ICIC Express Letters
Part B: Applications
Volume **8**, Number **3**, March **2017**

ICIC International ©2017 ISSN 2185-2766
pp. **615–621**

IMPULSIVE SYNCHRONIZATION CONTROL OF COUPLED NEURAL NETWORKS AND ITS APPLICATION ON THE EPILEPSY THERAPY

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Received August 2016; accepted November 2016

Abstract. *This letter is concerned with the synchronization of coupled neural networks and its application on the epilepsy therapy via impulsive control. Based on the Lyapunov theory and the method of the impulsive control, some simple criteria are derived for the synchronization of the coupled neural networks. The impulsive controller which we have obtained can cut down control cost and is more practical. The sufficient conditions of this letter for synchronization of the neural networks are less conservative and can be applied in the area of the epilepsy therapy. The numerical examples of the coupled neural networks are given to demonstrate the effectiveness of the control strategies. The design of controllers and control strategy may provide technical support for epilepsy treatment apparatus research and development.*
Keywords: Coupled neural networks, Synchronization, Impulsive control, Epilepsy therapy, Lyapunov stability

1. **Introduction.** It is well known that neurons in the brain connected with each other by synapse constitute the network, which is one of the most complex nonlinear networks in the world. The neural network is a fast growing field of research in both theoretical and application point of view. It has created an increasing interest and attracted much attention of many researchers, scientists in recent years due to their potential applications in physics, medicine, signal processing, image processing, bioengineering, associative memories, pattern recognition, and so on [1-5].

In addition, many efforts have been devoted to the control and synchronization of neural networks due to its potential practical applications. And many control approaches have been proposed to stabilize chaotic networks and nonlinear systems such as adaptive control [3,6], pinning control [7], intermittent control [8], and impulsive control [4,5].

There are many ways to cure the epilepsy, such as drug therapy, surgery therapy and deep brain stimulation. Unfortunately, drug therapy and surgery therapy typically have serious side effects. However, the two therapies have some defects, and more than 20% of the epilepsy people have no control effect [9]. Thus, it is urgent to study the mechanism of epilepsy, so that the abnormal epileptic discharge behavior can be controlled. Deep brain stimulation has received arising attention in treating epilepsy for the past few years [10]. Recently, the development of complex network dynamics accelerates the research of epilepsy. It is reported that epileptic seizures, diffusion and holding out are mostly due to the reciprocity of neuron network in the brain with small world functional connectivity. If we can fully understand the functional connectivity of brain when epileptic seizures by complex network analysis methods, then we will offer a new research approach for epileptic seizure prediction or controlling. It will greatly minimize risk or injury and right smart improve the quality of life for many people with epilepsy. It has the advantages of effectiveness and small side effects. It is helpful to find an effective method via brain stimulation to prevent the high frequency oscillation of neurons for treating some nervous system disease using the synchronous thought.

Motivated by the above discussion, the main contribution of this paper is to analyze the problem of impulsive synchronization of coupled neural networks and its application on the epilepsy therapy. Based on the Lyapunov stability theory combined with the method of impulsive control, some simple criteria are derived for the synchronization of the coupled delays neural networks. At last, simulation results show the effectiveness of the proposed control strategy. The design of controllers and control strategy may provide a potential electrical stimulation therapy on neurological diseases caused by abnormal synchronization.

The rest of the letter is organized as follows. In Section 2, the model of abnormal synchronization coupled neural networks is presented. And some hypotheses and preliminaries are given. In Section 3, impulsive control synchronization for the coupled neural networks is designed. The simple and novel synchronization criterion is obtained. In Section 4, a numerical example of coupled neural networks is given to demonstrate the effectiveness of the proposed controllers. Conclusions are given in Section 5.

