ICIC Express Letters
Part B: Applications
Volume **8**, Number **5**, May **2017**

ICIC International ©2017 ISSN 2185-2766
pp. 811–818

FUZZY APPROXIMATION-BASED COMMAND FILTERED DISCRETE-TIME ADAPTIVE POSITION TRACKING CONTROL FOR INTERIOR PERMANENT MAGNET SYNCHRONOUS MOTORS

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Received November 2016; accepted February 2017

Abstract. *Considering the problems of parameter uncertainties and load disturbance appearing in interior permanent magnet synchronous motor drive systems, a discrete-time position tracking control method is proposed in this paper. First, Euler method is used to describe the discrete-time model of interior permanent magnet synchronous motors (IPMSMs). Next, the fuzzy approximation technique is employed to approximate the unknown nonlinear functions. Furthermore, the "explosion of complexity" problem and noncausal problem emerged in traditional backstepping design is eliminated by command filtered control technique. Simulation results prove that tracking error can converge to a small area of the origin and illustrate the effectiveness of the proposed approach.*

Keywords: Discrete-time, Permanent magnet synchronous motor, Command filter, Backstepping, Fuzzy approximation

1. **Introduction.** In recent years, permanent magnet synchronous motor (PMSM) has received attention for high performance electric drive applications because of its considerable advantages. Especially for the interior permanent magnet synchronous motor (IPMSM) with many attractive characteristics such as wide-speed operation range, high-power density, large torque to inertia ratio, and free from maintenance [1], it is suitable for many applications such as electric vehicle drive system. Nevertheless, it is still a challenging problem to control the IPMSM drive systems to get the perfect dynamic performance because their dynamic models are usually multivariable, coupled, and highly nonlinear and they are also very sensitive to external load disturbances and parameter variations [2]. In order to achieve high performance of IPMSMs, many researchers have aimed to develop nonlinear control methods for the IPMSM and various algorithms have been proposed including nonlinear feedback linearization control [3], fuzzy logic control [4, 5], adaptive backstepping control [2], neural network control [6], sliding mode control [7], and disturbance-observer-based control [8]. Unfortunately, all those methods mentioned above were developed for continuous-time IPMSM drive systems and implemented on digital devices. Nonlinear discrete-time control design techniques for IPMSM drive system were seldom discussed. The discrete-time control system is regarded as typically superior to the continuous-time control system in terms of stability and achievable performances [9].

The backstepping control is considered to be one of the most popular techniques for controlling the nonlinear systems with linear parametric uncertainty. However, during the backstepping design procedure the problem of "explosion of complexity" and noncausal problem arise. To overcome these issues, a command filtered backstepping control method was proposed by introducing a second-order filtering of the virtual input at each step of the conventional backstepping approach. However, the command filter technique has not been applied to nonlinear discrete-time systems with unknown parameters. Recently, fuzzy-approximation method has attracted great attention in permanent magnet synchronous motor drive systems because of its inherent capability for modeling and controlling highly uncertain, nonlinear and complex systems.

From the above observations, an adaptive fuzzy command filtered backstepping control method based on discrete-time technique is proposed to position tracking for IPMSM in this paper. Compared with the existing achievements, the proposed discrete-time adaptive fuzzy command filtered backstepping control can solve the noncausal problem and "explosion of complexity" problem to alleviate the online calculational burden. The simulation results are provided to demonstrate the effectiveness of the proposed discrete-time adaptive position tracking control method.

The rest of the paper is organized as follows. Section 2 describes the mathematical model of IPMSM drive system. The fuzzy command filtered adaptive backstepping control is designed in Section 3. Section 4 shows stability analysis. In Section 5, the simulation results are given. Finally, some conclusions are presented.

