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KINEMATICS AND STATICS ANALYSES OF A PARALLEL MANIPULATOR WITH FIVE ACTIVE LIMBS

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Abstract. *A novel 5-DOF parallel manipulator (PM) with four SPS (spherical joint, prismatic joint, spherical joint) active limbs and one SPR (spherical joint, prismatic joint, revolute joint) active limb is proposed, and its kinematics are studied systematically. First, a virtual prototype of this PM is constructed and analytic formulas for solving the inverse displacement kinematics are derived. Second, analytic formulas for solving the inverse/forward velocities, inverse/forward accelerations, active forces and constrained forces of this mechanism are derived systematically. Third, an analytic example is given for solving the kinematics and statics of the proposed PM. Theoretical formulas and results provide a basis for its structure optimization design, control, dynamics performance analysis, manufacturing and applications.*
Keywords: Parallel mechanism, Kinematics, Inverse/forward velocities, Statics

1. **Introduction.** Currently, PMs have been studied and applied widely because of their advantages of high stiffness, good dynamic performances, simple in structures, small errors and easy to control [1-3]. Comparing with 6-DOF PMs, PMs with 5-DOF have attracted much attention because of their potential applications for medicine surgical operators, legs of walking robots, parallel machine tools, satellite surveillance platform, tunnel borer, and advantages such as low-cost, simple structure and less driver [4-6]. It is significant to create some novel 5-DOF PMs and investigate their kinematic characteristics in current industrial manufacturing and other fields. In this aspect, Piccin et al. [7] presented the architecture synthesis and kinematic modeling of a 3T2R 5-DOF PM which had been originally designed for a medical application. Sangveraphunsiri and Chooprasird [8] presented a design of a unique hybrid with 5-DOF mechanism based on an H-4 family PM and analyzed its kinematics and dynamics. Masouleh et al. [9] investigated the forward kinematic problem of 5-RPUR PM with identical limb structures. Li et al. [10] proposed a novel 5-DOF PM with 5PSS-type limbs and a central constraint UPU-type limb, and presented a statistics parameters optimization method for the novel 5-DOF PM. Borràs et al. [11] studied the kinematics and singularity of a line-plane 5-SPU PM in which the moving platform is rod with five universal joints. Liu et al. [12] developed a serial-parallel 5-DOF hybrid machine tool which had combined a 3-DOF PM with a 2-DOF serial tilting table.

The 5-DOF PMs above have their merits and different focuses, but they also have disadvantages which limit their applications in practice. For example, the 5-DOF PM proposed by Piccin et al. [7] and the 5-RPUR PM with identical limb structures proposed by Masouleh et al. [9] have large workspaces, but the stiffness is lower and they can be used in light-loading occasions only. The 5-DOF Gasbag Polishing Machine Tool with 5PSS-type limbs and a central constraint UPU-type limb proposed by Li et al. [10] has a complex structure and the workspace is small. The 5-SPU line-plane PM in which the moving platform is rod with five universal joints proposed by Borràs et al. [11] has advantages of high stiffness, but the workspace is limited. For this reason, a novel 5-DOF PM with four SPS active limbs and one SPR active limb is proposed and has the following merits by comparing with the 5-DOF PMs above. (1) The active limbs only with spherical joint S, prismatic joint P and revolute joint R have a simpler structure and it is easy to manufacture. (2) Since each of the active limbs with the linear actuator only bears the axial force along its own axis, it has a larger capacity of load bearing and it is easy to control. (3) The workspace and flexibility are clearly enlarged due to spherical joint and revolute joint having larger rotation range than universal joint before interference occurring.

This paper is organized as follows. The next section elaborates the composition of the proposed PM and its inverse displacement kinematics. Section 3 describes the derivation of inverse/forward velocity/acceleration and active/constrained forces. Section 4 gives an analytic example for solving the kinematics and statics of the proposed PM. Finally, Section 5 concludes the study and provides the recommendations for future research.