2. Model and Preliminaries.

Consider an abnormal neural networks model consisting of N abnormal identical nodes with high frequency oscillation (i.e., this kind of neurons easily leads to the occurrence of epilepsy), in which each node is an n-dimensional nontrivial periodic orbit neural network as follows

$$\dot{x}_i(t) = -Cx_i(t) + Af(x_i(t)) + Bf(x_i(t-\tau)) + I(t) + \sum_{j=1}^{N} g_{ij}\Gamma x_j(t) + u_i(t), \tag{1}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^{\mathrm{T}} \in \mathbb{R}^n$ $(i = 1, 2, \ldots, N)$ represents the state vector of the ith abnormal neural network at time t. $C = \mathrm{diag}\{c_1, c_2, \ldots, c_n\}$ with $c_k > 0$, $k = 1, 2, \ldots, n$, denotes the rate with which the cell k resets its potential to the resting state when isolated from other cells and inputs $A = (a_{pq})_{n\times n}$, $B = (b_{pq})_{n\times n}$ represent the connection weight matrix and the delayed connection weight matrix, respectively. a_{pq}, b_{pq} denote the strengths of connectivity between the cells p and q within the ith node at time t and $t-\tau$, respectively. $f(x_i(t)) = [f_1(x_{i1}(t)), f_2(x_{i2}(t)), \ldots, f_n(x_{in}(t))]^{\mathrm{T}}$, and $f_i(\cdot)$ $(i = 1, 2, \ldots, n)$ are activation functions. $I(t) = [I_1(t), I_2(t), \ldots, I_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ is an external input vector. The constant matrix $G = (g_{ij})_{N\times N}$ represents the linear coupling configuration of the whole network, which satisfies $g_{ij} \geq 0$, for $i \neq j$, and $g_{ii} = -\sum_{j=1,j\neq i}^{N} g_{ij}$, $\bar{g}_{ii} = -\sum_{j=1,j\neq i}^{N} g_{ji}$, $i = 1, 2, \ldots, N$. $\Gamma = (\gamma_{ij})_{n\times n}$ is inner-coupling matrix between nodes.

Definition 2.1. *Let $x_i(t; t_0; \phi)$, $i = 1, 2, \ldots, N$ be a solution of the delayed complex dynamical network (1), where $\phi = (\phi_1^{\mathrm{T}}, \phi_2^{\mathrm{T}}, \ldots, \phi_N^{\mathrm{T}})^{\mathrm{T}}$, $\phi_i = \phi_i(\theta) \in C([-\tau, 0], \mathbb{R}^n)$ are initial conditions. If there exist constants $\alpha > 0$, $\lambda > 0$ and a nonempty subset $\Lambda \subseteq \mathbb{R}^n$, such that ϕ_i take values in Λ and $x_i(t; t_0; \phi) \in \mathbb{R}^n$ for all $t \geq t_0$ and*

$$\|x_i(t; t_0; \phi) - s(t; t_0; s_0)\| \leq \alpha e^{-\lambda t} \sup_{-\tau\leq\theta\leq 0} \|\phi(\theta) - s_0\|, \quad i = 1, 2, \ldots, N, \tag{2}$$

where $s(t; t_0; s_0)$ is a solution of a membrane potential of health neuron from normal neural network with $s_0 \in \mathbb{R}^n$, i.e.,

$$\dot{s}(t) = -Cs(t) + A\bar{f}(s(t)) + B\bar{f}(s(t-\tau)) + I(t), \tag{3}$$

where $\bar{f}(x_i(t)) = \left[\bar{f}_1(x_{i1}(t)), \bar{f}_2(x_{i2}(t)), \ldots, \bar{f}_n(x_{in}(t))\right]^{\mathrm{T}}$*, and* $\bar{f}_i(\cdot)$ $(i = 1, 2, \ldots, n)$ *are activation functions of healthy neural cell with the chaotic attractor. Then the coupled neural network (1) is said to realize exponential synchronization, and* $\Lambda \times \Lambda \times \cdots \times \Lambda$ *is called the region of synchrony of the coupled neural network (1).*

Lemma 2.1. *For any vectors* $x, y \in \mathbb{R}^n$ *and positive definite matrix* $Q \in \mathbb{R}^{n\times n}$*, the following matrix inequality holds:*

$$2x^{\mathrm{T}}y \leq x^{\mathrm{T}}Qx + y^{\mathrm{T}}Q^{-1}y.$$

Define the error vector by

$$e_i(t) = x_i(t) - s(t), \quad i = 1, 2, \ldots, N. \tag{4}$$

Then the error system can be described by

$$\dot{e}_i(t) = -Ce_i(t) + AF(e_i(t)) + BF(e_i(t-\tau)) + \sum_{j=1}^{N} g_{ij}\Gamma e_j(t) + u_i(t),\ i = 1, 2, \ldots, N, \tag{5}$$

where $F(e_i(t)) = f(x_i(t)) - \bar{f}(s(t))$.