2. **Mathematical Model of Interior Permanent Magnet Synchronous Motor Drive System.** Interior permanent magnet synchronous motor's dynamic mathematical model can be described in the well known (d-q) frame as follows [10]:

$$\begin{cases} \dfrac{d\theta}{dt} = \omega, \\ \dfrac{d\omega}{dt} = \dfrac{3n_p\Phi}{2J}i_q - \dfrac{B}{J}\omega + \dfrac{3n_p(L_d-L_q)}{2J}i_d i_q - \dfrac{1}{J}T_L, \\ \dfrac{di_q}{dt} = -\dfrac{R_s}{L_q}i_q - \dfrac{n_p\Phi}{L_q}\omega - \dfrac{n_pL_d}{L_q}\omega i_d + \dfrac{1}{L_q}u_q, \\ \dfrac{di_d}{dt} = -\dfrac{R_s}{L_d}i_d + \dfrac{n_pL_q}{L_d}\omega i_q + \dfrac{1}{L_d}u_d, \end{cases}$$

where T_L, θ and ω denote the load torque, rotor position, and rotor angular velocity. i_d and i_q stand for the d-q axis currents. u_d and u_q are the d-q axis voltages. n_p denotes the pole pairs, the stator resistance R_s, L_d, and L_q are the d-q axis stator inductance, the rotor inertia J, the viscous friction coefficient B, and the magnetic flux Φ.

By using the Euler method, the dynamic model of IPMSM drivers can be described by the following equations:

$$\begin{aligned} x_1(k+1) &= x_1(k) + \Delta_t x_2(k) \\ x_2(k+1) &= a_1\Delta_t x_3(k) + (1-a_3\Delta_t)\, x_2(k) + a_2\Delta_t x_3(k)x_4(k) - a_4\Delta_t T_L \\ x_3(k+1) &= (1-b_1\Delta_t)\, x_3(k) - b_2\Delta_t x_2(k) - b_3\Delta_t x_2(k)x_4(k) + b_4\Delta_t u_q(k) \\ x_4(k+1) &= (1-c_1\Delta_t)\, x_4(k) + c_2\Delta_t x_2(k)x_3(k) + c_3\Delta_t u_d(k) \end{aligned} \tag{1}$$

where Δ_t is the sampling period and

$$\begin{aligned} &x_1(k) = \theta(k),\ x_2(k) = \omega(k),\ x_3(k) = i_q(k),\ x_4(k) = i_d(k), \\ &a_1 = \frac{3n_p\Phi}{2J},\ a_2 = \frac{3n_p\,(L_d-L_q)}{2J},\ a_3 = \frac{B}{J},\ a_4 = \frac{1}{J}, \\ &b_1 = \frac{R_s}{L_q},\ b_2 = \frac{n_p\Phi}{L_q},\ b_3 = \frac{n_pL_d}{L_q},\ b_4 = \frac{1}{L_q}, \\ &c_1 = \frac{R_s}{L_d},\ c_2 = \frac{n_pL_q}{L_d},\ c_3 = \frac{1}{L_d}. \end{aligned}$$

The control objective is to design an adaptive fuzzy controller such that the state variable $x_i(k)$ $(i = 1, 2, 3, 4)$ follows the given reference signal $x_{id}(k)$ and all the closed-loop signals are bounded. The approximation property of the fuzzy logic systems (FLSs) can be found in [9]. By using the FLSs, given a compact set $z = [z_1, z_2, \ldots, z_n] \in \Omega_z$, the unknown smooth function $\varphi(z)$ can be expressed as $\varphi(z) = W^T S(z) + \varepsilon(z)$, where $W \in R^N$ is the optimal parameter vector, $S(z) = \left[s^1(z), s^2(z), \ldots, s^N(z)\right]^T$ is a fuzzy basis function vector with $s^l(z) = \Pi_{i=1}^n \mu_{\phi_i^l}(z_i) / \Sigma_{l=1}^N \Pi_{i=1}^n \mu_{\phi_i^l}(z_i)$, and then $S(z)$ has the following properties: $\lambda_{\max}\left[S(z)S^T(z)\right] < 1$. And $\varepsilon(z) \in R$ is the approximation error satisfying $|\varepsilon(z)| \leq \bar{\varepsilon}$ with the constant $\bar{\varepsilon} > 0$. $\mu_{\phi_i^l}(z_i)$ is the fuzzy membership function and ϕ_i^l is fuzzy sets in R.