2. **The Proposed PM and Its Inverse Displacement Kinematics.** The proposed PM is composed of a moving platform, a fixed base, four SPS-type active limbs, and one SPR-type active limb, see Figure 1(a). The coordinate system O-XYZ is fixed at the centre of the fixed base at O, and p-xyz is fixed at the centre of the moving platform at p, see Figure 1(b). Among the five spherical joints A_i $(i = 1, 2, \dots, 5)$ on the fixed platform, the coordinate value of the first spherical joint A_1 is $(W, 0, 0)$, and the other four spherical joints are distributed around a circle with a radius of r_a uniformly. B_i $(i = 1, 2, \dots, 5)$ are uniformly distributed around a circle with a radius of r_b on the moving platform uniformly and the position angle is φ. At the initial time, the coordinate systems O-XYZ and p-xyz are parallel, and Z-axis is coincident with z-axis.

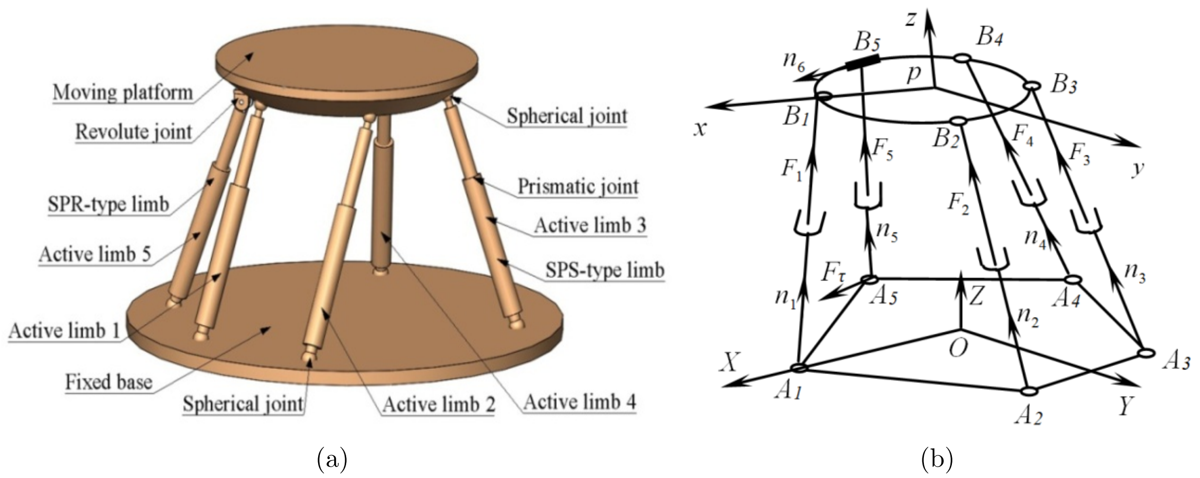

FIGURE 1. The proposed PM and its statics model

In this PM, the number of independent common constraints is $\lambda = 0$. The order of mechanism is $d = 6 - \lambda = 6$. The number of links is $n = 12$ including one moving platform, five cylinders, five piston-rods and one fixed base. The number of joints is $g = 15$ including five prismatic joints, nine spherical joints and one revolute joint. The dofs of the joints are $f_{\mathrm{P}} = f_{\mathrm{R}} = 1$ for prismatic or revolute joint, $f_{\mathrm{S}} = 3$ for spherical joint. The number of redudant constraints is $\nu = 0$. The redundancy dofs is $\zeta = 4$. Based on the revised Kutzbach-Grübler equation [13], the dof of the proposed PM is calculated as

$$\begin{aligned} M &= d(n-g-1) + \sum_{i=1}^{g} f_i + \nu - \zeta \\ &= 6 \times (12 - 15 - 1) + (5 \times 1 + 3 \times 9 + 1 \times 1) - 4 = 5 \end{aligned} \tag{1}$$