Then the synchronization problem of the coupled neural network (1) is equivalent to the problem of stabilization of the error dynamical system (5).

Concerning system (2), we list assumptions as follows.

Hypothesis 2.1. *There exist positive constants* l_k *such that*

$$\left|f_k(x(t)) - \bar{f}_k(s(t))\right| \leq l_k|x(t) - s(t)|, \quad k = 1, 2, \ldots, n.$$

3. **Impulsive Synchronization of Coupled Neural Networks.** In this section, we will introduce a useful impulsive synchronization control criterion for the coupled neural networks (1). In order to realize synchronization of the coupled neural networks (1) to the health neuron from normal neural network (3), we choose the impulsive controllers

$$u_i(t) = \begin{cases} -k_{ir}e_i(t_r^-), & t = t_r, r = 1, 2, \ldots; \\ 0, & t \neq t_r. \end{cases} \tag{6}$$

The general error neural networks with delays (5) become into the impulsive delays error neural networks (7). Thus, the system (5) can be rewritten as,

$$\begin{cases} \dot{e}_i(t) = -Ce_i(t) + AF(e_i(t)) + BF(e_i(t-\tau)) + \sum\limits_{j=1}^{N} g_{ij}\Gamma e_j(t) + u_i(t), & t \neq t_r \\ \Delta e_i(t_r) = e_i(t_r^+) - e_i(t_r^-) = k_{ir}e_i(t_r^-), & t = t_r. \end{cases} \tag{7}$$

It is clear that, if the zero solution of the error dynamical system (7) is globally exponentially stable, the exponential synchronization of the controlled coupled neural networks (1) is achieved, i.e., the abnormal neuron with high frequency oscillation becomes the health neurons with the chaotic attractor. Thus, the epilepsy has been inhibited.

Based on Hypothesis 2.1, the controlled coupled neural networks (1) synchronization criterion is deduced as follows.

Theorem 3.1. *Based on Hypothesis 2.1, if there exist positive constants* $a_1 > b^2L^2/q$*,* $a_2 < \lambda$ *and* q*, such that*

$$\left[q + a^2L^2/2q - C - (a_2 - a_1)/2\right] I_N + \sigma\widehat{G_0} \leq 0, \tag{8}$$

where $\lambda > 0$ *is the smallest real root of the equation* $a_1 - \lambda - b^2L^2/q \cdot \exp\{\lambda\tau\} = 0$*. Then the complex controlled coupled neural networks with delay (1) globally exponentially synchronize with the desired evolution* $s(t)$ *under impulsive controllers (6).*

Proof: Construct the following Lyapunov candidate function

$$V(t)=\frac{1}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t)+\frac{1}{2}\sum_{i=1}^{N}\exp\{-a_1t\}\frac{(k_i(t)-k)^2}{\alpha_i}, \tag{9}$$

and k is an undetermined sufficiently large positive constant.

In order to get less conservative synchronization criterion, we introduce the adaptive intermittent control as follows

$$u_i(t)=\begin{cases}-k_i(t)e_i(t), & t\in[\kappa T,\kappa T+h),\\ 0, & t\in[\kappa T+h,(\kappa+1)T),\end{cases} \tag{10}$$

and the updating laws $\dot{k}_i(t)=\alpha_i\exp\{a_1t\}\|e_i(t)\|^2$, where α_i $(i=1,2,\ldots,N)$ and a_1 are positive constants, $T>0$ denotes the control period, $0<h<T$ and $\kappa=0,1,2,\ldots$.

According to Hypothesis 2.1 and Lemma 2.1, then the derivative of $V(t)$ with respect to time t along the solutions of Equation (7) can be calculated as follows.