Lemma 2.1. *The command filter is defined as*

$$\begin{aligned} \dot{z}_1 &= \omega_n z_2 \\ \dot{z}_2 &= -2\zeta\omega_n z_2 - \omega_n (z_1 - \alpha_1) \end{aligned} \tag{2}$$

the input signal α_1 satisfies $|\dot{\alpha}_1| \leq \rho_1$, $|\ddot{\alpha}_1| \leq \rho_2$ for all $t \geq 0$, ρ_1, ρ_2 are a positive constant. And $z_1(0) = z_2(0) = \alpha_1(0) = 0$; then for any $\mu > 0$, there exist ζ, ω_n, we have $|z_1 - \alpha_1| \leq \mu$ and $|\dot{z}_1|$, $|\ddot{z}_1|$, $|\dddot{z}_1|$ are bounded.

3. **Adaptive Fuzzy Command Filtered Controller Design with Backstepping.** In this section, we will design the controllers for the approximate discrete-time IPMSM drive system via backstepping.

Step 1: For the reference signal x_d, define the tracking error variable as $e_1(k) = x_1(k) - x_d(k)$. From the first equation of (1), we can obtain $e_1(k+1) = x_1(k) + \Delta_t x_2(k) - x_d(k+1)$. Choose the Lyapunov function candidate as $V_1(k) = \frac{1}{2}e_1^2(k)$, and then the difference of $V_1(k)$ is computed by $\Delta V_1(k) = \frac{1}{2}\left[x_1(k) + \Delta_t x_2(k) - x_d(k+1)\right]^2 - \frac{1}{2}e_1^2(k)$. Construct the virtual control law $\alpha_1(k)$ as

$$\alpha_1(k) = \frac{-x_1(k) + x_{1d}(k+1)}{\Delta_t} \tag{3}$$

Define $e_2(k) = x_2(k) - x_{1c}(k)$, where $x_{ic}(k) = z_{i,1}(k)$, $(i = 1, 2)$ as the output of command filter. By using (3), $\Delta V_1(k)$ can be rewritten as

$$\Delta V_1(k) = \frac{1}{2}\Delta_t^2\left[e_2(k) + x_{1c}(k) - \alpha_1(k)\right]^2 - \frac{1}{2}e_1^2(k) \tag{4}$$

Step 2: From the second equation of (1), we can obtain

$$\begin{aligned} e_2(k+1) &= x_2(k+1) - x_{1c}(k+1) \\ &= a_1\Delta_t x_3(k) + (1 - a_3\Delta_t)\, x_2(k) + a_2\Delta_t x_3(k)\, x_4(k) - a_4\Delta_t T_L - x_{1c}(k+1) \end{aligned} \tag{5}$$

Choose the Lyapunov function candidate as $V_2(k) = \frac{1}{2}e_2^2(k) + V_1(k)$. Furthermore, differencing $V_2(k)$ yields

$$\Delta V_2(k) = \frac{1}{2}\left[f_1(k) + a_2\Delta_t x_3(k)\, x_4(k)\right]^2 - \frac{1}{2}e_2^2(k) + \Delta V_1(k) \tag{6}$$

where

$$f_1(k) = a_1\Delta_t x_3(k) - a_4\Delta_t T_L + (1 - a_3\Delta_t)\, x_2(k) - x_{1c}(k+1)$$

Construct the virtual control law $\alpha_2(k)$ as

$$\alpha_2(k) = \frac{a_4\Delta_t T_L - (1 - a_3\Delta_t)\, x_2(k) + x_{1c}(k+1)}{a_1\Delta_t} \tag{7}$$

By using equality (7), we can obtain

$$\Delta V_2(k) = \frac{1}{2}\left[a_1\Delta_t\left(e_3(k) + x_{2c}(k) - \alpha_2(k)\right) + a_2\Delta_t x_3(k)x_4(k)\right]^2 - \frac{1}{2}e_2^2(k) + \Delta V_1(k)$$

where $e_3(k) = x_3(k) - x_{2c}(k)$.

