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FAULT TOLERANT ATTITUDE CONTROL DESIGN FOR HYPERSONIC VEHICLES WITH ACTUATOR FAULTS

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Abstract. *In this study, a fault tolerant control approach is presented for the hypersonic vehicle attitude systems with unknown actuator faults and external disturbance by using terminal sliding mode technique. Firstly, the nonlinear attitude systems of hypersonic vehicles are given, which contains dynamics equations and kinematics equations. For the aim of fault tolerant control, the actuator fault model is introduced. Secondly, to obtain the estimation value of the effects of unknown actuator fault, a nonlinear sliding mode observer is developed and an adaptive fault estimation law is also developed. Then, the fault tolerant tracking controller is designed for the faulty attitude systems of hypersonic vehicles by using terminal sliding mode technique and the estimated fault information. Meanwhile, the Lyapunov theory is used to analyze the stability of the closed-loop attitude systems. Finally, simulation for the attitude dynamics models shows the feasibility of the proposed fault tolerant scheme.*
Keywords: Fault tolerant control, Hypersonic vehicle, Actuator faults, Terminal sliding mode control

1. **Introduction.** It is well known that near space is a space which has not been developed by human beings and its potential application value is gradually being recognized by the people in last ten years. A hypersonic vehicle is a class of typical near space vehicle, whose flight speed is more than 5 Mach in near space domain. Compared to traditional aerospace planes, the hypersonic vehicle has lots of advantages, such as strategy monitoring, tactical attack, and high utility of consuming ratio. Therefore, it possesses the important military application values [1]. For adapting the complex near space flight environment, hypersonic vehicles make use of the engine-airframe integration technology. This configuration has a strong coupling among the elastic airframe, propulsion system and structural dynamics. This brings highly nonlinear dynamic characteristics to the dynamic models. Considering its special and complicated flight circumstances, its controller needs better reliability and robustness [2].

In recent years, many results have been published about control design and stability analysis of hypersonic vehicles. In [3], an attitude controller design approach is given to the attitude control systems of hypersonic vehicles in control input saturation case by using the classical nonlinear control scheme – backstepping control method. In [4], the authors introduce a reference model to a flexible hypersonic vehicle, and then an adaptive sliding mode control approach is proposed, which could guarantee the asymptotical stability of the closed-loop dynamical systems. In [5], a guidance law is designed for hypersonic vehicles using feedback linearization technology to realize high precision guidance; meanwhile, a sliding mode controller design approach is proposed for hypersonic vehicles in dive phase to realize the quick maneuvering flight. In [6], an auxiliary error compensation control strategy is provided for the longitudinal dynamics of a flexible hypersonic vehicle by using adaptive neural networked technology, which guarantees that the good velocity tracking and altitude tracking are satisfied in actuator saturation case. In [7], a tracking controller is designed for the longitudinal dynamics of hypersonic vehicle with parameter uncertainty, strong nonlinearity and control constraints, which could guarantee that the satisfactory tracking and the good robustness are achieved. In [8], the authors utilize the functional decomposition methodology to the dynamic model of hypersonic vehicles, such that the dynamic model of a hypersonic vehicle is decomposed into both velocity subsystem and altitude subsystem, and then two adaptive neural controllers are designed for altitude subsystem and velocity subsystem, respectively. In [9], a nonlinear robust control strategy is proposed for the hypersonic vehicle by using both small-gain arguments and adaptive control techniques, such that the asymptotic velocity tracking is achieved, and the flight-path angle reference trajectories belong to a given class of vehicle maneuvers. It is noted that the attitude control approaches developed in [3-9] do not consider the effects of all kinds of unknown faults to the closed-loop attitude control systems; in other words, the results obtained in [3-9] have not the capability of being fault tolerant.

For improving the safety and reliability of the plants, fault tolerant control (FTC) technology is often used to design all kinds of controller of the plants. For the hypersonic vehicle as a modern complex aircraft, FTC technology has become one of the alternative ways to enhance the safety and reliability of that. As we all know, fault tolerant control can be classified into two categories: passive approach and active approach. An advantage of passive approach is that a fixed controller has relatively modest hardware and software requirements. Active approach differs from passive approach in that they can adapt on-line to fault information. This on-line adaptation allows active FTC to deal with more faults and generally achieve better performance than passive FTC. Recently, some results about fault tolerant controller design for hypersonic vehicle have been published. In [10], the authors make use of sliding mode control theory to study the fault tolerant attitude controller design problem of hypersonic vehicles, and the asymptotic attitude angles tracking in actuator faulty case is guaranteed under the designed FTC strategy. In [11], a decentralized fault tolerant control scheme is proposed for the attitude dynamics of hypersonic vehicles, but it must rely on a complex fault diagnosis and identification module for estimating the unknown actuator fault. In [12], a robust passive fault tolerant tracking controller is designed for the longitudinal dynamic model of hypersonic vehicles with unknown sensor faults by utilizing H_∞ observer control approach and linear matrix inequality techniques. It is noted that most results described above are concerned with passive FTC, and that active FTC method for hypersonic vehicle attitude systems have not been fully investigated yet, which remains challenging and motivates us to do this study.

