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IMPROVED LANDWEBER ITERATION-BASED ANGULAR SUPERRESOLUTION FOR AIRBORNE FORWARD-LOOKING RADAR

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Abstract. *Airborne forward-looking radar (AFLR) has a wide range of civil and military applications, such as runway obstacle detection, low-altitude flight, autonomous landing. However, its angular resolution is low and determined by 3 dB beam width of the radar antenna beam. This paper studies an angular superresolution method based on Landweber iteration for AFLR systems. Firstly, signal model of the AFLR system in azimuth is built as a mathematical convolution of the antenna pattern and the targets' scattering. Secondly, maximum likelihood criterion and Landweber iteration are chosen to resolve the deconvolution inverse problem. Furthermore, simulations show that the method can effectively realize angular superresolution. Finally, the proposed method is validated by real radar imagery data processing, and compared with the improved Wiener filter technique that is the classic deconvolution superresolution method.*
Keywords: Airborne forward-looking radar (AFLR), Angular superresolution, Landweber iteration

1. **Introduction.** Airborne forward-looking radar (AFLR) has a wide range of applications in civilian and military fields, such as runway obstacle detection, low-altitude flight, autonomous landing sea/air warning and battlefield detection. In such applications, radar resolution is not only the most important parameter to evaluate performance of the radar, but also plays a decisive role in the accuracy of detection [1,2]. The high range resolution can be obtained by sending wideband signal and using pulse compression technique. However, conventional Doppler beam sharpening (DBS) and synthetic aperture radar (SAR) techniques fail to improve the angular resolution because the Doppler width is almost zero in azimuth as radar detecting the forward-looking area [3,4]. Geometry model of the AFLR systems is shown in Figure 1. To obtain good angular resolution in the flight direction, many methods are researched to obtain finer resolution in angle than the limit of the antenna beam, and we call these methods angular superresolution.

There is also a class method that is based on the inverse filtering theory because radar echo signal could be regarded as convolution of the radar antenna pattern and targets' scattering information in azimuth approximately [3]. Ill-posedness is inherent for the inverse problem. Selecting a regularization criterion appropriately makes the problem well-posed that is critical to deal with the inverse problem.

A direct inverse filtering method has been proposed in [5], [6] has presented a constrained iterative method, and [3,7] have studied an improved Wiener filtering method. These approaches can improve the angular resolution for AFLR systems to some extent. However, algorithms referred to are sensitive to noise, and they are difficult to be promoted in practical applications. For the low noise tolerance, [8-10] have proposed superresolution algorithm based on statistical optimization in Poisson noise.

An angular superresolution algorithm in Gaussian noise is presented in this paper. It is suitable for radar data that is affected by Gaussian noise in many cases. The rest of

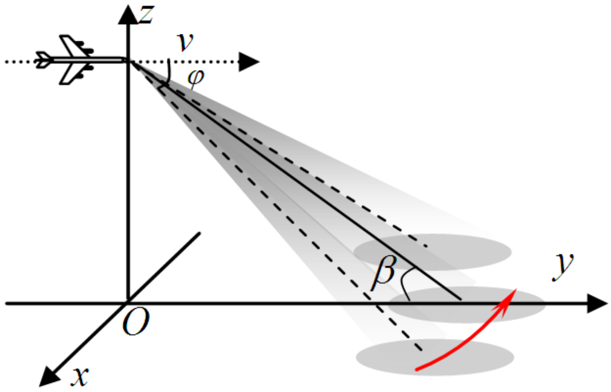

FIGURE 1. Geometry model of AFLR systems

this paper is organized as follows. In Section 2, signal model in the azimuth angle of AFLR systems is illustrated. Principle of the angular superresolution algorithm based on Landweber iteration in Gaussian noise is proposed in Section 3. In Section 4, simulations analysis and experiments prove the effectiveness of the method. Finally, the paper concludes in Section 5.

2. **Signal Model in Angle.** Amplitude of the received signal changes with the spatial dimensions of range and angle while the radar antenna beam scans different targets. The received signal can be equivalent to the output of a linear filter, and the targets' scattering into the range of the radar is linear filter input. Therefore, amplitude of the received signal shows the distribution of the observed targets' scattering, and is a filtered and weighted version of the actual scattering distribution [2,11]. While angle variation is only considered, signal model in angle is demonstrated

$$s(\theta) = h(\theta) * \sigma(\theta) \tag{1}$$

where $s(\theta)$ is the receiver output signal, $h(\theta)$ is the antenna power pattern, $*$ is the convolution operation, $\sigma(\theta)$ is the effective scattering. Considering the Gaussian noise $n(\theta)$, (1) is recast

$$s(\theta) = h(\theta) * \sigma(\theta) + n(\theta) \tag{2}$$

The generalization of the received signal is a convolution of the targets' scattering and the actual antenna pattern. The antenna pattern is equivalent to the angular impulse response.