When $\kappa T\le t<\kappa T+h$, for $\kappa=0,1,2,\ldots$

$$\begin{aligned}
\dot{V}(t)&=\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\dot{e}_i(t)-\frac{1}{2}a_1\sum_{i=1}^{N}\exp\{-a_1t\}\frac{(k_i(t)-k)^2}{\alpha_i}+\sum_{i=1}^{N}(k_i(t)-k)e_i^{\mathrm{T}}(t)e_i(t)\\
&=-\sum_{i=1}^{N}Ce_i^{\mathrm{T}}(t)e_i(t)+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)AF(e_i(t),t)+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)BF(e_i(t-\tau),t-\tau)\\
&\quad+\sum_{i=1}^{N}\sum_{j=1}^{N}g_{ij}e_i^{\mathrm{T}}(t)\Gamma e_j(t)-\sum_{i=1}^{N}k_i(t)e_i^{\mathrm{T}}(t)e_i(t)-\frac{1}{2}a_1\sum_{i=1}^{N}\exp\{-a_1t\}\frac{(k_i(t)-k)^2}{\alpha_i}\\
&\quad+\sum_{i=1}^{N}(k_i(t)-k)e_i^{\mathrm{T}}(t)e_i(t)\\
&\le\sum_{i=1}^{N}\left(q+\frac{a^2L^2}{2q}-c\right)e_i^{\mathrm{T}}(t)e_i(t)+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\sigma\xi_ig_{ij}\|e_i(t)\|_2\|e_j(t)\|_2\\
&\quad+\sum_{i=1}^{N}\lambda\xi_ig_{ii}e_i^{\mathrm{T}}(t)e_i(t)+\frac{b^2L^2}{2q}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t-\tau)e_i(t-\tau)-\sum_{i=1}^{N}ke_i^{\mathrm{T}}(t)e_i(t)\\
&\quad-\frac{1}{2}a_1\sum_{i=1}^{N}\exp\{-a_1t\}\frac{(k_i(t)-k)^2}{\alpha_i}\\
&\le\bar{e}^{\mathrm{T}}(t)\left[\left(q+\frac{a^2L^2}{2q}-C+\frac{1}{2}a_1\right)I_N+\sigma\hat{G}-K\right]\bar{e}(t)-\frac{1}{2}a_1\bar{e}^{\mathrm{T}}(t)\bar{e}(t)\\
&\quad+\frac{b^2L^2}{2q}\bar{e}_i^{\mathrm{T}}(t-\tau_l)\bar{e}_i(t-\tau_l)-\frac{1}{2}a_1\sum_{i=1}^{N}\exp\{-a_1t\}\frac{(k_i(t)-k)^2}{\alpha_i}\\
&\quad+\frac{b^2L^2}{2q}\sum_{i=1}^{N}\exp\{-a_1t\}\frac{(k_i(t-\tau)-k)^2}{\alpha_i},
\end{aligned} \tag{11}$$

where $\bar{e}^{\mathrm{T}}(t)=(\|e_1(t)\|_2,\|e_2(t)\|_2,\ldots,\|e_N(t)\|_2)^{\mathrm{T}}$, $K=kI_N$. Because k is an undetermined sufficiently large positive constant, we can select k as

$$k>q+\frac{a^2L^2}{2q}-C+\frac{1}{2}a_1+\sigma\lambda_{\max}\hat{G}, \tag{12}$$

so we have

$$\dot{V}(t)\le-a_1V(t)+V(t-\tau). \tag{13}$$

Similarly, when $\kappa T + h \le t < (\kappa + 1)T$, using condition in Equation (8), one has

$$\dot{V}(t) \le (a_2 - a_1)V(t) + \sum_{l=1}^{m} \frac{b^2 L^2}{q} V(t-\tau). \tag{14}$$

According to the method of [8], we can attain similarly

$$V(t) \le \bar{Q} \exp\left\{-\left[\lambda - a_2\left(1 - \frac{h}{T}\right)\right] t\right\}, \quad t \ge 0, \tag{15}$$

where $\bar{Q} = \sup_{-\tau \le s \le 0} V(s)$. Take $\varepsilon = \lambda - a_2$ and $h \to 0$, so we obtain

$$V(t) \le \sup_{-\tau \le s \le 0} V(s) \exp\{-\varepsilon t\}, \quad t \ge 0. \tag{16}$$

Apparently, when $h \to 0$, the periodically intermittent controller (10) degenerates into the impulsive controller (6) right now. It follows from condition in Equation (9) that the zero solution of the error dynamical system (7) is globally exponentially stable. This completes the proof of Theorem 3.1.