In this study, an active fault tolerant tracking controller is designed for the faulty attitude control systems of hypersonic vehicles by using terminal sliding mode control technique. Firstly, the nonlinear attitude dynamical system with unknown actuator faults is introduced. To obtain the estimation of actuator faults effects, a nonlinear sliding mode observer is designed. Then an active fault tolerant tracking control design approach is given in the frame of sliding mode control structure, which guarantees that the closed-loop attitude system is asymptotically stable. Finally, the effectiveness of FTC approach is demonstrated by a simulation example.

2. **The Attitude Control Systems Description.** In 1990, NASA's Langley Research Center developed a winged-cone simulation model for a general hypersonic vehicle, which is widely used in research on attitude control. The attitude dynamical model of hypersonic vehicle, which is simplified from the six degrees of freedom vehicle model, can be described

in the body coordinate frame as [2]:

$$\dot{\phi} = \omega_y \sin\gamma + \omega_z \cos\gamma \tag{1}$$

$$\dot{\psi} = (\omega_y \cos\gamma + \omega_z \sin\gamma)\sec\phi \tag{2}$$

$$\dot{\gamma} = \omega_x - (\omega_y \cos\gamma - \omega_z \sin\gamma)\tan\phi \tag{3}$$

$$\dot{\omega}_x = J_{xx}^{-1} M_x + J_{xx}^{-1}(J_{yy} - J_{zz})\omega_z\omega_y \tag{4}$$

$$\dot{\omega}_y = J_{yy}^{-1} M_y + J_{yy}^{-1}(J_{zz} - J_{xx})\omega_x\omega_z \tag{5}$$

$$\dot{\omega}_z = J_{zz}^{-1} M_z + J_{zz}^{-1}(J_{xx} - J_{yy})\omega_y\omega_x \tag{6}$$

where ω_x, ω_y and ω_z are the pitch rate, yaw rate, and roll rate, respectively. ϕ, ψ and γ are the pitch angle, yaw angle, and roll angle, respectively. J_{xx}, J_{yy} and J_{zz} are the moment of inertia, and M_x, M_y and M_z are the total rolling moment, total yawing moment, and total pitching moment, respectively, and can be written as follows:

$$M_x = qbSm_x, \quad M_y = qbSm_y, \quad M_z = qbSm_z \tag{7}$$

where $q = 0.5\rho\nu^2$ is the dynamic pressure, ν is the velocity of the vehicle, and ρ is the atmosphere density, S is the reference area, b is the lateral-directional reference length, and m_x, m_y and m_z are the total rolling moment coefficient, total yawing moment coefficient, and total pitching moment coefficient, respectively.

The state variable ω of the rotation motion model, the output variable θ, and the control torque τ used in this study are given by

$$\omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \quad \theta = \begin{bmatrix} \phi \\ \psi \\ \gamma \end{bmatrix}, \quad \tau = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} \tag{8}$$

Thus, the rotation motion given in (1) and (2) can be rewritten as

$$\dot{\theta} = F_\theta \omega, \quad \dot{\omega} = J^{-1} F_\omega + J^{-1}\tau \tag{9}$$

where

$$F_\theta = \begin{bmatrix} 0 & \sin\gamma & \cos\gamma \\ 0 & \cos\gamma\sec\phi & -\sin\gamma\sec\phi \\ 1 & -\cos\gamma\tan\phi & \sin\gamma\tan\phi \end{bmatrix}, \quad F_\omega = \begin{bmatrix} (J_{yy} - J_{zz})\omega_z\omega_y \\ (J_{zz} - J_{xx})\omega_x\omega_z \\ (J_{xx} - J_{yy})\omega_x\omega_y \end{bmatrix}, \quad J = \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix}$$

The control torque τ is generated by the aerodynamic surfaces, which are described by the following,

$$\tau = D(\cdot)u = D(\cdot)[u_r,\ u_e,\ u_l]^T \tag{10}$$

where $D(\cdot) \in R^{3\times 3}$ is a control input distributed matrix, and $u \in R^3$ is the vector of aerodynamic surface deflections. u_r, u_e, and u_l are the control surface deflections of right elevon, rudder and left elevon, respectively.