Utilizing the fact that

$$\left[a_1\Delta_t\left(e_3(k) + x_{2c}(k) - \alpha_2(k)\right) + a_2\Delta_t x_3(k)\, x_4(k)\right]^2$$

$$\leq 2a_1^2\Delta_t^2\left(e_3(k)+x_{2c}(k)-\alpha_2(k)\right)^2+2a_2^2\Delta_t^2x_3^2(k)x_4^2(k)$$

we can obtain

$$\Delta V_2(k)\leq a_1^2\Delta_t^2\left(e_3(k)+x_{2c}(k)-\alpha_2(k)\right)^2+a_2^2\Delta_t^2x_3^2(k)x_4^2(k)-\frac{1}{2}e_2^2(k)+\Delta V_1(k) \tag{8}$$

Step 3: From the third equation of (1), we can obtain

$$\begin{aligned}e_3(k+1)&=x_3(k+1)-x_{2c}(k+1)\\&=\left(1-b_1\Delta_t\right)x_3(k)-b_2\Delta_tx_2(k)-b_3\Delta_tx_2(k)x_4(k)+b_4\Delta_tu_q(k)-x_{2c}(k+1)\\&=f_3(k)+b_4\Delta_tu_q(k)\end{aligned}$$

where $f_3(k)=\left(1-b_1\Delta_t\right)x_3(k)-b_2\Delta_tx_2(k)-b_3\Delta_tx_2(k)x_4(k)-x_{2c}(k+1)$.

Remark 3.1. *The variable $x_{2c}(k+1)$ contains future information. If we continue to construct the real controller via backstepping, we will end up with a controller containing more future information, and make it possibly infeasible in practice. This drawback was called noncausal problem [9]. The existing result to solve this problem is to transform the systems into a predictor form, which will add the control complexity. In this paper, we use the command filter to gain the expression of time k to indicate $x_{2c}(k+1)$, and thus the noncausal problem can be overcome.*

Choose the Lyapunov function candidate as $V_3(k)=\frac{1}{2}e_3^2(k)+V_2(k)$. Furthermore, differencing $V_3(k)$ yields

$$\begin{aligned}\Delta V_3(k)&=\frac{1}{2}e_3^2(k+1)-\frac{1}{2}e_3^2(k)+\Delta V_2(k)\\&=\frac{1}{2}\left[f_3(k)+b_4\Delta_tu_q(k)\right]^2-\frac{1}{2}e_3^2(k)+\Delta V_2(k)\end{aligned} \tag{9}$$

By using the approximation property of the FLSs, for any given $\varepsilon_3>0$, there must be an FLS $W_3^TS_3(z_3(k))$ such that $f_3\left(k\right)=W_3^TS_3(z_3(k))+\varepsilon_3$, where ε_3 is the approximation error. In general, W_3 is bounded and unknown. Define $||W_3||=\eta_3$, where $\eta_3>0$ is unknown constant. Let $\hat{\eta}_3(k)$ be as the estimate of η_3 and $\tilde{\eta}_3(k)=\eta_3-\hat{\eta}_3(k)$. At this present stage, we define the control law $u_q(k)$ and adaptive law $\hat{\eta}_3(k+1)$ as the following equations

$$u_q(k)=-\frac{1}{b_4\Delta_t}\hat{\eta}_3(k)||S_3(z_3(k))|| \tag{10}$$

$$\hat{\eta}_3(k+1)=\hat{\eta}_3(k)+\gamma_3||S_3(z_3(k))||e_3(k+1)-\delta_3\hat{\eta}_3(k)$$

where γ_3 and δ_3 are positive parameters.

Using (10), $\Delta V_3(k)$ can be rewritten as

$$\begin{aligned}\Delta V_3(k)\leq{}&\frac{1}{2}[\tilde{\eta}_3(k)||S_3(z_3(k))||+\varepsilon_3]^2-\frac{1}{2}e_3^2(k)+a_1^2\Delta_t^2\left(e_3(k)+x_{2c}(k)-\alpha_2(k)\right)^2\\&+a_2^2\Delta_t^2x_3^2(k)x_4^2(k)-\frac{1}{2}e_2^2(k)+\frac{1}{2}\Delta_t^2\left[e_2(k)+x_{1c}(k)-\alpha_1(k)\right]^2-\frac{1}{2}e_1^2(k)\end{aligned} \tag{11}$$