For mathematical simplicity, the signal model in (2) can be rewritten with a matrix-vector form by boldface symbols

$$\boldsymbol{s} = H\boldsymbol{\sigma} + \boldsymbol{n} \tag{3}$$

The matrix H is the convolution operation matrix which is a circular matrix model that could be used to realize convolution by fast Fourier transformation (FFT). Assuming that size of H is $L \times L$, then $\boldsymbol{s}$, $\boldsymbol{\sigma}$, $\boldsymbol{n}$ are vectors of size $L \times 1$. $\boldsymbol{s}$, $\boldsymbol{\sigma}$, $\boldsymbol{n}$ present the received signal, the targets' scattering, and noise respectively.

3. **Angular Superresolution Algorithm.** Based on the Bayes formula the maximum likelihood criterion finds the estimated value $\tilde{\sigma}$ which satisfies $s(\theta)$

$$P(\boldsymbol{s}\,|\tilde{\sigma}\,) = \max_{\sigma}(P(\boldsymbol{s}\,|\sigma\,)) \tag{4}$$

where $P(\cdot)$ represents the probability density function. The signal received $\boldsymbol{s}$ is the convolution of the antenna pattern and the targets' scattering. Then the original value at the ith element in the signal received is

$$E(s_i) = \sum_j H_{ij}\sigma_j \quad j = 1 \cdots L \tag{5}$$

where H_{ij} is the (i, j)th element of matrix H, σ_j is the jth element of vector $\boldsymbol{\sigma}$.

Ignoring other noise, the actual ith pixel value s_i in $\boldsymbol{s}$ is the result of Gaussian distribution of the mean $\sum_j H_{ij}\sigma_j$. Thus, the relation of s_i and ε is written as follows

$$P(s_i|\boldsymbol{\sigma}) = \frac{1}{\sqrt{2\pi\varepsilon}} \exp\left(-\frac{[s_i - (\boldsymbol{H\sigma})_i]^2}{2\varepsilon^2}\right) \tag{6}$$

Each element in the received signal $\boldsymbol{s}$ is regarded as an independent Gaussian distribution and has the same variance ε^2, and

$$P(\boldsymbol{s}|\boldsymbol{\sigma}) = \prod_{i=1}^{L} \frac{1}{\sqrt{2\pi\varepsilon}} \exp\left(-\frac{[s_i - (\boldsymbol{H\sigma})_i]^2}{2\varepsilon^2}\right) \tag{7}$$

Then the value estimated $\tilde{\sigma}$ which satisfies the maximum likelihood criterion is

$$\tilde{\boldsymbol{\sigma}} = \arg\max_{\boldsymbol{\sigma}} \left[\prod_{i=1}^{M} \frac{1}{\sqrt{2\pi\varepsilon}} \exp\left(-\frac{[s_i - (\boldsymbol{H\sigma})_i]^2}{2\varepsilon^2}\right)\right] \tag{8}$$

Function $\ln(x)$ is a monotonic function in terms of x, and $\ln P(\boldsymbol{s}|\boldsymbol{\sigma})$ is much easier to deal with. The logarithm of $P(\boldsymbol{s}|\boldsymbol{\sigma})$ will be considered. Maximizing the logarithm of $P(\boldsymbol{s}|\boldsymbol{\sigma})$ is equal to minimizing the objective function

$$J(\boldsymbol{\sigma}) = \frac{(||\boldsymbol{s} - \boldsymbol{H\sigma}||_2)^2}{2} \tag{9}$$

where $|| \bullet ||_2$ is l_2 norm. To acquire the maximum value of $P(\boldsymbol{s}|\boldsymbol{\sigma})$, differentiating $J(\boldsymbol{\sigma})$ with respect to $\boldsymbol{\sigma}$ and the gradient vector is:

$$\frac{\partial J(\boldsymbol{\sigma})}{\partial \boldsymbol{\sigma}} = \boldsymbol{H}^{\mathrm{T}}\boldsymbol{H\sigma} - \boldsymbol{Hs} = 0 \tag{10}$$

where $\boldsymbol{H}^{\mathrm{T}}$ is transposition of $\boldsymbol{H}$. The corresponding Euler equation is

$$\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H\sigma} - \boldsymbol{Hs} = 0 \tag{11}$$