Before analyzing the inverse kinematics of the proposed PM, the position of A_i ($i = 1, 2, \ldots, 5$) in O-XYZ, B_i ($i = 1, 2, \ldots, 5$) in p-xyz, and B_i ($i = 1, 2, \ldots, 5$) in O-XYZ must be determined, and they can be expressed as:

$$\boldsymbol{A}_i^O = \begin{bmatrix} A_{iX} \\ A_{iY} \\ A_{iZ} \end{bmatrix} \quad \boldsymbol{B}_i^p = \begin{bmatrix} B_{ix} \\ B_{iy} \\ B_{iz} \end{bmatrix} \quad \boldsymbol{B}_i^O = \begin{bmatrix} B_{iX} \\ B_{iY} \\ B_{iZ} \end{bmatrix} \quad R_p^O = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix} \tag{2}$$

$$\boldsymbol{p}^O = \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix}, \quad \boldsymbol{B}_i^O = R_p^O \boldsymbol{B}_i^P + \boldsymbol{p}^O \quad (i = 1, 2, \ldots, 5) \tag{3}$$

where $\boldsymbol{p}^O = \begin{bmatrix} X_o & Y_o & Z_o \end{bmatrix}^T$ is the position vector of p-xyz in O-XYZ. R_p^O is a rotation transformation matrix from p-xyz to O-XYZ. Let α, β, γ be three Euler angles of p-xyz. R_p^O is formed by three Euler rotations of ($Z\ Y_1\ X_2$), namely, a rotation of α about Z-axis, followed by a rotation of β about Y_1-axis, and then a rotation of γ about X_2-axis. And Y_1 is formed by Y rotating about Z by α, X_2 is formed by X_1 rotating about Y_1 by β. Let λ be one of (α, β, γ) and set $c\lambda = \cos\lambda$, $s\lambda = \sin\lambda$, and thus, R_p^O is derived as below

$$R_p^O = \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\lambda + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}$$

Each active limbs of $\boldsymbol{l}_i$ ($i = 1, 2, \ldots, 5$) can be expressed as:

$$\boldsymbol{l}_i = \boldsymbol{B}_i^O - \boldsymbol{A}_i^O \quad (i = 1, 2, \ldots, 5) \tag{4}$$

The unit vector $\boldsymbol{n}_i$ of $\boldsymbol{l}_i$ ($i = 1, 2, \ldots, 5$) can be derived as

$$\boldsymbol{n}_i = \frac{\boldsymbol{l}_i}{\|\boldsymbol{l}_i\|} = \frac{1}{\|\boldsymbol{l}_i\|} \begin{bmatrix} B_{iX} - A_{iX} \\ B_{iY} - A_{iY} \\ B_{iZ} - A_{iZ} \end{bmatrix} \quad (i = 1, 2, \ldots, 5) \tag{5}$$

3. Inverse/Forward Velocity/Acceleration and Static Analysis.

3.1. Inverse/forward velocity.

Let $\boldsymbol{V}$ be the velocity of moving platform at p and $\boldsymbol{v}_i$ be the linear velocity of moving platform at B_i ($i = 1, 2, \ldots, 5$) respectively. They can be expressed as follows

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix}_{6\times 1}, \quad \boldsymbol{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}, \quad \boldsymbol{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \quad \boldsymbol{v}_i = \boldsymbol{v} + \boldsymbol{\omega} \times \boldsymbol{r}_i \quad (i = 1, 2, \ldots, 5) \tag{6}$$

where $\boldsymbol{v}$ and $\boldsymbol{\omega}$ are the linear velocity and angular velocity of p-xyz at p, $\boldsymbol{r}_i$ ($i = 1, 2, \ldots, 5$) are the vector from point p to B_i. By means of Equation (6), the velocities along the active limbs l_i ($i = 1, 2, \ldots, 5$) are derived as below

$$v_{l_i} = \boldsymbol{v}_i \cdot \boldsymbol{n}_i = (\boldsymbol{v} + \boldsymbol{\omega} \times \boldsymbol{r}_i) \cdot \boldsymbol{n}_i = \boldsymbol{n}_i \cdot \boldsymbol{v} + (\boldsymbol{r}_i \times \boldsymbol{n}_i) \cdot \boldsymbol{\omega} \quad (i = 1, 2, \ldots, 5) \tag{7}$$

$$\Rightarrow \begin{bmatrix} v_{l_1} \\ v_{l_2} \\ v_{l_3} \\ v_{l_4} \\ v_{l_5} \end{bmatrix} = \begin{bmatrix} \boldsymbol{n}_1^T & (\boldsymbol{r}_1 \times \boldsymbol{n}_1)^T \\ \boldsymbol{n}_2^T & (\boldsymbol{r}_2 \times \boldsymbol{n}_2)^T \\ \boldsymbol{n}_3^T & (\boldsymbol{r}_3 \times \boldsymbol{n}_3)^T \\ \boldsymbol{n}_4^T & (\boldsymbol{r}_4 \times \boldsymbol{n}_4)^T \\ \boldsymbol{n}_5^T & (\boldsymbol{r}_5 \times \boldsymbol{n}_5)^T \end{bmatrix}_{5\times 6} \cdot \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix}_{6\times 1} \tag{8}$$