4. **Numerical Example.** In this section, a numerical example of neural networks is given to demonstrate the effectiveness of the impulsive controllers. First, choose the membrane potential of abnormal synchronization neural networks from pathological neural network (1). Here $I(t) = (0,0)^{\mathrm{T}}$ is external input vector, and $\tau = 1.1$ is time delay. Take $C = \begin{pmatrix} 1.5 & 0 \\ 0 & 1 \end{pmatrix}$, $A = \begin{pmatrix} 2.1 & -0.1 \\ -5.2 & 3.2 \end{pmatrix}$, $B = \begin{pmatrix} -1.6 & -0.2 \\ -0.3 & -2.5 \end{pmatrix}$, $f(x_i(t)) = \tanh(x)$. Then the dynamical behavior of a solution of a membrane potential of abnormal neuron from pathological neural network (1) with initial condition $x_1(t_0) = 0.8$, $x_2(t_0) = 0.2$, which can be seen in Figure 1. Evidently, this pathological neural network (1) has a nontrivial periodic orbit with high frequency oscillation. The abnormal neural network is in a state of an abnormal discharge. If these abnormal neurons are synchronous, epilepsy is likely to happen.

Then, choose the membrane potential of health neuron from normal neural network (3). Taking $\bar{C} = \begin{pmatrix} 1.1 & 0 \\ 0 & 1 \end{pmatrix}$, all other parameters are the same as the abnormal neuron above. Then the dynamical behavior of model (3) with initial condition $x_1(t_0) = 0.8$, $x_2(t_0) = 0.2$, which can be seen in Figure 2. Obviously, this normal neuron (3) has a chaotic attractor.

