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AN OPTIMIZATION LENGTH MODEL FOR RAPID TRANSIT SYSTEM

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Abstract. *Traffic congestion becomes more serious in big cities. Rapid transit system is an effective method to solve it by improving travelling efficiency for passengers. Through analyzing construction cost, operation cost and benefit of urban rapid transit, an optimal length model of rapid transit is built based on passenger flow spatial distribution model. The model, with the objective of maximum benefit and constraints of the urban rapid transit length and construction fund, can be used to optimize the length of urban rail transit (URT) and bus rapid transit (BRT) simultaneously. A numerical example is given in the paper. The results show that, with the constraints of total length (50 Km) and construction investment capital (20 billion RMB Yuan), the optimal length is 20.7 Km (URT) and 29.3 Km (BRT); and the total benefit is up to 496.342 billion RMB Yuan within 50 years. Rapid transit system not only alleviates the traffic congestion, but also creates more benefit.*

Keywords: Rapid transit system, Urban rail transit, Bus rapid transit, Operation cost, Construction cost

1. **Introduction.** Traffic congestion of big city becomes more and more serious in the world. With the traffic congestion problem, government adopts various methods to solve it. The methods include setting traffic signals, building more roads, limiting development of cars and so on. Urban mass rapid transit system can also be used to solve this problem and improve travelling efficiency. However, the construction and operation cost of this kind of traffic infrastructure is huge. With constraints of resources and investment capital, the problems of where and how long of urban rapid transit system become important issues in the process of urban transportation planning for government.

With huge construction and operation cost, the reasonable length of urban rapid transit becomes worth investigating. The studies of urban rapid transit length may generally fall into two categories: URT length optimization and BRT length optimization. The perspectives of optimal URT length include urban planning, passenger flow demand and economic benefit. Jin analyzed the reasonable length of URT from the prospective of the integration of urban planning, urban transport planning and so on [1]. From aspect of passenger flow demand characteristics, considering the impact of long and short lines on passenger flow and operational benefit, Lei and Luo analyzed how to determine the optimal length of urban rail lines [2]. From an economic point of view, Luo and Chen built the optimal length model considering investment and benefit with the maximal relative net benefit as objective, and pointed out that optimal length of URT lines is between 14.5-37 Km under normal passenger flow intensity [3]. Gong and Gu analyzed the influencing factors of URT line length and established a model with the maximal net benefit as objective [4]. Based on passenger flow demand density function, Wang and Liang [5] and Lu and Zhu [6] built an urban rail line length effectiveness function and a cost function to determine the reasonable length of rail transit lines. Contrasting to optimal length of bus line, Lin and Taylor analyzed the length of bus lines with the objective of maximum profit [7] and Liu et al. studied feeder bus network design based on effective path [8].

All above studies only optimized one kind of urban rapid transit from perspectives of economic benefit or passenger flow demand, which do not consider the URT and BRT simultaneously. Black built a mathematical model to analyze length of bus and subway lines with passenger flow demand function following the exponential distribution [9]. However, Black did not solve the problem where to build the urban rapid transit system.

In order to establish efficient, economical and rational urban rapid transit system, an optimization model is established from the aspects of construction cost, operating cost and environmental benefit, which can optimize URT and BRT system simultaneously. In the model, the passenger flow spatial distribution model is used to predict the passenger demand which can determine the urban rapid transit line direction [10]. The urban rapid transit system optimized by the model can not only satisfy traffic demand and save the investment, but also solve the problem where to build urban rapid transit line.

The paper is arranged as follows. Section 2 analyzes objective function and impact factors. Section 3 establishes the optimal model of rapid transit system which can optimize the length of URT and BRT simultaneously. Section 4 gives a numerical example to explain the optimal model. Section 5 gives the conclusions.