Let us consider that an actuator fault occurs and the time-varying unknown external disturbance exists. In particular, consider the situation in which the actuator loses complete or partial control power and the actuator lock-in-place fault. The actuator fault model can be modeled as

$$u_i = u_{ci} + e_i(t)(\bar{u}_{ci} - u_{ci}) \tag{11}$$

where u_{ci} is the desired output torques, u_i denotes the actual control signal of the ith actuator, with $i = 1, 2, 3$. e_i is an unknown constant, which represents the effectiveness factor of the ith control surface. In our study, we mainly introduce three kinds of situations. The first case $e_i = 0$ means the ith actuator is healthy. The case $e_i(t) = 1$ and $\bar{u}_{ci} \neq 0$ implies that the ith actuator lock-in-place fault. $0 < e_i(t) < 1$ and $\bar{u}_{ci} = 0$ correspond to the case in which the ith actuator partially loses its actuating power, but still works all the time. According to (11), the actuator control torque u in faulty case can be described as follows

$$u = u_c + E(\bar{u}_c - u_c) = u_c + u_f \tag{12}$$

where $E = \text{diag}\{e_i\}$ and u_f can be seen as an unknown additive signal. Substituting the faulty control torque (6) into (4), the general nonlinear spacecraft attitude dynamics models with actuator fault can be rewritten as

$$\begin{cases} \dot{\theta} = F_\theta \omega \\ \dot{\omega} = J^{-1}F_\omega + J^{-1}Du_c + J^{-1}Du_f + d(t) \\ y = \theta \end{cases} \tag{13}$$

The control objective is to design an active FTC strategy for the hypersonic vehicle attitude systems, such that the desired attitude commands could be followed asymptotically by the actual attitude angles in spite of unknown actuator faults occurrence. Figure 1 shows the structure of fault tolerant attitude control systems.

Assumption 2.1. *For the attitude motion model (6) of the hypersonic vehicle, matrix F_θ is invertible.*

Assumption 2.2. *The time-varying external disturbance $d(t)$ in the system is unknown but bounded and satisfies the following inequality: $\|d\| < \gamma$, where γ is a positive scalar.*

Assumption 2.3. *The nonlinear function F_ω is locally Lipschitz nonlinear, i.e., there exists a constant β that satisfied $\left\|F_\omega - \hat{F}_\omega\right\| \le \beta\|\omega - \hat{\omega}\|$.*

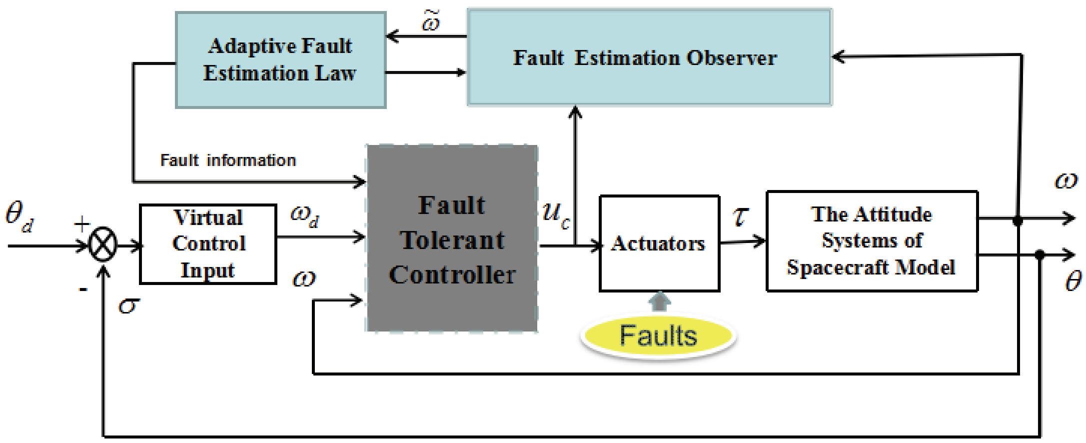

FIGURE 1. Fault tolerant control systems diagram

3. **Faults Estimation Observer Design.** In this section, a nonlinear robust fault estimation observer is designed for the dynamic equation of the attitude motion model to obtain the estimated actuator fault value. We can design the following observer for the information of the actuator fault by the angular velocity loop:

$$\dot{\hat{\omega}} = -L\tilde{\omega} + J^{-1}\hat{F}_\omega + J^{-1}Du_c + J^{-1}D\hat{u}_f + \gamma\varrho \tag{14}$$

