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DISRUPTION RECOVERY MODELS FOR REPLACEMENT SERVICE DECISION OF LIGHT RAIL SYSTEMS

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Abstract. *This research focuses on the decision analysis of the replacement service type for disruption recovery service in urban public light rail systems. The traditional approach – bus replacement service, the new approach – taxi replacement service, and the taxi-bus hybrid method are examined. The involving parties' decision functions taking the passengers' behaviors into account are formulated. Both theoretical and numerical sensitivity analyses are conducted to shed lights on efficient replacement service decisions, the key factors affecting the choice from three tools, and the balance between recovery cost and passenger loss.*
Keywords: Light rail systems, Disruption recovery, Replacement service, Collaboration decision

1. **Introduction.** The modern light rail system, as a typical easy going and environmentally friendly public transport mode, is widely used in various parts of the world. Unfortunately, this light rail train system invariably encounters disruptions during its daily operations; for example, rail/track failures, traffic jams, accidents along the route, and vehicle breakdowns are all possible sources for disruptions. Hence, dealing with unplanned events quickly and efficiently is an imperative task, which could not only mitigate the commuters' impact during disruption, but also affect the light rail system's long-term productivity and sustainable development.

Light rail companies have attempted to employ a range of methods, out of which buses are frequently used as a recovery service tool. Recently, several German cities have implemented a new approach – working with the local taxi providers to provide replacement services [1]. Evidently, taxis can be redistributed much faster than a bus service. From a financial point of view, when taxis are used to recover the transport service during disruptions, the light rail company has to absorb the entire recovery service fee, while the passengers have no obligation to pay. Consequently, the light rail operators need to balance between the recovery cost and the passenger loss. Therefore, it is not always efficient to use taxis for providing recovery service. In this paper, we examine and compare three recovery tools: taxi-only, bus-only, and taxi-bus hybrid. We develop a series of mathematical models to answer the questions as (1) whether to provide a replacement service; (2) what are the key factors affecting the choice of the three tools for recovery service; (3) how to balance between recovery cost and passenger loss.

The study of disruption management can be found in many industrial fields, such as airline operations, production planning and control, supply chain and logistics, project management, and transportation systems [2]. Generally, the method to mitigate the disruption impacts in public transport system can be classified into three categories: proactive planning [3,4], optimization-based re-scheduling [5,6], and collaboration efforts in system recovery.

Collaboration is an important mechanism for disruption management [7]. The bus collaborative mechanism, which is also called as bus bridging service, is widely adopted to respond to disruptions in urban transit rail systems with focus on how to set bus routes, how to design an efficient bus bridging network and how to allocate resources [8]. De-Los-Santos et al. [4] compared the robustness of a railway system with and without bus bridging service. Darmanin et al. [9] addressed the key deficiencies of bus bridging service, namely slow response, deployment difficulty, and new uncertainties caused by buses. The taxi collaborative recovery service is also investigated in previous literature. The study by Westerlund and Cazemier [1] presented several cases about how the public transport systems in Sweden and the Netherlands collaborated with taxi companies to provide demand responsive transport services. Yang et al. [11] pointed out that the customer waiting time is generally considered as an important measure of service quality, since it can obviously affect passengers' decisions as to whether to take a taxi or not.

The existing findings provide some fundamental elements for our research; at the same time, our work goes beyond the existing studies in the following three ways. Firstly, the relationship between passengers' waiting rate and recovery service time is modeled in such a way that it captures both complete and partial passenger loss. Secondly, the trade-off between the replacement service level and the cost in the presence of different passenger behaviors is simultaneously taken into consideration in the light rail company's decision making.

The remainder of the paper is organized as follows. Section 2 gives a description of the decision problem and the preliminaries such as assumptions and basic terms. Section 3 presents the mathematical decision models and their theoretical results and implications. A set of numerical examples is conducted in Section 4. Finally, concluding remarks are provided in Section 5.