Step 4: Define the tracking error variable as $e_4(k)=x_4(k)$. From the fourth equation of (1), we have $e_4(k+1)=x_4(k+1)=\left(1-c_1\Delta_t\right)x_4(k)+c_2\Delta_tx_2(k)x_3(k)+c_3\Delta_tu_d(k)$. Choose the Lyapunov function candidate as $V_4(k)=\frac{P}{2}e_4^2(k)+V_3(k)$ with $P>0$, and then the difference of $V_4(k)$ is computed by

$$\Delta V_4(k)=\frac{P}{2}\left[f_4(k)+c_3\Delta_tu_d\left(k\right)\right]^2-\frac{P}{2}e_4^2(k)+\Delta V_3(k) \tag{12}$$

where $f_4(k)=\left(1-c_1\Delta_t\right)x_4(k)+c_2\Delta_tx_2(k)x_3(k)$.

Similarly, the fuzzy logic system $W_4^T S_4(z_4(k))$ is utilized to approximate the nonlinear function $f_4(k)$ such that for given $\varepsilon_4 > 0$, $f_4(k) = W_4^T S_4(z_4(k)) + \varepsilon_4$. Now choose the following control law $u_d(k)$ and adaptive law $\hat{\eta}_4(k+1)$ as

$$u_d(k) = -\frac{1}{c_3\Delta_t}\hat{\eta}_4(k)||S_4(z_4(k))|| \tag{13}$$

$$\hat{\eta}_4(k+1) = \hat{\eta}_4(k) + \gamma_4||S_4(z_4(k))||e_4(k+1) - \delta_4\hat{\eta}_4(k) \tag{14}$$

where γ_4 and δ_4 are positive parameters. In general, W_4 is bounded and unknown and let $||W_4|| = \eta_4$, where $\eta_4 > 0$ is unknown constant. Let $\hat{\eta}_4(k)$ estimate η_4 and we have $\tilde{\eta}_4(k) = \eta_4 - \hat{\eta}_4(k)$. By using $|x_{ic}(k) - \alpha_i(k)| \leq \mu_i$, $(i = 1, 2)$ and substituting (13) into (12), we have

$$\begin{aligned}\Delta V_4(k) \leq\ & P[\tilde{\eta}_4(k)||S_4(z_4(k))||]^2 + P\varepsilon_4^2 - \frac{P}{2}e_4^2(k) + [\tilde{\eta}_3(k)||S_3(z_3(k))||]^2 + \varepsilon_3^2 \\ & -\frac{1}{2}e_3^2(k) + 2a_1^2\Delta_t^2 e_3^2(k) + 2a_1^2\Delta_t^2\mu_2^2 + a_2^2\Delta_t^2 x_3^2(k)x_4^2(k) \\ & -\frac{1}{2}e_2^2(k) + \Delta_t^2 e_2^2(k) + \Delta_t^2\mu_1^2 - \frac{1}{2}e_1^2(k)\end{aligned} \tag{15}$$

4. **Stability Analysis.** To address the stability of the closed-loop system, choose the Lyapunov function candidate as $V(k) = V_4(k) + \frac{1}{2\gamma_3}\tilde{\eta}_3^2(k) + \frac{P}{2\gamma_4}\tilde{\eta}_4^2(k)$, where γ_3, γ_4, P are positive parameters. Furthermore, differencing $V(k)$ yields

$$\Delta V(k) = \Delta V_4(k) + \frac{1}{2\gamma_3}\left[\tilde{\eta}_3^2(k+1) - \tilde{\eta}_3^2(k)\right] + \frac{P}{2\gamma_4}\left[\tilde{\eta}_4^2(k+1) - \tilde{\eta}_4^2(k)\right] \tag{16}$$

