system will arrive at the sliding surface despite of actuator faults. The asymptotic consensus ability has been proved by using Lyapunov theory. The proposed fault-tolerant consensus protocol approach can inherit the robustness of the sliding mode control, so the designed controllers improve the robustness abilities to system uncertainty. Finally, an example has been provided to illustrate validity of the developed approach to those fault-tolerant consensus control in leader-following multi-agent systems. It is shown that the combination of adaptive technique and sliding mode principle has provided holistic advantages, such as systematical and convenient controller design procedure, and robustness to system uncertainty. Future works include the study of fault-tolerant consensus of multi-agent systems in the presence of sensor faults and the related application on the robotic system.

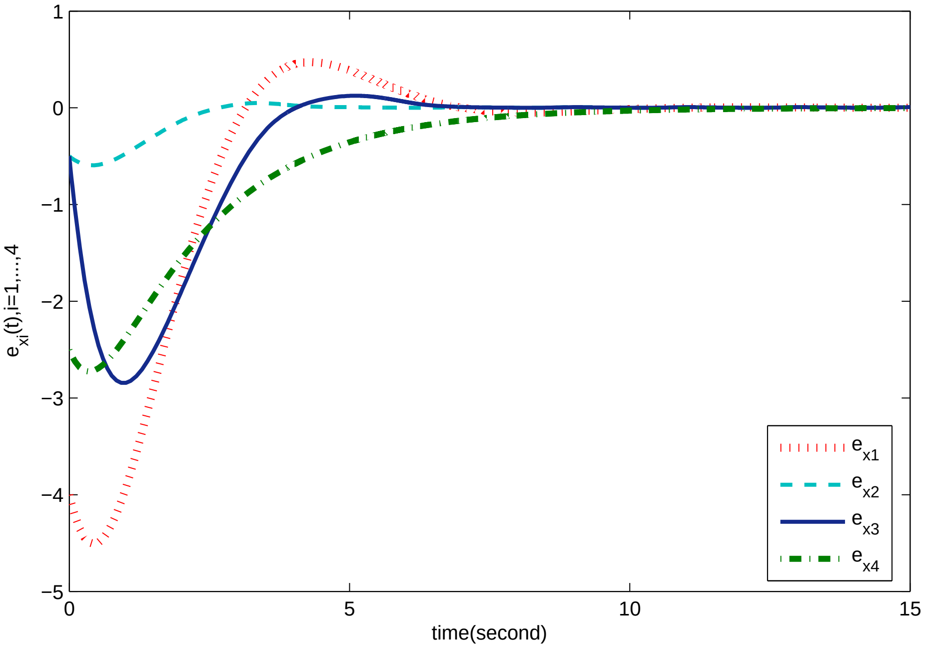

Figure 3. Trajectory of position consensus error with fault-tolerant consensus protocol

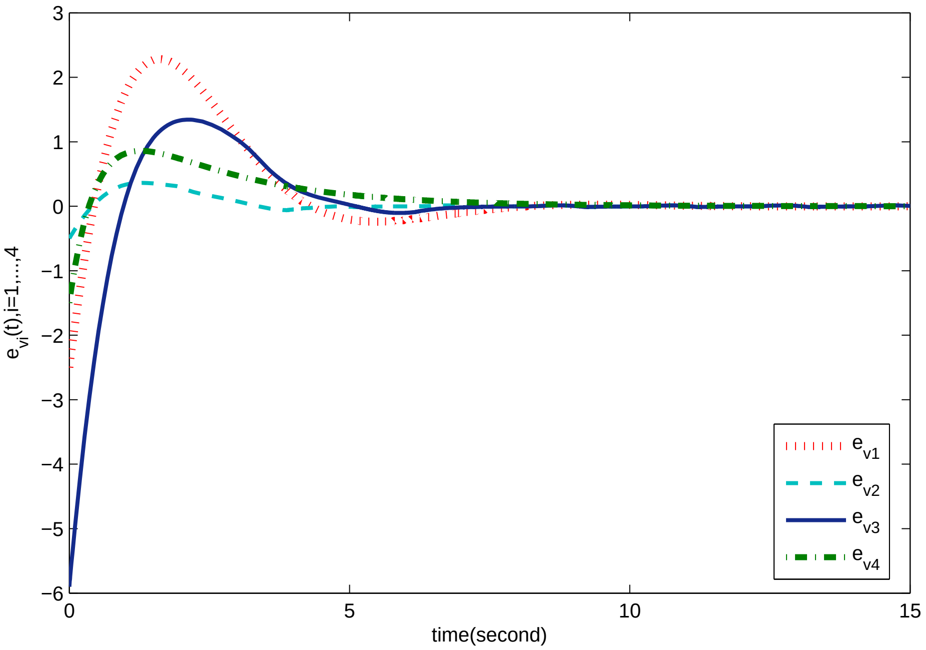

Figure 4. Trajectory of velocity consensus error with fault-tolerant consensus protocol