The forces situation of the proposed PM is shown in Figure 1(b). Constrained force $\boldsymbol{F}_\tau$, applied on the spherical joint S of SPR active limb, is in parallel with revolute joint R of the SPR limb.

Since constrained force $\boldsymbol{F}_\tau$ does not do any work during the movement of the PM, there must be

$$F_\tau \boldsymbol{n}_6 \cdot \boldsymbol{v} + (F_\tau \boldsymbol{n}_6 \times \boldsymbol{r}_6) \cdot \boldsymbol{\omega} = 0 \Rightarrow \begin{bmatrix} \boldsymbol{n}_6^T & (\boldsymbol{r}_6 \times \boldsymbol{n}_6)^T \end{bmatrix}_{1\times 6} \cdot \boldsymbol{V} = [0] \tag{9}$$

where $\boldsymbol{n}_6$ is the unit vector of revolute joint R in O-XYZ, $\boldsymbol{r}_6 = \boldsymbol{A}_5^O - \boldsymbol{p}^O$.

By combining Equation (8) with Equation (9), the formulas for solving the inverse/forward velocities and a Jacobian matrix $\boldsymbol{G}$ are derived as below

$$\begin{bmatrix} v_{l_1} \\ v_{l_2} \\ v_{l_3} \\ v_{l_4} \\ v_{l_5} \\ 0 \end{bmatrix} = \boldsymbol{G} \cdot \boldsymbol{V}, \quad \boldsymbol{V} = \boldsymbol{G}^{-1} \cdot \begin{bmatrix} v_{l_1} \\ v_{l_2} \\ v_{l_3} \\ v_{l_4} \\ v_{l_5} \\ 0 \end{bmatrix}, \quad \boldsymbol{G} = \begin{bmatrix} \boldsymbol{n}_1^T & (\boldsymbol{r}_1 \times \boldsymbol{n}_1)^T \\ \boldsymbol{n}_2^T & (\boldsymbol{r}_2 \times \boldsymbol{n}_2)^T \\ \boldsymbol{n}_3^T & (\boldsymbol{r}_3 \times \boldsymbol{n}_3)^T \\ \boldsymbol{n}_4^T & (\boldsymbol{r}_4 \times \boldsymbol{n}_4)^T \\ \boldsymbol{n}_5^T & (\boldsymbol{r}_5 \times \boldsymbol{n}_5)^T \\ \boldsymbol{n}_6^T & (\boldsymbol{r}_6 \times \boldsymbol{n}_6)^T \end{bmatrix}_{6\times 6} \tag{10}$$

3.2. **Inverse/forward acceleration.** Let $\boldsymbol{A}$ be an acceleration of the moving platform at p. Let $\boldsymbol{a}$ and $\boldsymbol{\varepsilon}$ be a linear acceleration and an angular acceleration of the moving platform at p, respectively. They can be expressed as follows

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{\varepsilon} \end{bmatrix}_{6\times 1}, \quad \boldsymbol{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}, \quad \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} \tag{11}$$

Suppose there are two vectors $\boldsymbol{\eta}$, $\boldsymbol{\zeta}$ and a skew-symmetric matrix $\hat{\boldsymbol{\eta}}$. There must be the following relevant equations [13]

$$\boldsymbol{\eta} = \begin{bmatrix} \eta_x \\ \eta_y \\ \eta_z \end{bmatrix}, \quad \boldsymbol{\zeta} = \begin{bmatrix} \zeta_x \\ \zeta_y \\ \zeta_z \end{bmatrix}, \quad \hat{\boldsymbol{\eta}} = \begin{bmatrix} 0 & -\eta_z & \eta_y \\ \eta_z & 0 & -\eta_x \\ -\eta_y & \eta_x & 0 \end{bmatrix}, \ \boldsymbol{\eta} \times \boldsymbol{\zeta} = \hat{\boldsymbol{\eta}}\boldsymbol{\zeta}, \ \hat{\boldsymbol{\eta}}^T = -\hat{\boldsymbol{\eta}} \tag{12}$$