The Landweber algorithm is equal to the simplest iterative method for approximating the least-square solutions of Equation (11). The iterative process is

$$\boldsymbol{\sigma}^{k+1} = \boldsymbol{\sigma}^k + \tau\boldsymbol{H}^{\mathrm{T}}(\boldsymbol{s} - \boldsymbol{H\sigma}^k) \tag{12}$$

where k denotes the kth iteration, τ is a relaxation parameter controlling the convergence of the iterative process [12]. In order to guarantee the convergence, the value of τ is given by

$$0 < \tau < \frac{2}{\eta_1^2} \tag{13}$$

where η_1 is the largest singular value of matrix $\boldsymbol{H}$. The value of τ is usually set to 0.01.

Because the Landweber iteration algorithm is a linear iteration method, it has limited capability of superresolution from the frequency domain explanation. The algorithm does not realize spectral extrapolation. Furthermore, for the received radar signal which contains intensities near zero, negative intensities will be introduced to the following iteration step, due to the lack of high spatial frequencies [13].

Thus, the iteration process must guarantee the positivity of the estimated signal. Spectral extrapolation is reliable estimation of high frequencies. Utilization of a priori known information during the iteration processing is an effective method. Projection on convex sets (POCS) can modify the Landweber algorithm to take account of priori information of the solution [14]. Combining with the POCS method, the improved Landweber algorithm is

$$\boldsymbol{\sigma}_p^{k+1} = p_c\left[\boldsymbol{\sigma}^{k+1}\right] \tag{14}$$

where p_c is the projection operator onto the constraint set c. It produces frequencies beyond the passband because p_c is non-linear operator. The improved Landweber iteration-based algorithm can enhance the convergence rate and stabilize the solution. The simple and natural convex set is nonnegative constraint of the solution [15]. We can impose the constraint that the radar signal is nonnegative on the iteration.

$$p_c\boldsymbol{\sigma} = \begin{cases} \boldsymbol{\sigma} & \text{if } \boldsymbol{\sigma} > 0 \\ 0 & \text{if } \boldsymbol{\sigma} \leq 0 \end{cases} \tag{15}$$

Using FFT and inverse fast Fourier transformation (IFFT) computation achieves iterative process. Each of iteration mainly includes two FFT, one IFFT and two element-by-element multiplications.

4. **Simulations and Experiments.**

4.1. **Simulations.**
Referring to signal modeling, angular superresoluiton, points simulations are made to evaluate effectiveness of the proposed algorithm. The simulation parameters are listed in Table 1. The radar signal transmitted is LFM signal.

TABLE 1. Simulation parameters

Parameter	Value
Carrier frequency	10 GHz
Chirp bandwidth	30 MHz
Sample frequency	90 MHz
Pulse duration	5 μs
Platform velocity	100 m/s
Pulse repletion frequency (PRF)	2000 Hz
Beam width	3°
Antenna scanning speed	30 °/s
Range	9-11 Km
Azimuth	$-6°$-6°

Figure 2 is the original scene that consists of eight point targets, the interval of 100 meters in the range, the interval of 1 degrees in the azimuth. Figure 2(a) is the radar echo signal referring to signal modeling. Figure 2(b) is the echo signal after pulse compression. It is clear that there is range migration in range, and targets are blurred by the radar antenna pattern. Figure 2(c) is the signal scene after range migration correction. Angular superresolution processing result where targets are distinguished easily is demonstrated in Figure 2(d).

4.2. **Experiments.**
The proposed algorithm is validated by the real radar imagery data, and compared with the improved Wiener filter technique [3,5]. Figure 3 is a real millimeter wave radar imagery obtained from the airborne forward-looking scanning radar. It includes an airport apron, five aircrafts, and other buildings. The airport apron and aircrafts are shown as marking in the figure. Original real beam radar imagery is shown in the top part of Figure 3(a). Aircrafts are difficult to be distinguished, and boundary of the airport apron is blurred obviously. In Figure 3(b), the improving angular resolution result processed by the Wiener filter technique is illustrated in the middle part. Then five aircrafts become clearer, and are separated approximately. In Figure 3(c), it shows the result processed by the proposed algorithm. It proves that the proposed method sharpens the boundary and clearly distinguishes the adjacent aircrafts.