2. **Problem Description and Preliminaries.** Consider the scenario when an unexpected interruption breaks down one or a few links between rail stations causing a temporary closure of the rail line. Meanwhile, commuters along the closed area need to find alternative ways to arrive at their destination either by leaving the system or waiting for the light rail company to arrange for a replacement service by taxis or buses. A common response of the light rail company to such disruptions is to divert the passengers to the nearest transfer station for buses or subways depending on the situation. Then the light rail company will consider three recovery service choices – bus, taxi, and both. If the light rail company takes no action, all of the passengers would have to find alternatives on their own to reach their destinations and will likely require ticket refund, compensation, or even never use the light rail again. We refer to this situation as complete passenger loss. However, if the light rail company provides a replacement service, a portion of passengers will accept and wait for such a service. We consider this situation as partial loss. The difference between the two scenarios, called reduced loss, is adopted to measure the benefit of the light rail company and support the decision making during disruption recovery. A few critical terms and assumptions are defined and explained as listed below.

(1) Light Rail Company's Decision Function: The light rail company's reduced loss, $O(t_a)$, is used as a basis for analysis and defined as a function of the replacement vehicle arrival time (t_a) with the format: $O(t_a)$ = Complete loss of doing nothing – (Recovery service cost + Partial passenger loss). Note that t_a is the recovery service provider's

decision variable and that if $O(t_a) < 0$, the light rail company should not provide recovery service, and that the service method resulting in higher reduced loss is better.

(2) Taxi Company's Decision Function: The taxi company's recovery service profit, $\Pi(t_{a\_t})$, is a function of the average taxi arriving time, t_{a_t}, and serves as a basis for decision making. The function is defined as $\Pi(t_{a\_t})$ = recovery service payment − total service cost.

(3) Cost of Arranging Taxis: The taxi company's cost of arranging needed number of taxi trips, $C_t(t_{a\_t})$, includes the drivers' efforts and all the associated service costs and is a non-linear decreasing function with respect to t_{a_t} as follows: $C_t(t_{a\_t}) = k_t t_{a\_t}^{-1}$, where k_t is a scale factor and t_{a_t} is a decision variable. This function captures the fact that the effort and the incurred service cost increase quickly as the requested arrival time shortens.

(4) Cost of Arranging Buses: Similar to the taxi company, the bus company incurs various costs such as internal communication, finding drivers, vehicle scheduling and opportunity loss when arranging needed number of bus trips. Let k_b be a scale factor. The cost function with respect to t_{a_b} is assumed as $C_b(t_{a\_b}) = k_b t_{a\_b}^{-1}$.

(5) Passengers' Willingness to Wait: The aggregate passengers' willingness to wait, $w(t_a)$, is assumed to decrease linearly as the replacement vehicle's arrival time (t_a) increases and can be calculated in (1). This assumption stems from the fact that if the replacement service is provided instantaneously, all the passengers will stay for the service, but if the arrival time of the recovery vehicles is about the same as the disruption duration, only a small portion of passengers (α: $0 \le \alpha < 1$) will wait for the service.

$$w(t_a) = 1-(1-\alpha)(t_a/t_d),\ \forall 0 \le t_a \le t_d,\ 0 \le \alpha < 1; \text{ and } \begin{cases} w(t_a) \to 1, & \text{if } t_a \to 0 \\ w(t_a) \to \alpha, & \text{if } t_a \to t_d \end{cases} \quad (1)$$

In addition, the following input parameters are needed
P = the total blocked passenger volume;
t_s = vehicle recovery service time;
c_l = unit passenger loss ($/person);
p_0 = light rail company's payment rate ($/unit time) for taxi replacement service;
c_t = taxi service cost per unit time ($/unit time);
c_b = average cost of bus service per unit time ($/unit time);
V = standard vehicle capacity (people/vehicle), with the subscript "t" representing one taxi capacity and "b" indicating one bus capacity.

3. Decision Functions under Different Service Types.

3.1. Single-type-vehicle approaches: Taxi-only and bus-only.

With these parameters and terms defined above, we can provide the decisions functions in Table 1. Note that under the bus-only method, the light rail and the bus companies are usually from the same network (This is true at least in both Germany and China), which will absorb all the recovery service costs. Taking the light rail company's profit function as an example, the first item $c_l P$ is the "Complete Loss" for providing none replacement service. The second one $p_0 t_s P \left(1-(1-\alpha)(t_{a\_t}/t_d)\right)/V_t$ is the financial payment to the taxi company, which is considered as the part of loss according to assumption (1). And the last term $c_l P(1-\alpha)(t_{a\_t}/t_d)$ is the partial passenger loss before taxis' arrival.