By differentiating the first five rows of Equation (10) with respect to time, five accelerations $\boldsymbol{a}_{li}$ along the ith active limb are expressed as below

$$\boldsymbol{a}_{l_i} = \begin{bmatrix} \boldsymbol{n}_i^T & (\boldsymbol{r}_i \times \boldsymbol{n}_i)^T \end{bmatrix} \boldsymbol{A} + \boldsymbol{V}^T \boldsymbol{H}_i \boldsymbol{V} \quad (i = 1, 2, \ldots, 5) \tag{13}$$

where

$$\boldsymbol{H}_i = \frac{1}{l_i} \begin{bmatrix} \boldsymbol{E}_{3\times 3} & -\hat{\boldsymbol{r}}_i \\ \hat{\boldsymbol{r}}_i & \hat{\boldsymbol{r}}_i^2 \end{bmatrix}_{6\times 6} - \frac{1}{l_i} \begin{bmatrix} \boldsymbol{n}_i \\ \boldsymbol{r}_i \times \boldsymbol{n}_i \end{bmatrix} \begin{bmatrix} \boldsymbol{n}_i^T \, (\boldsymbol{r}_i \times \boldsymbol{n}_i)^T \end{bmatrix} + \begin{bmatrix} \boldsymbol{0}_{3\times 3} & \boldsymbol{0}_{3\times 3} \\ \boldsymbol{0}_{3\times 3} & \hat{\boldsymbol{r}}_i \hat{\boldsymbol{n}}_i \end{bmatrix}_{6\times 6}.$$

By differentiating the sixth row of Equation (10) with respect to time, we obtain

$$\boldsymbol{0}_{1\times 6} = \begin{bmatrix} \boldsymbol{n}_6^T & (\boldsymbol{r}_6 \times \boldsymbol{n}_6)^T \end{bmatrix} \boldsymbol{A} + \boldsymbol{V}^T \boldsymbol{H}_6 \boldsymbol{V} \tag{14}$$

where $\boldsymbol{H}_6 = \begin{bmatrix} \boldsymbol{0}_{3\times 3} & -\hat{\boldsymbol{n}}_6 \\ \hat{\boldsymbol{n}}_6 & -\hat{\boldsymbol{n}}_6 \hat{\boldsymbol{r}}_6 \end{bmatrix}_{6\times 6}$.

Combining Equation (13) with Equation (14), an inverse acceleration $\boldsymbol{a}_{in}$ and a forward acceleration $\boldsymbol{A}$ and a Hessian matrix $\boldsymbol{H}$ are derived as follows:

$$\boldsymbol{a}_{in} = \boldsymbol{G}\boldsymbol{A} + \boldsymbol{V}^T \boldsymbol{H} \boldsymbol{V}, \quad \boldsymbol{A} = \boldsymbol{G}^{-1} \left(\boldsymbol{a}_{in} - \boldsymbol{V}^T \boldsymbol{H} \boldsymbol{V} \right) \tag{15}$$

where $\boldsymbol{a}_{in} = \begin{bmatrix} \boldsymbol{a}_{l_1} & \boldsymbol{a}_{l_2} & \boldsymbol{a}_{l_3} & \boldsymbol{a}_{l_4} & \boldsymbol{a}_{l_5} & 0 \end{bmatrix}^T$, $\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{H}_2 & \boldsymbol{H}_3 & \boldsymbol{H}_4 & \boldsymbol{H}_5 & \boldsymbol{H}_6 \end{bmatrix}^T$.