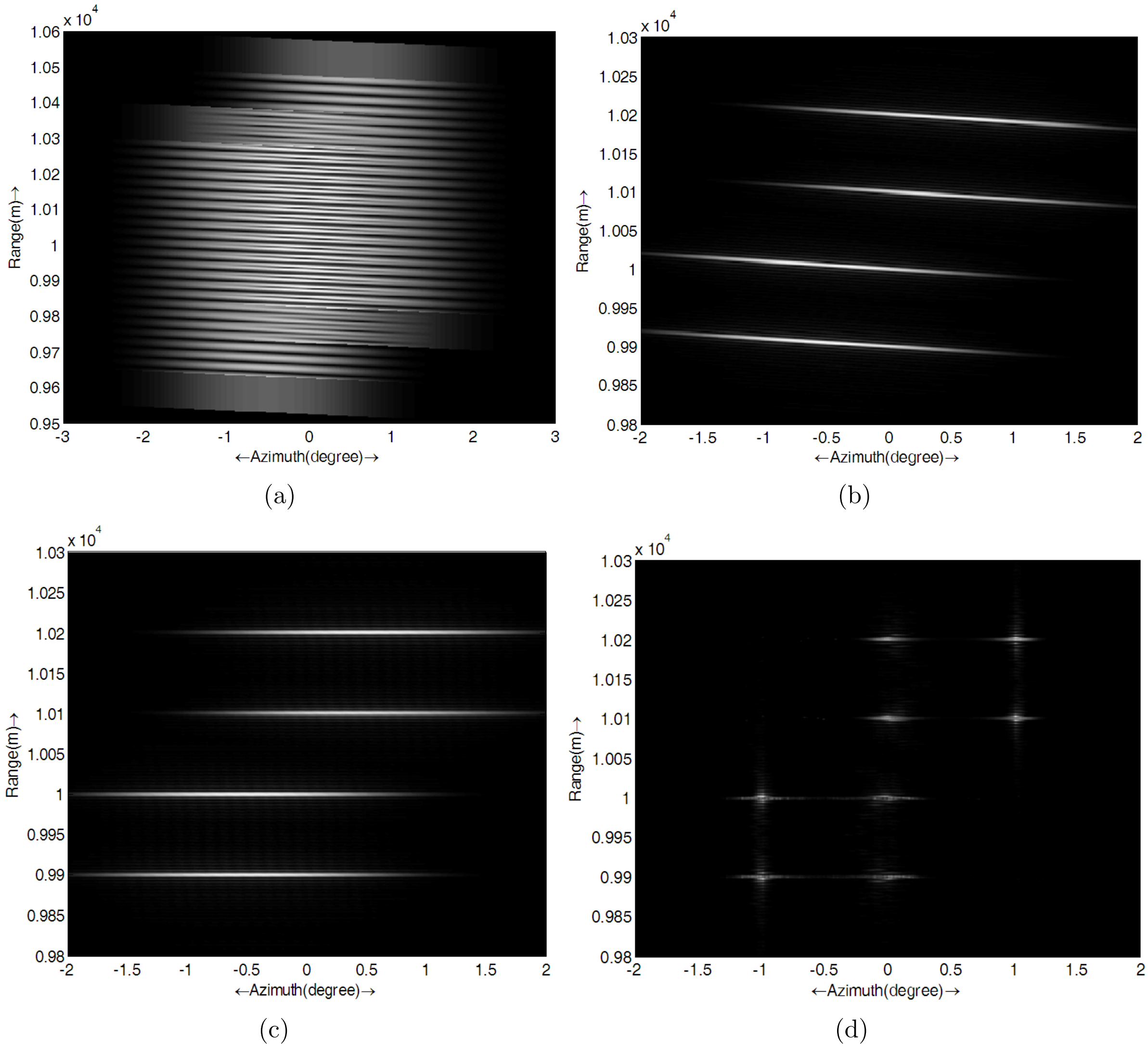

Figure 2. Simulations of point targets: (a) radar echo signal, (b) echo signal after pulse compression, (c) signal scene after range migration correction, (d) angular superresolution processing result