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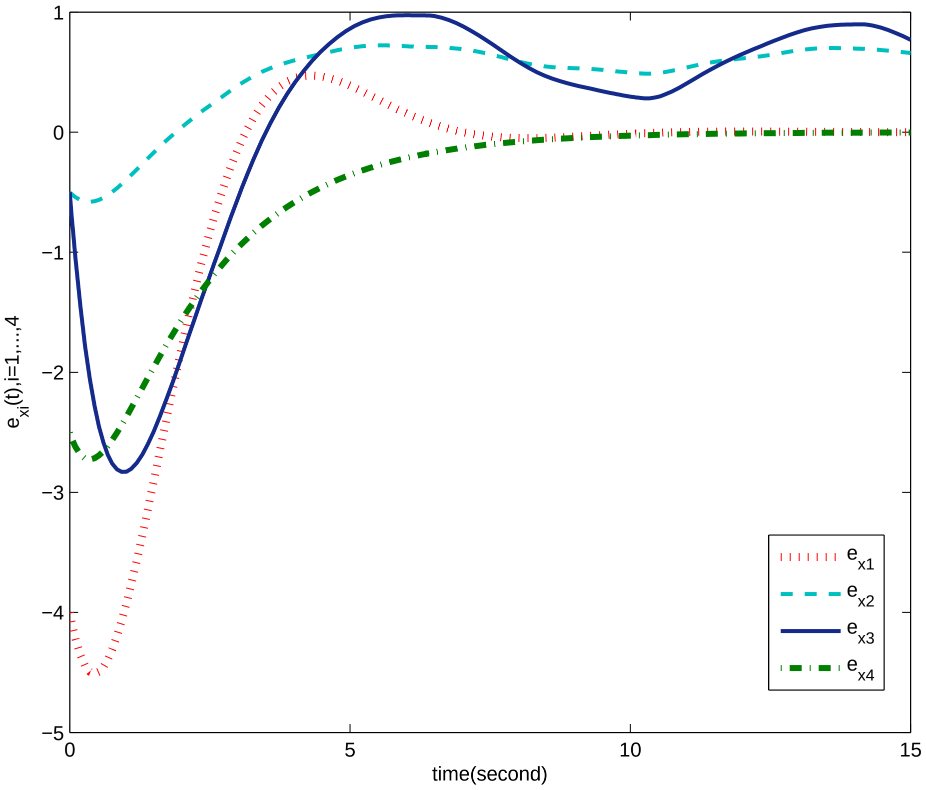

FIGURE 5. Trajectory of position consensus error with consensus protocol in [7]

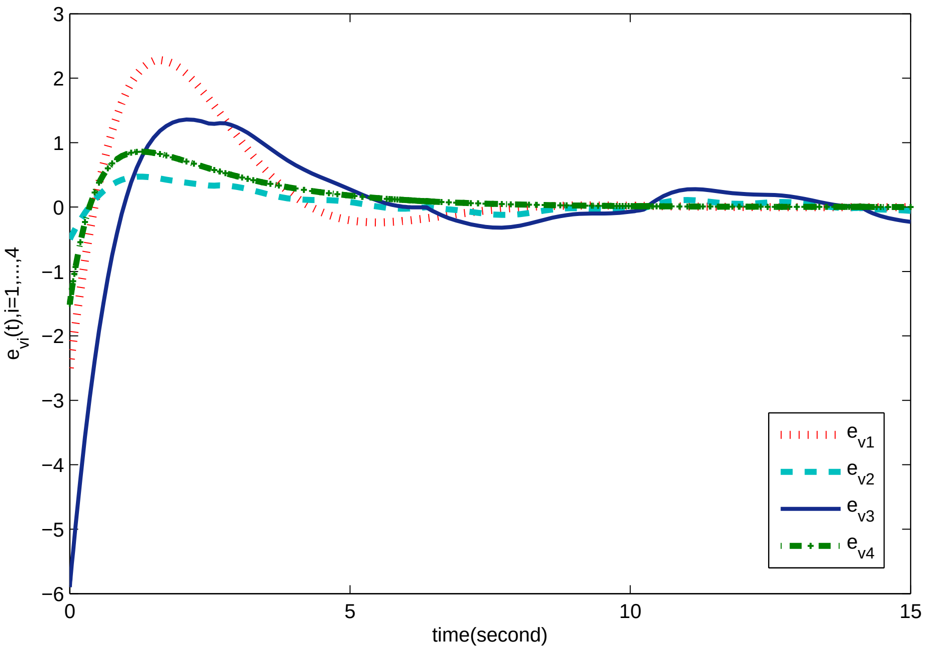

FIGURE 6. Trajectory of velocity consensus error with consensus protocol in [7]

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