3.3. **Active/constraint forces and torque.** When ignoring gravity and friction of all links, the proposed PM is an ideal constrained system, the workloads of it can be simplified as a wrench ($\boldsymbol{F}$, $\boldsymbol{T}$) applied on the moving platform at p, where $\boldsymbol{F}$ is a central force and $\boldsymbol{T}$ is a central torque. ($\boldsymbol{F}_X$, $\boldsymbol{F}_Y$, $\boldsymbol{F}_Z$, $\boldsymbol{T}_X$, $\boldsymbol{T}_Y$, $\boldsymbol{T}_Z$) are the components of ($\boldsymbol{F}$, $\boldsymbol{T}$) and they are balanced by five active forces $\boldsymbol{F}_i$ ($i = 1, 2, \ldots, 5$) and one constrained force $\boldsymbol{F}_\tau$. Each of $\boldsymbol{F}_i$ is applied on and along the active limbs l_i ($i = 1, 2, \ldots, 5$).

Based on the principle of virtual work, by means of Equation (10), the formulas for solving the active/constrained force are derived as

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_\tau \end{bmatrix}^T \begin{bmatrix} v_{r1} \\ v_{r2} \\ v_{r3} \\ v_{r4} \\ v_{r5} \\ 0 \end{bmatrix} + \begin{bmatrix} F \\ \boldsymbol{T} \end{bmatrix}^T \boldsymbol{V} = 0, \quad \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_\tau \end{bmatrix}^T \boldsymbol{G} + \begin{bmatrix} F \\ \boldsymbol{T} \end{bmatrix}^T = 0$$

$$\Rightarrow \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_\tau \end{bmatrix} = -\left(\boldsymbol{G}^{-1}\right)^T \begin{bmatrix} F \\ \boldsymbol{T} \end{bmatrix} = -\left(\boldsymbol{G}^T\right)^{-1} \begin{bmatrix} F \\ \boldsymbol{T} \end{bmatrix} \tag{16}$$

where $-(\boldsymbol{G}^T)^{-1}$ is defined as force Jacobian matrix.

According to the analyses above, a simple approach for solving the inverse/forward velocity/acceleration and active/ constrained forces of the proposed PM was derived based on the principle of virtual work. The equations of inverse/forward velocity/acceleration and active/constrained forces are unified and simple and this approach can be used on other PMs with linear active legs.

4. **Analytic Solved Example.** Optimization design and performance analysis have been an important issue on robotics; a good performance distribution means that the performance is steady and uniform in the workspace. The purpose of the performance analysis is to find a set of optimal parameters of r_a and r_b. Based on the novel approach proposed by Gosselin [14] to evaluate the performance of mechanism, that is, $r_a = 400$ mm, $r_b = 100$ mm, the output errors between the joint space and cartesian space are small, and both of velocity and acceleration will have good movement performance. Therefore, set $r_a = 400$ mm, $r_b = 100$ mm, $\boldsymbol{F} = (0\ 10\ \ -10)^T$ N, $\boldsymbol{T} = (10\ 0\ 10)^T$ N·mm. The pose parameters X_o, Y_o, Z_o, α, β, γ of the moving platform are given in Figure 2(a) and Figure 2(b). These pose parameters are used to verify the analytic formulas above and the proposed PM has the same characteristics with other parameters. The analytic formulas above can be used on other PMs with linear active legs. A program is compiled in Matlab based on the relevant analytic equations of (5), (10), (15) and (16), the length, velocity, acceleration and statics of the five active limbs that vary with time are solved using the compiled program, which are shown in Figures 2(c)-2(f).

The characteristics of the proposed PM are found based on the solved results and are analyzed as follows.

(1) When the displacement components of moving platform are varied within $-169.2 \sim 74.5$ mm, $0 \sim 535.0$ mm, $459.1 \sim 580.2$ mm for X_o, Y_o, Z_o, respectively, the orientation components of moving platform are varied within $0 \sim 52.5°$, $0 \sim 35°$, $-39.4 \sim 0°$ for α, β, γ, respectively, see Figure 2(a) and Figure 2(b). The length of active limbs are varied within $670.4 \sim 951.3$ mm, $587.9 \sim 859.0$ mm, $520.0 \sim 646.1$ mm, $627.3 \sim 933.7$ mm, $633.4 \sim 1020.8$ mm for active limb 1, active limb 2, active limb 3, active limb 4 and active limb 5, see Figure 2(c). It implies that the proposed PM has a large reachable workspace.