Proposition 3.1. *If the unit passenger loss ($/person) meets the condition: $c_{l,t} > p_0 t_s/V_t$, the light rail company will adopt the taxi recovery service. If the unit passenger loss satisfies the requirement: $c_{l,b} > \frac{k_b t_{ab}^{-1}}{P(1-(1-\alpha)t_{ab}/t_d)} + \frac{c_b t_s}{V_b}$, the light rail company will adopt the bus recovery service.*

Proposition 3.2. *Regardless of the vehicle type selected for recovery service, the taxi company's recovery service profit is a strictly concave function with respect to the vehicle*

TABLE 1. The two parties decision functions

Party involved	Taxi-only method	Bus-only method
Light rail company	$O(t_{a\_t})$ $= c_l P - p_0 t_s P\left(1-(1-\alpha)(t_{a\_t}/t_d)\right)/V_t$ $-c_l P(1-\alpha)(t_{a\_t}/t_d)$ $= P\left(1-(1-\alpha)(t_{a\_t}/t_d)\right)(c_l - p_0 t_s/V_t)$	$O(t_{a\_b})$ $= c_l P - c_b t_s P\left(1-(1-\alpha)(t_{a\_b}/t_d)\right)/V_b$ $-c_l P(1-\alpha)(t_{a\_b}/t_d) - k_b t_{a\_b}^{-1}$ $= P\left(1-(1-\alpha)(t_{a\_b}/t_d)\right)(c_l - c_b t_s/V_b)$ $-k_b t_{a\_b}^{-1}$
Taxi company	$\Pi(t_{a\_t})$ $= (p_0 - c_t) t_s P\left(1-(1-\alpha)(t_{a\_t}/t_d)\right)/V_t$ $-k_t t_{a\_t}^{-1}$	N/A

arrival time. The optimal average taxi arrival time and the optimal average bus arrival time are listed below, respectively:

$$t_{a\_t}^* = A\left(k_t V_t / \left((p_0 - c_t)t_s\right)\right)^{1/2}; \quad t_{a\_b}^* = A\left(k_b V_b / \left(c_l V_b - c_b t_s\right)\right)^{1/2} \tag{2}$$

where $A = (t_d/(P(1-\alpha)))^{1/2}$. It can be also shown that the optimal bus arrival time is larger than the taxi arrival time if $\Theta = \rho_1\rho_2\rho_3 > 1$, where $\rho_1 = k_b/k_t$, $\rho_2 = V_b/V_t$, and $\rho_3 = (p_0 - c_t)/(c_l V_b/t_s - c_b)$.

3.2. **Hybrid-vehicle approach.** It is evident that the taxi-only approach is advantageous in providing fast response to unplanned disruptions, whereas the bus-only approach is preferable in terms of capacity and cost efficiency. Would the hybrid approach outperform the single-type-vehicle approach? We answer this question in this section.

In the hybrid approach, we propose the following recovery procedure: (1) once an unplanned disruption is confirmed, both taxi and bus centers are notified and will then start arranging the replacement service; (2) the taxis will arrive first to pick up a portion (denoted as γ) of the affected passenger volume, which is computed in Equation (1), and the average arrival time, t_{a_t} ($t_{a\_t} \neq t_{a\_t}^*$), is subjectively determined; (3) the buses will arrive t minutes after the taxis to pick up the rest of the passengers, where t is unknown and to be examined for its critical value; thus, the average bus arrival time is $t_{a\_b} = t_{a\_t} + t$ ($t_{a\_t} < t_{a\_b} \leq t_d$). Since the bus arrival time and capacity are guaranteed and will be conveyed to the passengers, we assume that even though the buses arrive a few minutes after the taxis, the rest of the passengers will be all willing to wait. Under such a procedure, the light rail company's reduced loss is computed as follows:

$$\begin{aligned} O_h(\gamma, t) = Pc_l - \big\{ & Pc_l(1 - w(t_{a\_t})) + \gamma P w(t_{a\_t}) p_0 t_s / V_t \\ & + (1-\gamma) P w(t_{a\_t}) c_b t_s / V_b + k_b/(t_{a\_t} + t) \big\} \end{aligned} \tag{3}$$

Thus, the question to be answered is that whether there exist values for the pair, (γ, t), such that the hybrid approach will outperform both single-type-vehicle methods, i.e., $O_h(\gamma, t) > \max\left(O(t_{a\_t}^*), O(t_{a\_b}^*)\right)$. Using the optimal solutions expressed in (2) and the light rail company's decision functions in Table 1, we arrive at the following results.