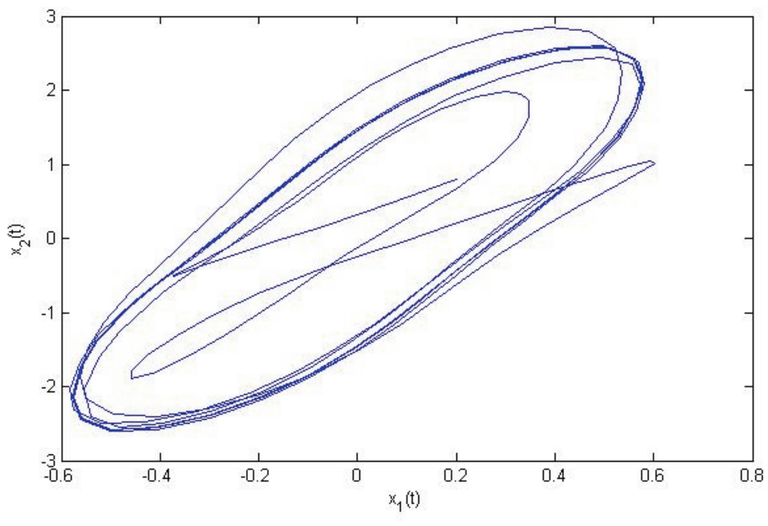


Figure 1. Chaotic trajectory of abnormal neuron

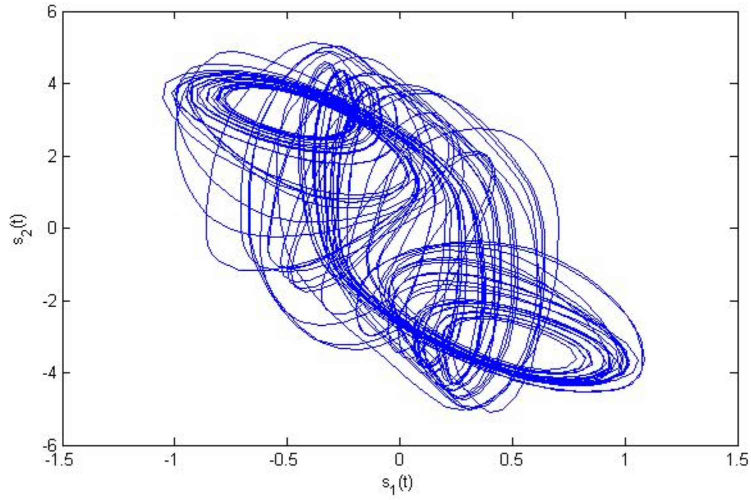
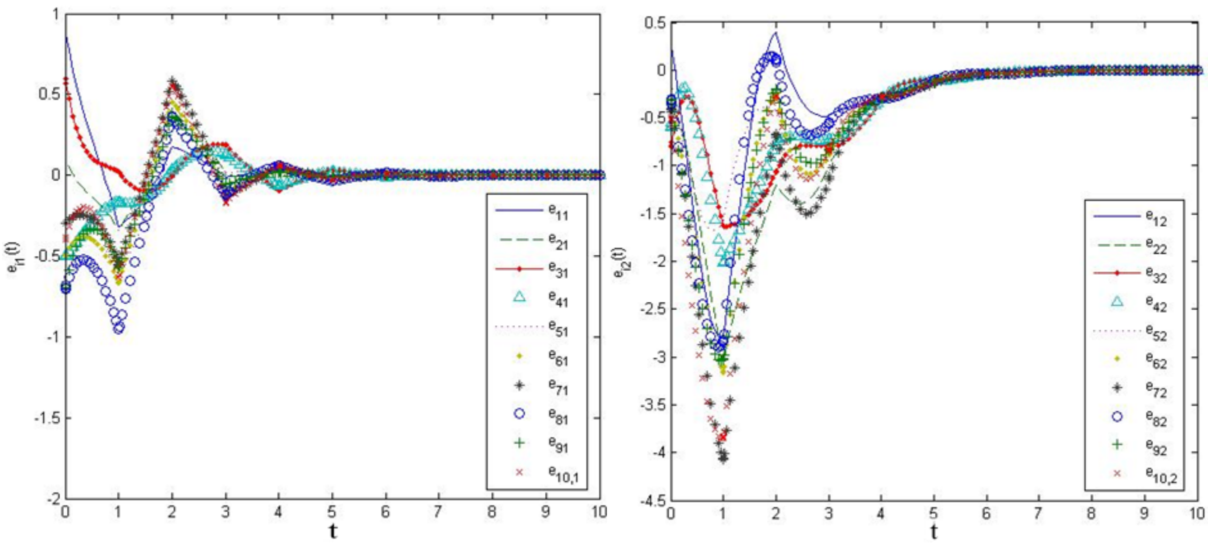


FIGURE 2. Periodic orbit of health neuron



(a) (b)

FIGURE 3. Errors e_{i1}, e_{i2} $(i = 1, 2, \ldots, 10)$ of network (1) under the impulsive controllers (6)

In the simulation, the asymmetric coupling matrix as G is random and satisfied with the coupling condition. According to Theorem 3.1, it is found (8) is satisfied under the impulsive controllers (6). The errors of neural network (7) by impulsive controllers are numerically demonstrated as Figure 3. Clearly, all abnormal neurons are rapidly achieving synchronization with the state of health neuron. It follows that the impulsive controller is designed such that the abnormal neuron can synchronize and recover a health neuron.

5. **Conclusions.** In this paper, the synchronization of coupled neural networks and its application on the epilepsy therapy via impulsive control have been investigated in detail. Based on the method of the impulsive control and the Lyapunov theory, a simple criterion which is derived for the synchronization of the neural networks with delay is less conservative. The conditions of this paper for synchronization of the neural networks can be applied in the area of the epilepsy therapy. The simulation example of the coupled neural

networks is given to demonstrate the effectiveness of the control strategies. The controllers designed in this paper provide technical support for the research and development of epilepsy treatment apparatus.

In this letter, the synchronization of the linear coupled neural networks was considered via impulsive control. However, nonlinear coupling relationship among the nodes is inevitable in realistic neural networks and can impact behaviors of neural networks. Our future research topics mainly consider the impulsive control of neural networks with nonlinear coupling relationship. In addition, the application of our results on the epilepsy therapy only stays in the laboratory. So we want to apply it to clinic in the further work.

Acknowledgments. This work was partially supported by the Fundamental Research Funds for the Central Universities (grant no. N140504009), Liaoning BaiQianWan Talents Program (grant no. 2017076), the Discipline Content Education Project of Shenyang Jianzhu University (grant no. XKHY2-104), and the 9th Group of Education Scientific Research Project Topics of Shenyang Jianzhu University (grant no. 20160124).

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