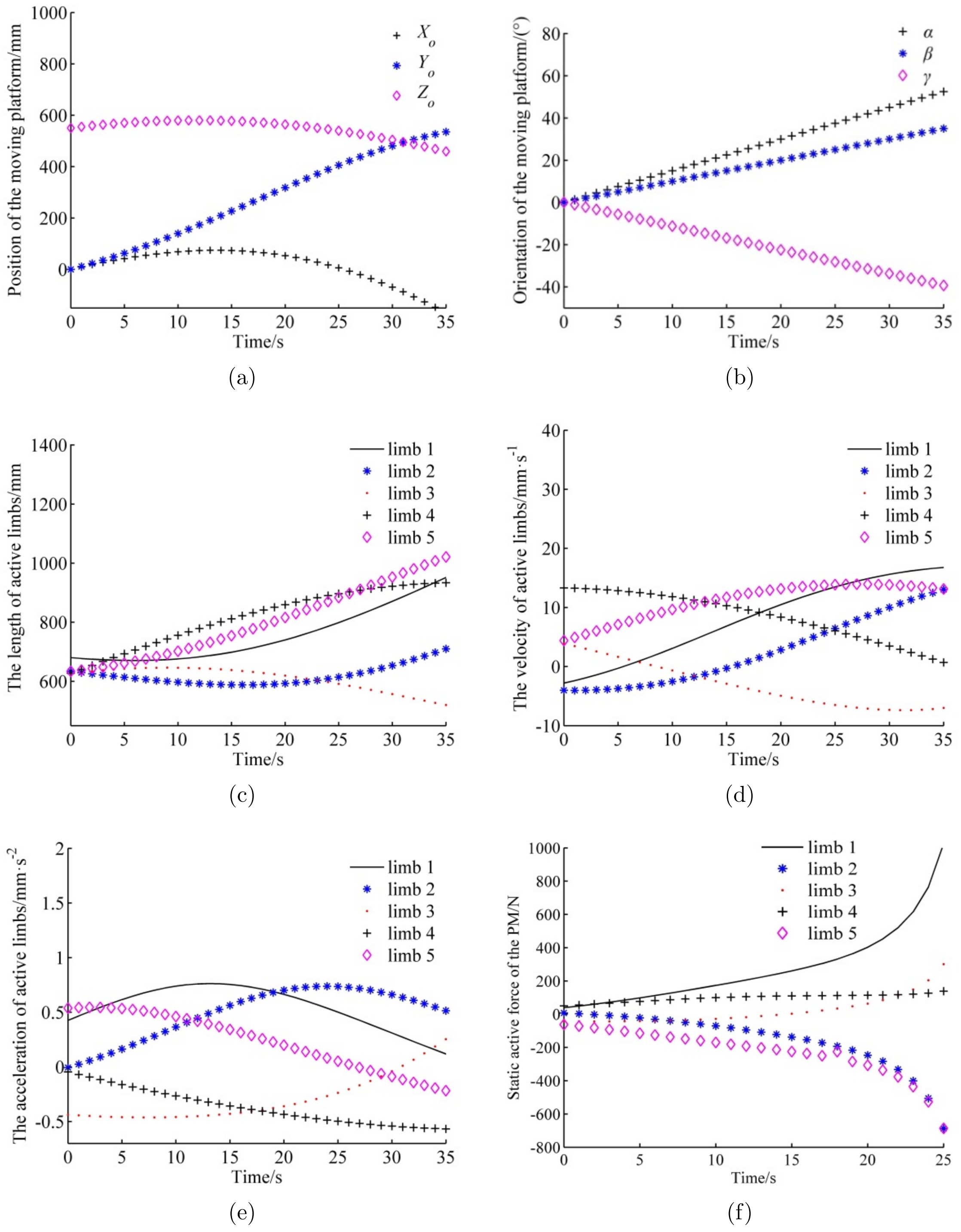

FIGURE 2. Analytic results of kinematics and statics

(2) When the center p of the moving platform is close to Z axis, the active forces of the proposed PM are lowered. When the center p of the moving platform is far from Z axis, the active forces of the proposed PM are increased largely, see Figure 2(f).

(3) When the displacement and orientation of the moving platform are varied smoothly, the displacement, velocity, acceleration, and active forces of the active limbs are varied smoothly with no sudden changes or breakpoints. It implies that the proposed PM has good characteristics of kinematics and statics.