where $\hat{\omega}$ is the estimation value of ω, $\tilde{\omega} = \hat{\omega} - \omega$ is the observer error variable. $L = \text{diag}\{\lambda_1, \lambda_2, \lambda_3\} > 0$ is a positive definite matrix to be defined in advance and $\varrho = [1, 1, 1]^T$. $\hat{u}_f$ denotes the estimation of the fault factor, which is obtained as

$$\dot{\hat{u}}_f = -\frac{1}{\rho}D^T J^{-T}\tilde{\omega} \tag{15}$$

where $\rho > 0$. Let $\tilde{u}_f = \hat{u}_f - u_f$ and $k = \gamma\varrho - d$. By using the aforementioned adaptive fault estimation observer, the resulting state estimation error dynamic is

$$\dot{\tilde{\omega}} = -L\tilde{\omega} + J^{-1}D\tilde{u}_f + J^{-1}\left(F_\omega - \hat{F}_\omega\right) - k \tag{16}$$

Hence, we obtain the following results for the aforementioned error dynamic.

Theorem 3.1. *Considering the adaptive fault estimation observer in (14) and (15), the state estimation error dynamic given by (16) is globally asymptotically stable, i.e., for any initial conditions* $\tilde{\omega}(0)$*, we have* $lim_{t\to\infty}\tilde{\omega}(t) = 0$*, and* $\tilde{u}_f(t)$ *is bounded.*

Proof: Consider the following Lyapunov function

$$V_1 = \frac{1}{2}\tilde{\omega}^T\tilde{\omega} + \frac{\rho}{2}\tilde{u}_f^T\tilde{u}_f \tag{17}$$

The derivative of V_1 along the trajectory of the augmented state error dynamic (17) can be written as

$$\dot{V}_1 = \tilde{\omega}^T\dot{\tilde{\omega}} + \rho\tilde{u}_f^T\dot{\tilde{u}}_f = -\tilde{\omega}^TL\tilde{\omega} + \tilde{\omega}^TJ^{-1}D\tilde{u}_f - \tilde{\omega}^Tk + \rho\tilde{u}_f^T\dot{\tilde{u}}_f + \tilde{\omega}^TJ^{-1}\left(F_\omega - \hat{F}_\omega\right) \tag{18}$$

Substituting adaptive update law (15) into (18) yields the following equation:

$$\dot{V}_1 = -\tilde{\omega}^TL\tilde{\omega} - \tilde{\omega}^Tk + \tilde{\omega}^TJ^{-1}\left(F_\omega - \hat{F}_\omega\right) \tag{19}$$

In terms of Assumption 2.3, we select appropriate positive definite matrix L, such that

$$\dot{V}_1 \le -\left(||L|| - \beta||J^{-1}||\right)||\tilde{\omega}||^2 - ||\tilde{\omega}||\cdot||k|| \le 0 \tag{20}$$

Therefore, the systems of adaptive fault estimation observer are ultimately uniformly bounded. Namely, the designed observer (14) could asymptotically estimate the unknown actuator fault. This proof is completed.

4. **Fault Tolerant Attitude Control Design.** In this section, a fault tolerant attitude tracking controller is designed for the faulty plant (13) based on the terminal sliding mode technique. Firstly, it is defined that $\tilde{\theta} = \theta - \theta_d$, θ_d is the desired attitude command signal. For the attitude angle loop, the first terminal sliding mode surface is given by,

$$s_1 = \tilde{\theta} + \int_0^t \left(A_1\tilde{\theta} + B_1\tilde{\theta}^{q_1/p_1}\right)\mathrm{d}t \tag{21}$$

where $A_1 = \mathrm{diag}\{a_{11}, a_{12}, a_{13}\}$, $B_1 = \mathrm{diag}\{b_{11}, b_{12}, b_{13}\}$, $(a_{1i} > 0,\ b_{1i} > 0,\ i = 1, 2, 3)$, p_1, q_1 $(q_1 < p_1)$ are positive odd integers.

Taking the time differentiating of s_1 in (21), we have

$$\dot{s}_1 = \dot{\tilde{\theta}} + A_1\tilde{\theta} + B_1\tilde{\theta}^{q_1/p_1} = F_\theta\tilde{\omega} + F_\theta\omega_d - \dot{\theta}_d + A_1\tilde{\theta} + B_1\tilde{\theta}^{q_1/p_1} \tag{22}$$

where $\tilde{\omega} = \omega - \omega_d$.