By using $\tilde{\eta}_i(k) = \eta_i - \hat{\eta}_i(k)$, we can obtain

$$\tilde{\eta}_i^2(k+1) - \tilde{\eta}_i^2(k) = \eta_i^2 + \hat{\eta}_i^2(k+1) - 2\eta_i\hat{\eta}_i(k+1) - \tilde{\eta}_i^2(k) \tag{17}$$

$$\begin{aligned}\hat{\eta}_i^2(k+1) =\ & \gamma_i^2 e_i^2(k+1)||S_i(z_i(k))||^2 + (1-\delta_i)^2\hat{\eta}_i^2(k) \\ & +2(1-\delta_i)\gamma_i||S_i(z_i(k))||e_i(k+1)\hat{\eta}_i(k)\end{aligned} \tag{18}$$

Replacing (18) into (17) yields

$$\begin{aligned}\tilde{\eta}_i^2(k+1) - \tilde{\eta}_i^2(k) =\ & \eta_i^2 + (1-\delta_i)^2\hat{\eta}_i^2(k) + \gamma_i^2 e_i^2(k+1)||S_i(z_i(k))||^2 \\ & -2(1-\delta_i)\eta_i\hat{\eta}_i(k) + 2(1-\delta_i)\gamma_i||S_i(z_i(k))||e_i(k+1)\hat{\eta}_i(k) \\ & -\tilde{\eta}_i^2(k) - 2\gamma_i||S_i(z_i(k))||e_i(k+1)\eta_i\end{aligned} \tag{19}$$

Then, with $||S_i(z_i(k))||^2 \leq 1$ and according to the Young's inequality [9], we have

$$\begin{aligned}\tilde{\eta}_i^2(k+1) - \tilde{\eta}_i^2(k) \leq\ & \left(16\gamma_i^2 - 8\gamma_i^2\delta_i + 9\gamma_i - \delta_i + 2\right)\eta_i^2 + \left(\delta_i^2 - 4\delta_i + 3\right)\hat{\eta}_i^2(k) \\ & + \left(4\gamma_i^2 - 2\gamma_i^2\delta_i + 2\gamma_i - 1\right)\tilde{\eta}_i^2(k) + \left(4\gamma_i^2 - 2\gamma_i^2\delta_i + 2\gamma_i\right)\varepsilon_i^2,\ (i = 3, 4)\end{aligned} \tag{20}$$

Define $x_3^2(k) \leq \mathrm{M}$, where M is a positive constant. Substituting (20) and (15) into (16), one has

$$\begin{aligned}\Delta V_4(k) \leq\ & \left[a_2^2\Delta_t^2 M - \frac{P}{2}\right]e_4^2(k) + \left[2a_1^2\Delta_t^2 - \frac{1}{2}\right]e_3^2(k) + \left[\Delta_t^2 - \frac{1}{2}\right]e_2^2(k) - \frac{1}{2}e_1^2(k) \\ & +\frac{1}{2\gamma_3}\left[\left(\delta_3^2 - 4\delta_3 + 3\right)\hat{\eta}_3^2(k) + \beta_3 + \left(4\gamma_3^2 - 2\gamma_3^2\delta_3 + 4\gamma_3 - 1\right)\tilde{\eta}_3^2(k)\right] \\ & +\frac{P}{2\gamma_4}\left[\left(\delta_4^2 - 4\delta_4 + 3\right)\hat{\eta}_4^2(k) + \beta_4 + \left(4\gamma_4^2 - 2\gamma_4^2\delta_4 + 4\gamma_4 - 1\right)\tilde{\eta}_4^2(k)\right]\end{aligned}$$

$$\beta_3 = \left(4\gamma_3^2 - 2\gamma_3^2\delta_3 + 4\gamma_3\right)\varepsilon_3^2 + \left(16\gamma_3^2 - 8\gamma_3^2\delta_3 + 9\gamma_3 - \delta_3 + 2\right)\eta_3^2 + 2\gamma_3\Delta_t^2\mu_1^2$$

$$\beta_4 = \left(4\gamma_4^2 - 2\gamma_4^2\delta_4 + 4\gamma_4\right)\varepsilon_4^2 + \left(16\gamma_4^2 - 8\gamma_4^2\delta_4 + 9\gamma_4 - \delta_4 + 2\right)\eta_4^2 + \frac{4\gamma_4 a_1^2 \Delta_t^2 \mu_2^2}{P}$$