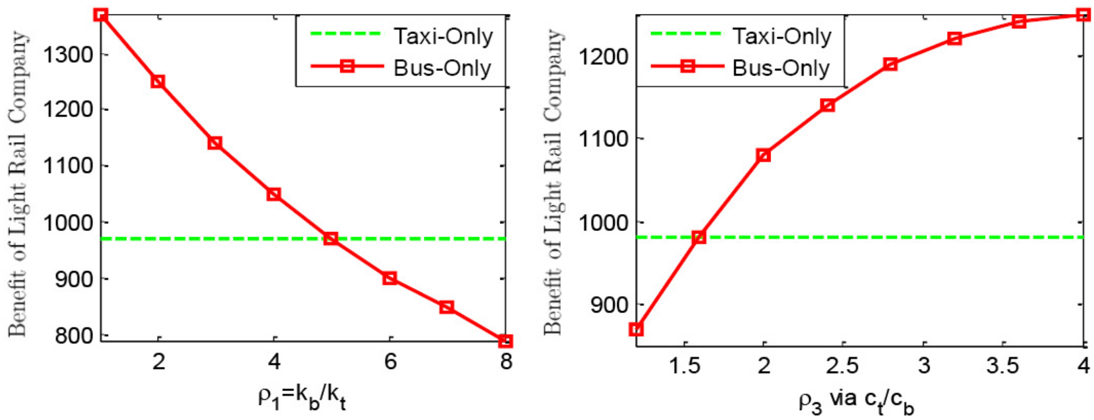
Proposition 3.3. *The critical values of (γ, t) for the hybrid method to outperform the taxi-only and the bus-only methods are given below, respectively:*

$$t_t > \frac{k_b}{P\left\{w(t_{a\_t})K_h - w(t_{a\_t}^*)K_t\right\}} - t_{a\_t}; \quad t_b > \frac{k_b}{P\left(w(t_{a\_t})K_h - w(t_{a\_b}^*)K_b\right) - k_b/t_{a\_b}^*} - t_{a\_t},$$

where $K_t = c_l - p_o t_s/V_t$, $K_b = c_l - c_b t_s/V_b$, and $K_h = c_l - \gamma p_o t_s/V_t - (1-\gamma)c_b t_s/V_b$.

4. Numerical Experiment and Managerial Implications.

4.1. Comparisons between taxi-only and bus-only services. According to Proposition 3.2, the effects of ρ_1 and ρ_3 to the selection of taxis or buses are worthy of investigating. Here, k_t is set to be 1,000, while ρ_1 is changed with k_b, as shown in Figure 1.1. The effect of ρ_3 is conducted by changing with c_t/c_b, where c_t keeps stable. The other base values of input parameters are listed below in Figure 1.



1.1: $c_b = 10$ 1.2: $k_b = 5{,}000$

(Fixed values: $P = 150$, $c_l = 15$, $t_s = 15$, $t_d = 60$, $\alpha = 0.1$, $k_t = 1{,}000$, $V_t = 4$, $p_0 = 2$, $c_t = 0.5$, $V_b = 40$)

FIGURE 1. Comparisons based on the light rail company's reduced loss

In Figure 1, the light rail company's decision function is used as the main basis for comparisons. From the numerical results, we have noticed that (1) the required unit passenger loss for using the bus-only replacement service is usually smaller than that for using the counterpart service; (2) the preference between buses and taxis is heavily determined by the cost of arranging the vehicles (k_t, k_b) and the service cost (c_t, c_b). The breaking points for the two types of recovery services exist and can be identified numerically. For example, when ρ_1 is higher than 5, which means the bus company's cost factor is higher, the taxi-only service will be an appropriate choice.