In this study, the following exponential reaching law is chosen as,

$$\dot{s}_1 = -k_1s_1 - \varepsilon_1\mathrm{sgn}(s_1) \tag{23}$$

where k_1 and ε_1 are two positive constant scalars.

According to (22) and (23), the virtual control input ω_d is selected as

$$\omega_d = -F_\theta^{-1}\left(A_1\tilde{\theta} + B_1\tilde{\theta}^{q_1/p_1} - \dot{\theta}_d + k_1s_1 + \varepsilon_1\mathrm{sgn}(s_1)\right) \tag{24}$$

Based on the angular velocity error $\tilde{\omega}$, the second terminal sliding mode surface is designed for the inner loop systems, namely,

$$s_2 = \tilde{\omega} + \int_0^t \left(A_2\tilde{\omega} + B_2\tilde{\omega}^{q_2/p_2}\right)dt \tag{25}$$

where $A_2 = \text{diag}\{a_{21}, a_{22}, a_{23}\}$, $B_2 = \text{diag}\{b_{21}, b_{22}, b_{23}\}$, $(a_{2j} > 0,\ b_{2j} > 0,\ j = 1, 2, 3)$, p_2, q_2 $(q_2 < p_2)$ are positive odd integers.

Taking the time differentiating of s_2 in (25), we have

$$\begin{aligned}\dot{s}_2 &= \dot{\tilde{\omega}} + A_2\tilde{\omega} + B_2\tilde{\omega}^{q_2/p_2} \\ &= J^{-1}F_\omega + J^{-1}Du_c + J^{-1}Du_f + d(t) - \dot{\omega}_d + A_2\tilde{\omega} + B_2\tilde{\omega}^{q_2/p_2} \end{aligned} \tag{26}$$

Here, the exponential reaching law is given by the following,

$$\dot{s}_2 = -k_2 s_2 - \varepsilon_2 \text{sgn}(s_2) \tag{27}$$

where k_2 and ε_2 are two positive constant scalars.

According to (26) and (27), an active fault tolerant tracking controller is proposed for the faulty attitude systems

$$u_c = -\hat{u}_f - D^{-1}F_\omega - (J^{-1}D)^{-1}\left(\gamma l_3 + \dot{\omega}_d + A_2\tilde{\omega} + B_2\tilde{\omega}^{q_2/p_2} + k_2 s_2 + \varepsilon_2 \text{sgn}(s_2)\right) \tag{28}$$

In terms of the analysis described above, the main result of this study is summarized as follows.

Theorem 4.1. *For the attitude control systems (13) in actuator faults case, it is supposed that Assumptions 2.1-2.3 are satisfied, the proposed active fault tolerant tracking control law u_c described in (28) guarantees that all the signals of the whole closed-loop attitude systems are uniformly ultimately bounded and the attitude angle θ could asymptotical tracks the desired attitude command θ_d.*

Proof: For the whole closed-loop attitude control systems (13), a Lyapunov function is firstly given by

$$V = \frac{1}{2}s_1^T s_1 + \frac{1}{2}s_2^T s_2 \tag{29}$$

Taking the time derivative of V, it can be obtained that

$$\begin{aligned}\dot{V} = s_1^T\dot{s}_1 + s_2^T\dot{s}_2 = s_1^T &\left(F_\theta\tilde{\omega} + F_\theta\omega_d - \dot{\theta}_d + A_1\tilde{\theta} + B_1\tilde{\theta}^{q_1/p_1}\right) \\ &+ s_2^T\left(J^{-1}F_\omega + J^{-1}Du_c + J^{-1}Du_f + d - \dot{\omega}_d + A_2\tilde{\omega} + B_2\tilde{\omega}^{q_2/p_2}\right)\end{aligned} \tag{30}$$

Substituting the virtual control input ω_d described in (24) and the active fault tolerant controller u_c described in (28) into the above equality, we have

$$\begin{aligned}\dot{V} \le\ & -s_1^T k_1 s_1 - \varepsilon_1 s_1^T \text{sgn}(s_1) + s_1^T F_\theta\tilde{\omega} - s_2^T k_2 s_2 - \varepsilon_2 s_2^T \text{sgn}(s_2) \\ & + s_2^T J^{-1}Ds_2 + \tilde{u}_f^T J^{-1}D\tilde{u}_f\end{aligned} \tag{31}$$

According to the famous Young inequality, the following inequalities can be obtained,

$$s_1^T F_\theta\tilde{\omega} \le \frac{1}{2}s_1^T F_\theta F_\theta^T s_1 + \frac{1}{2}\tilde{\omega}^T\tilde{\omega} \le \frac{1}{2}\left(s_1^T F_\theta F_\theta^T s_1 + s_2^T s_2\right) \tag{32}$$