By choosing a suitable parameter P and sampling period Δ_t, we can get $a_2^2\Delta_t^2 M - \frac{P}{2} < 0$, $2a_1^2\Delta_t^2 - \frac{1}{2} < 0$, $\Delta_t^2 - \frac{1}{2} < 0$. If we choose the design parameters as follows: $\delta_i^2 - 4\delta_i + 3 < 0$, $4\gamma_i^2 - 2\gamma_i^2\delta_i + 4\gamma_i - 1 < 0$, for $i = 3, 4$. Then $\Delta V(k) \leq 0$ once the error $|e_4(k)| > \sqrt{\frac{P\beta_4}{-2\gamma_4 a_2^2 \Delta_t^2 M + P\gamma_4}}$ and $|e_3(k)| > \sqrt{\frac{\beta_3}{\gamma_3 - 4\gamma_3 a_1^2 \Delta_t^2}}$. $\lim\limits_{k\to\infty} ||x_1(k) - x_d(k)|| \leq \sigma$, where σ is a small positive constant.

5. **Simulation Results.** To illustrate the effectiveness of the proposed control approach, the simulation is run for IPMSM with the parameters: $J = 0.00379\text{Kg}\cdot\text{m}^2$, $R_s = 0.68\Omega$, $L_d = 0.00315\text{H}$, $L_q = 0.00285\text{H}$, $\Phi = 0.1245\text{Wb}$, $B = 0.001158\text{Nm}/(\text{rad/s})$, $n_p = 3$. The reference signal is chosen as $x_d(k) = 2\cos(\Delta_t k\pi/2)$ with the load torque being $T_L = \begin{cases} 0.5, & 0 \leq k < 2000, \\ 1.0, & k \geq 2000. \end{cases}$ The initial values of the states are chosen as $x_1 = (0) = x_2(0) = x_3(0) = x_4(0) = 0$. The sampling period is chosen as $\Delta_t = 0.005\text{s}$. The values of the control parameters are selected as $\delta_3 = 0.8$, $\delta_4 = 0.65$, $\zeta = 1.1$, $\omega_n = 230$, $\gamma_3 = 0.76$ and $\gamma_4 = 0.65$.