4.2. Comparisons between hybrid-vehicle approach and single-type-vehicle approaches. In this section, a numerical example is used to compare the hybrid service and the single vehicle services. As shown in Figure 2, the values of the light rail company's reduced loss against t are plotted. Two values of the proportion of the passengers served by taxis: $\gamma = 0.5$ and 0.7, are considered, and the associated critical values of t are summarized below the diagram. In Figure 2, the optimal values of the reduced loss under the bus-only and taxi-only methods are shown as benchmarks, and with the chosen inputs, the taxi replacement is better than the bus service. This example indicates that (1) higher values of γ make it more possible for the hybrid approach to outperform the other two; (2) with a careful selection of the pair, (γ, t), the hybrid method could reduce the total recovery costs.

5. Conclusions.

This study has analyzed the replacement service options for providing quick response and efficient service during unplanned disruptions in urban public light rail systems. We have developed a systematic decision tool for light rail operating centers to answer three key questions: (1) whether to provide replacement service; (2) which service type should be adopted: the traditional bus service, the new service by taxis, or a combination of both; (3) how to consider the service level and the recovery cost. The decision functions under the three services are formulated from the operational management perspective. Moreover, our work is a successful application of the collaborative partnership

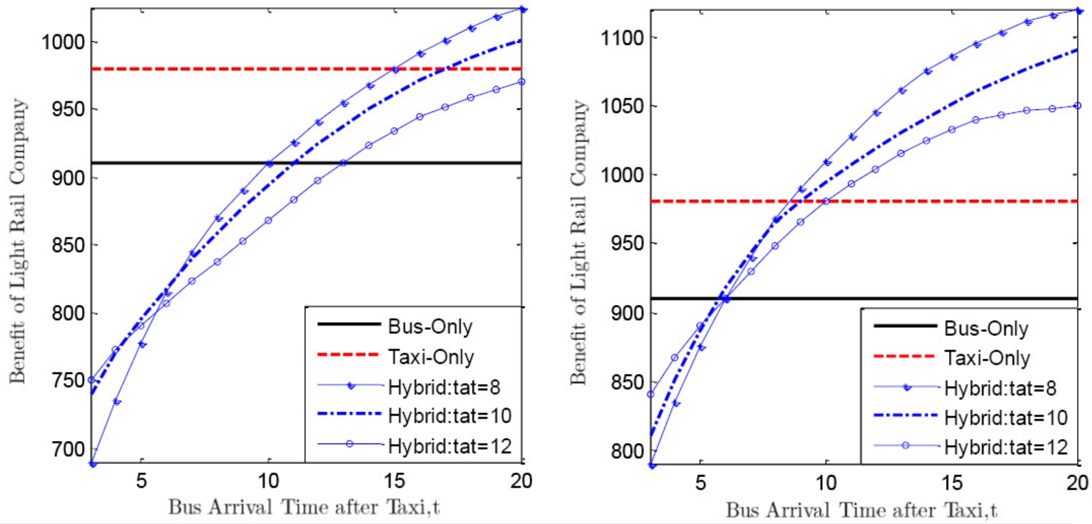


	Figure 2.1 $\gamma = 0.5$		**Figure 2.2 $\gamma = 0.7$**	
	Critical t_t	*Critical* t_b	*Critical* t_t	*Critical* t_b
$t_{a\_t} = 8$	15	10	8.5	6
$t_{a\_t} = 10$	17	11	9	6
$t_{a\_t} = 12$	22	13	10	6

($P = 150$, $c_l = 15$, $t_s = 15$, $t_d = 60$, $\alpha = 0.1$, $k_t = 1,000$, $V_t = 4$, $p_0 = 2$, $c_t = 0.5$, $k_b = 6,000$, $V_b = 40$, $c_b = 10$)

FIGURE 2. Comparisons of the hybrid method with bus-only and taxi-only methods

to the public transport systems. The passengers' behavior is considered as the basis for the light rail company to balance between the recovery cost and replacement service level. The promising directions of future study could focus on how to build a reliable long-term collaboration relationship between the taxi, bus and light rail companies, and how to incorporate the stochastic parameters into the decision models.

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