Substituting (32) into (31), we have that

$$\begin{aligned}\dot{V} \le\ & -s_1^T k_1 s_1 - \varepsilon_1 s_1^T \text{sgn}(s_1) - s_2^T k_2 s_2 - \varepsilon_2 s_2^T \text{sgn}(s_2) \\ & + \frac{1}{2}\left(s_1^T F_\theta F_\theta^T s_1 + s_2^T s_2\right) + s_2^T J^{-1}Ds_2 + \tilde{u}_f^T J^{-1}D\tilde{u}_f \\ \le\ & -s_1^T\left(k_1 I_3 - F_\theta F_\theta^T\right)s_1 - s_2^T\left(k_2 I_3 + \frac{1}{2}I_3 + J^{-1}D\right)s_2 \\ & -\varepsilon_1 s_1^T \text{sgn}(s_1) - \varepsilon_2 s_2^T \text{sgn}(s_2) + \tilde{u}_f^T J^{-1}D\tilde{u}_f\end{aligned} \tag{33}$$

It is noted that $\tilde{u}_f = \hat{u}_f - u_f$ is the fault estimation error, by selecting the appropriation fault estimation observer and adaptive fault estimation law, the fault estimation error $\tilde{u}_f$

could asymptotically converge to zero. Meanwhile, we chose the large enough k_i and ε_i, such that the inequality holds,

$$\dot{V} \leq -\left\|k_1 I_3 - F_\theta F_\theta^T\right\| \cdot \|s_1\|^2 - \varepsilon_1\|s_1\| - \varepsilon_2\|s_2\| - \left\|k_2 I_3 + \frac{1}{2}I_3 + J^{-1}D\right\| \|s_2\|^2 < 0 \quad (34)$$

Therefore, we can know that the output signals asymptotically track the reference commands under the active fault tolerant tracking controller (28). This proof is completed.

Remark 4.1. *In order to eliminate the chattering in the control surface deflection, virtual control input (24) and actual control input (28) are revised as the continuous format*

$$\omega_d = -F_\theta^{-1}\left(A_1\tilde{\theta} + B_1\tilde{\theta}^{q_1/p_1} - \dot{\theta}_d + k_1 s_1 + \varepsilon_1 \mathrm{sat}\left(\frac{s_1}{\vartheta}\right)\right)$$

$$u_c = -\hat{u}_f - D^{-1}F_\omega - (J^{-1}D)^{-1}\left(\gamma l_3 + \dot{\omega}_d + A_2\tilde{\omega} + B_2\tilde{\omega}^{q_2/p_2} + k_2 s_2 + \varepsilon_2 \mathrm{sat}\left(\frac{s_2}{\varpi}\right)\right)$$

where

$$\mathrm{sat}\left(\frac{s_{1i}}{\vartheta_i}\right) = \begin{cases} 1, & s_{1i} > \vartheta_i \\ s_{1i}/\vartheta_i, & s_{1i} \leq \vartheta_i \\ -1, & s_{1i} < -\vartheta_i \end{cases}, \quad \mathrm{sat}\left(\frac{s_{2i}}{\varpi_i}\right) = \begin{cases} 1, & s_{2i} > \varpi_i \\ s_{2i}/\varpi_i, & s_{2i} \leq \varpi_i \\ -1, & s_{2i} < -\varpi_i \end{cases} \quad (i = 1, 2, 3)$$

Remark 4.2. *In terms of singular perturbation theory, the inner angular rate loop sliding mode dynamics in Equation (25) must be much faster than the outer attitude angular loop sliding mode dynamics in Equation (21) to preserve sufficient time-scale separation between two loops. The parameters of controllers should satisfy $k_2 \geq 3k_1$ and $\varepsilon_2 \geq 3\varepsilon_1$.*

Remark 4.3. *In contrast to a passive FTC, the active FTC reacts to the system failures by properly accommodating its control actions so that the stability of the entire system can still be acceptable [10]. However, active FTC must rely on the accurate fault diagnosis information [11]. It is assumed that actuator fault occurs in the attitude control systems of hypersonic vehicle in this study, the proposed faults estimation observer can provide the satisfactory fault diagnosis information for the next fault tolerant control design, such that the designed active FTC can accommodate the effect of actuator fault effectively.*