5. **Conclusions.** This paper presents a novel 5-DOF PM with four SPS-type active limbs and one SPR-type active limb. The whole structure is simple and the interference among

the active legs and moving platform can be avoided easily. Each of the active legs with the linear actuator has a relatively large capability of load bearing as it only bears an active force along its own axis.

The analytic formulas for solving inverse displacement, inverse/forward velocities, inverse/forward accelerations and active/constrained forces of the proposed PM have been derived. The analytic formulas above are unified and simple and this approach can be used on other PMs with linear active legs.

The solved results show that the proposed 5-DOF PM has a large reachable workspace, good kinematic and static characteristics. In addition, this paper provides a theoretical basis for its future studies and this novel parallel mechanism has some potential applications for 5-DOF parallel machine tools, parallel sensor, precision measurement and so on. For the future research, the prototype of the proposed PM should be manufactured and the stiffness and elastic deformation should be analysed.

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REFERENCES

[1] J. X. Qing, J. F. Li and B. Fang, Drive optimization of tricept parallel mechanism with redundant actuation, *Journal of Mechanical Engineering*, vol.46, no.5, pp.8-14, 2010.

[2] J. He, F. Gao, X. D. Meng and W. Z. Guo, Type synthesis for 4-DOF parallel press mechanism using G_F set theory, *Chinese Journal of Mechanical Engineering*, vol.28, no.4, pp.851-859, 2015.

[3] H. Lim, S. H. Lee, B. R. So and B. J. Yi, Design of a new 6-DOF parallel mechanism with a suspended platform, *International Journal of Control, Automation, and Systems*, vol.13, no.4, pp.942-950, 2015.

[4] M. Siahmansouri, A. Ghanbari and M. M. S. Fakhrabadi, Design, implementation and control of a fish robot with undulating fins, *International Journal of Advanced Robotic Systems*, vol.8, no.5, pp.61-69, 2011.

[5] M. T. Masouleh and C. Gosselin, Singularity analysis of 5-RPUR parallel mechanisms (3T2R), *International Journal of Advanced Manufacturing Technology*, vol.57, no.9-12, pp.1107-1121, 2011.

[6] X. L. Chen, X. X. Liang, Y. Deng and Q. Wang, Rigid dynamic model and analysis of 5-DOF parallel mechanism, *International Journal of Advanced Robotic Systems*, vol.12, pp.1-9, 2015.

[7] O. Piccin, B. Bayle, B. Maurin and M. D. Mathelin, Kinematic modelling of a 5-DOF parallel mechanism for semi-spherical workspace, *Mechanism and Machine Theory*, vol.44, no.8, pp.1485-1496, 2009.

[8] V. Sangveraphunsiri and K. Chooprasird, Dynamics and control of a 5-DOF manipulator based on an H-4 parallel mechanism, *International Journal of Advanced Manufacturing Technology*, vol.52, nos.1-4, pp.343-364, 2011.

[9] M. T. Masouleh, C. Gosselin, M. Husty and D. R. Walter, Forward kinematic problem of 5-RPUR parallel mechanisms (3T2R) with identical limbs structures, *Mechanism and Machine Theory*, vol.46, no.7, pp.945-959, 2011.

[10] Y. B. Li, D. P. Tan, D. H. Wen et al., Parameters optimization of a novel 5-DOF gasbag polishing machine tool, *Chinese Journal of Mechanical Engineering*, vol.26, no.4, pp.680-688, 2013.

[11] J. Borràs, F. Thomas and C. Torras, Singularity-invariant families of line-plane 5-SPU platforms, *IEEE Trans. Robotics*, vol.27, no.5, pp.837-848, 2011.

[12] H. T. Liu, T. Huang and J. P. Mei, Kinematic design of a 5-DOF hybrid robot with large workspace/limb-stroke ratio, *Journal of Mechanical Design*, vol.129, no.5, pp.530-537, 2007.

[13] Z. Huang, L. F. Kong and Y. F. Fang, *Theory on Parallel Robotics and Control*, Machinery Industry Press, Beijing, China, 1997.

[14] C. Gosselin and J. Angeles, A global performance index for the kinematic optimization of robotic manipulators, *Journal of Mechanical Design*, vol.113, no.3, pp.220-226, 1991.