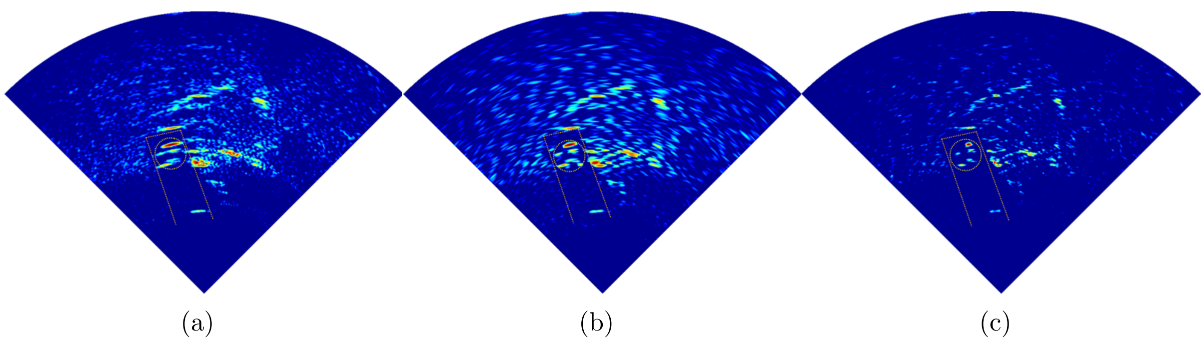

Figure 3. A real radar imagery from the airborne forward-looking scanning radar: (a) original radar imagery, (b) imagery processed by the improved Wiener filter technique, (c) imagery processed by the proposed algorithm

5. **Conclusions.** This paper studies the angular superresolution method based on Landweber iteration for AFLR system. Maximum likelihood criterion is chosen to regularize the deconvolution inverse problem.

Simulations show that the proposed method can more effectively realize angular superresolution. Real radar imagery data processing validates the effectiveness of this method. Furthermore, the angular superresolution method based on maximum likelihood is compared with the Wiener filter technique that is the classic deconvolution superresolution method. It shows that the proposed method can endure more noise disturbance than conventional approaches.

The proposed algorithm has practical and extensive applications such as surveillance, autonomous landing of aircraft, navigation, guidance. The next step in our research work will consist in improving algorithm that reduces the computational complexity and achieving real time processing in our research work.

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REFERENCES

[1] M. I. Skolnik, *Radar Handbook*, 3rd Edition, McGraw-Hill, New York, 2008.
[2] G. W. Stimson, *Introduction to Airborn Radar*, 2nd Edition, Scitech Publishing, Inc., New Jersey, 1998.
[3] T. Suzuki, Radar beamwidth reduction techniques, *Aerospace and Electronic Systems Magazine IEEE*, vol.13, no.5, pp.43-48, 1998.
[4] W. Li, J. Yang and Y. Huang, Keystone transform-based space-variant range migration correction for airborne forward-looking scanning radar, *Electronics Letters*, vol.48, no.2, pp.121-122, 2012.
[5] S. Senmoto and D. G. Childers, Signal resolution via digital inverse filtering, *IEEE Trans. Aerospace and Electronic Systems*, vol.5, pp.633-640, 1972.
[6] M. A. Richards, Iterative noncoherent angular superresolution [radar], *Proc. of the 1988 IEEE National Radar Conference*, 1988.
[7] F. Sadjadi, Radar beam sharpening using an optimum FIR filter, *Circuits, Systems and Signal Processing*, vol.19, no.2, pp.121-129, 2000.
[8] D. Zhou, Y. Huang and J. Yang, Radar angular superresolution algorithm based on Bayesian approach, *IEEE the 10th International Conference on Signal Processing (ICSP)*, 2010.
[9] J. Guan et al., Improving angular resolution based on maximum a posteriori criterion for scanning radar, *Radar Conference (RADAR)*, 2012.
[10] J. Guan, J. Yang, Y. Huang et al., Maximum a posteriori-based angular superresolution for scanning radar imaging, *IEEE Trans. Aerospace and Electronic Systems*, vol.50, no.3, pp.2389-2398, 2014.
[11] D. Li, Y. Zhong and P. Li, An improved maximum likelihood-based angular superresolution algorithm for scanning radar, *ICIC Express Letters, Part B: Applications*, vol.6, no.11, pp.3011-3016, 2015.
[12] Y. Li and W. Yang, Image reconstruction by nonlinear Landweber iteration for complicated distributions, *Measurement Science & Technology*, vol.19, no.9, 2008.
[13] A. Neubauer, Some generalizations for Landweber iteration for nonlinear ill-posed problems in Hilbert scales, *Journal of Inverse and Ill-Posed Problems*, vol.24, no.4, pp.393-406, 2016.
[14] C. Wang, The convergence properties for regularized Landweber method, *Journal of Optimization Theory & Applications*, pp.1-14, 2016.
[15] R. Li, Z. Gan, Z. Cui, M. Wu and X. Zhu, Distributed adaptive compressed video sensing using smoothed projected landweber reconstruction, *China Communications*, vol.10, no.11, pp.58-69, 2013.