5. **Numerical Simulation.** In this section, numerical simulations are carried out to verify the effectiveness of the proposed control technique. The normal inertial matrix J is given in [13]. The initial values are chosen as $\phi_0 = 1$deg, $\psi_0 = 2$deg, $\gamma_0 = 1.5$deg and $\omega_{x0} = \omega_{y0} = \omega_{z0} = 0$deg/$s$. $y_d = \theta_d$ is the desired system output signal. The desired system output signals $y_d = \theta_d = [\phi_d, \psi_d, \gamma_d]$ are given by

$$\phi_d = 1\text{deg}, \quad \psi_d = 3\text{deg}, \quad \gamma_d = 2\text{deg}.$$

The periodic disturbance torque is given by as

$$d = 10^6\left[\cos(0.3t) + 0.1, \sin(0.2t) + 0.1, \sin(0.1t)\right]^T \ \text{N·m}$$

In the simulation study, the gain parameters of virtual controller ω_d and actual controller δ are chosen as

$$A_1 = A_2 = \mathrm{diag}\{1, 1, 1\}, \quad B_1 = B_2 = \mathrm{diag}\{1, 1, 1\}$$
$$q_1 = q_2 = 6.5, \quad p_1 = p_2 = 9.5$$
$$k_1 = 2, \ k_2 = 7, \ \varepsilon_1 = 1.5, \ \varepsilon_2 = 5$$
$$\rho_1 = \rho_2 = 1, \ \gamma = 0.01, \ \beta = 0.5, \quad l = 2.5$$

To show the effectiveness of the FTC control scheme, the necessary simulation comparisons are given. Firstly, we use the controller developed in [2] as a baseline controller when the vehicle is fault free and before the faults have been estimated. Its function is to accommodate actuator faults and stabilize the system before observer provides accurate and reliable information about fault. The baseline controller, in fact, bears the responsibilities of the passive fault-tolerant controller [14,15]. In fault free case, the attitude angle tracking responses are shown in Figure 2, and it can be seen that the closed-loop attitude systems have the satisfactory dynamic performance. We assumed actuator stuck faults occurred in the second actuator, in which corresponding parameters of fault model are chosen as $e_1 = e_3 = 0$, $e_2 = 1$, $\bar{u}_c = [0; 0.8; 0]$ in simulation 20th second, the corresponding simulation results are given Figures 3, and it is not difficult to find that the design approach obtained in [2] does not have the fault tolerant accommodation capability if we do not take the fault tolerant control strategy to the faulty systems. By using the active FTC approach presented in this paper, the corresponding simulation results are given in Figures 4 and 5. Figure 4 is the fault estimation curve and it can be seen that actuator fault could be estimated accurately. Figure 5 is the attitude angle output curve under fault tolerant control; it is found that the attitude output can still track the desired commands when the fault occurred. Therefore, simulation results demonstrate that the proposed active FTC strategy could deal with the effect of actuator fault effectively, and guarantees asymptotical stability of the closed-loop control systems in the event of actuator faults.

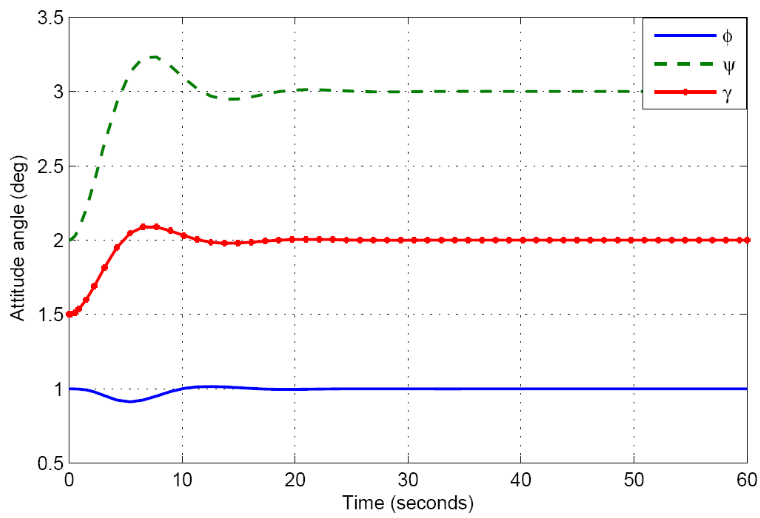

Figure 2. Attitude angle responses in healthy case using the controller in [2]