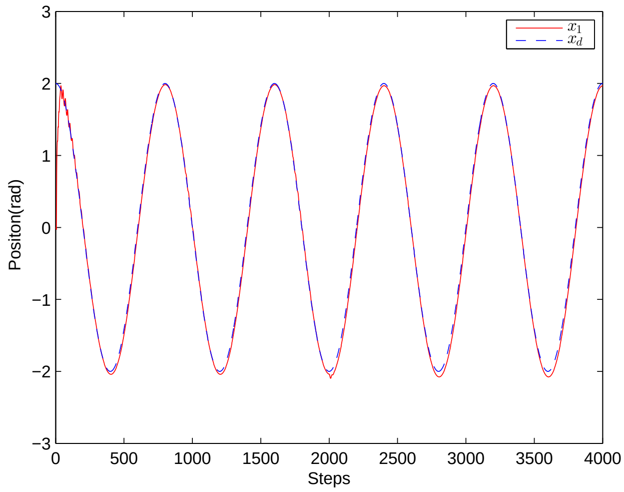

FIGURE 1. x_1 and x_d

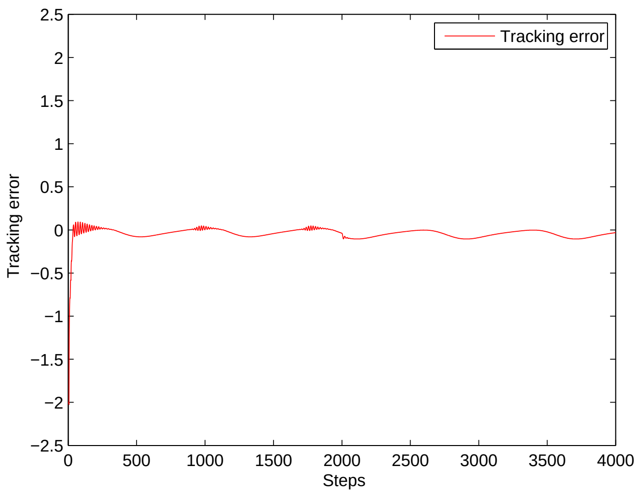

FIGURE 2. The tracking error

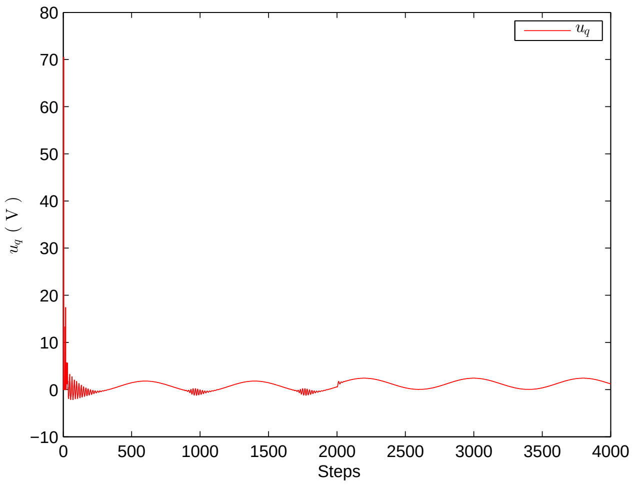

FIGURE 3. The control law u_q

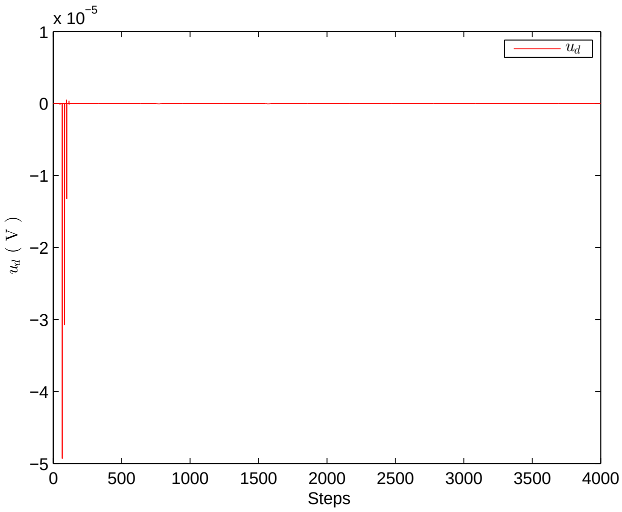

FIGURE 4. The control law u_d

Simulation results in Figures 1-4 are obtained by using the proposed scheme. The trajectories of $x_1(k)$ and $x_d(k)$ are given in Figure 1, in which the solid line represents $x_1(k)$, and the dashed line represents $x_d(k)$. It can be observed that the system output can track the desired reference signal well. The dynamics of the tracking error is shown in Figure 2 and it can be seen that the tracking error converges to a small neighborhood of the origin. The trajectories of $u_q(k)$ and $u_d(k)$ are shown in Figure 3 and Figure 4. From Figures 3 and 4, we can seen that $u_q(k)$ and $u_d(k)$ are bounded into a certain area. The controllers can guarantee the robustness against the system parameter variations and load disturbances. In this simulation, it should be remarked that when the load torque changes, the controllers can cope with the sudden change of the load torque and provide a fast tracking response.

6. **Conclusions.** In this paper, based on command filtered backstepping technique, a fuzzy adaptive discrete-time method is proposed to solve the position tracking problem for IPMSM drive system. The designed controllers guarantee that the tracking error converges to a small neighborhood of the origin. Simulation results are provided to demonstrate the effectiveness and robustness of the proposed approach. Future research will be focused on adaptive fuzzy control of induction motors based on command filter.

Acknowledgment. This work was partially supported by the National Natural Science Foundation of China (61573204, 61573203, 61501276, 61603204), Shandong Province Outstanding Youth Fund (ZR2015JL022), the China Postdoctoral Science Foundation (2014T70620, 2013M541881, 201303062, 2016M592139), Qingdao Postdoctoral Application Research Project (2015120), Qingdao Application Basic Research Project (16-5-1-22-jch) and Taishan Scholar Special Project Fund.

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