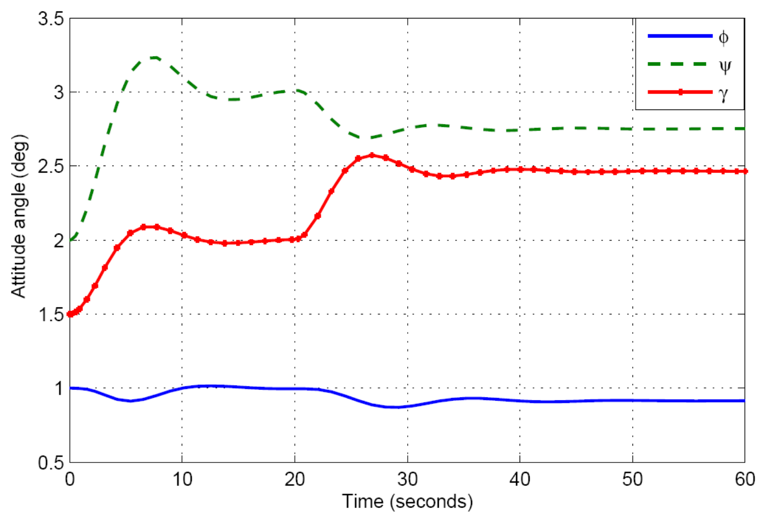

Figure 3. Attitude angle responses in faulty case using the controller in [2]

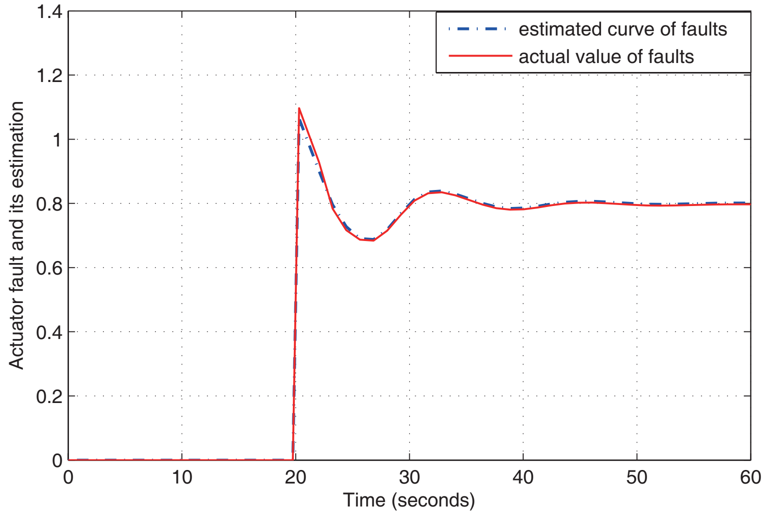

FIGURE 4. The actuator fault effect estimation curve

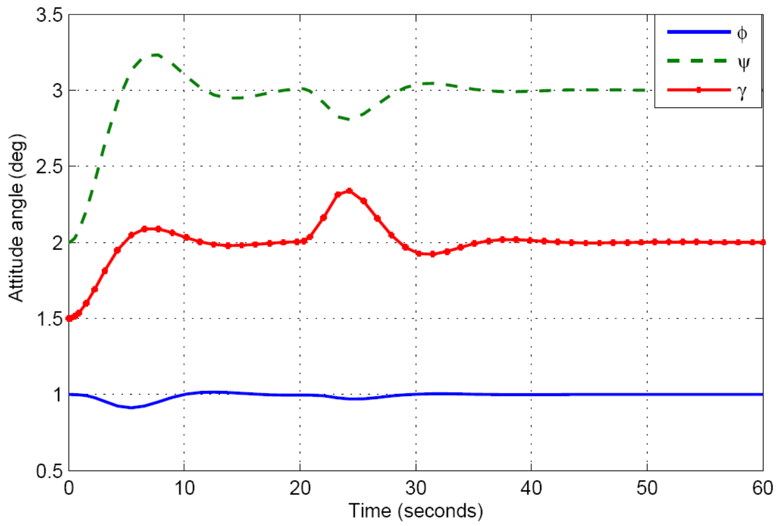

FIGURE 5. Attitude angle responses in faulty case using FTC

6. **Conclusions.** In this study, a active fault tolerant tracking control scheme has been proposed for the attitude systems of hypersonic vehicles in the presence of external disturbance and actuator faults. A nonlinear sliding mode observer with adaptive fault estimation law is designed to obtain the estimation value of the impact caused by actuator faults. By utilizing timescale separation principle, the two-loop terminal sliding mode controllers are designed. Meanwhile, the stability of the whole closed-loop attitude systems is analyzed using the Lyapunov theory. In the simulation example, the effectiveness of the proposed FTC method is verified. It is noted that we only considered actuator faults in this paper, and other forms of faults, such as sensor faults and structure faults, were not considered. The active fault tolerant control design for hypersonic vehicles with other kinds of faults is our studied work in the future.

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