2. **Objective Function Analysis.** Assume that a city builds a rapid transit system that extends from the city center to city border, which consists of two parts : URT and BRT. Let L be the total length of urban rapid transit system, x be URT length, y be BRT length. We assume that BRT system uses fuel vehicles. Then if government built BRT system, environment treatment cost will be considered; if URT system will be built, the environment treatment cost will be omitted. Several factors, such as construction cost, operating cost and revenue of URT and BRT, have important impact on rapid transit planning and construction. Then we assume that objective function is composed of cost and investment revenue. The cost includes construction and operation cost of URT and BRT and the cost of air pollution treatment of BRT vehicles; the benefit function includes toll charge revenue of URT and BRT, the value of passengers' saving time and pollution treatment saving cost.

The construction length of rapid transit system and construction fund are two constraints in the optimization model. Construction length constraint includes URT length and total length of rapid transit system. Other constraints can be set up when needed. Specific contents are shown in Table 1.

TABLE 1. Model explanation

objective = benefit − cost		
objective function	benefit	The toll charge revenue of URT and BRT, URT passenger saving time value, BRT passenger saving time value, pollution treatment cost saving
	cost	The construction cost of URT and BRT, the operating cost of URT and BRT, pollution treatment cost
Constraint		The length of URT and BRT, the total length, construction cost

2.1. **Cost analysis.** The rapid transit system cost is sum of URT and BRT construction cost, URT and BRT operating cost and pollution treatment cost caused by BRT operation. It is calculated as follows.

$$C_{to} = C_1 \cdot x + C_2 \cdot x \cdot T + C_3 \cdot y + C_4 \cdot y \cdot T + C_5 \cdot y \cdot T \quad (1)$$

where C_{to} is total cost of urban rapid transit system (RMB Yuan); C_1 is construction cost factor of URT (RMB Yuan/Km); C_2 is operation cost factor of URT (RMB Yuan/(Km·Year)); C_3 is construction cost factor of BRT (RMB Yuan/Km); C_4 is operation cost factor of BRT (RMB Yuan/(Km·Year)); C_5 is pollution treatment cost factor that is pollution treatment cost per BRT bus per mile, (RMB Yuan/(Km·Year)); T is design life time of urban rapid transit system (year). We assume that BRT and URT have the same life time.

The first and third terms on the right side of Equation (1) are construction cost of URT and BRT respectively. The second and fourth terms of equation are operation cost of URT and BRT during the life time T. The fifth term is BRT air pollution treatment cost.

2.2. **Benefit analysis.** Using a faster, more comfortable travel mode, travelers will get more benefit. Since both URT and BRT run in the separate systems, its speed and travelling time can be guaranteed with reducing the air pollution. Therefore, the benefit of urban rapid transit system includes URT and BRT operating revenue, the value of passenger saving time and saved environmental cost by URT operation. Note that travelling time value by URT or BRT is smaller than that of ordinary bus. It is calculated as follows.

$$\begin{aligned} M_{to} = T \cdot \{D \cdot [&B_1 \cdot P_{urt} + B_2 \cdot P_{urt} \cdot (t_{orb} - t_{urt}) \cdot s_{urt} \\ &+ B_3 \cdot P_{brt} + B_4 \cdot P_{brt} \cdot (t_{orb} - t_{brt}) \cdot s_{brt}] + B_5 \cdot x\} \end{aligned} \quad (2)$$

where M_{to} is total benefit of urban rapid transit system (RMB Yuan); B_1 is URT toll charge (RMB Yuan/people); B_2 is URT saving time factor (RMB Yuan/h); B_3 is BRT toll charge, (RMB Yuan/people); B_4 is saving time factor of BRT, (RMB Yuan/h); B_5 is pollution treatment cost saving of URT (RMB Yuan/(Km·Year)). If government builds URT system instead of BRT system, the government will not need to deal with environment pollution by vehicle emission; P_{urt} is passenger flow of URT (people/day); P_{brt} is passenger flow of BRT (people/day); t_{urt} is average travel time of URT passenger, (h/Km); t_{brt} is average travel time of BRT passenger, (h/Km); t_{orb} is average travel time of ordinary bus passenger, (h/Km); s_{urt} is average travel distance by URT passenger, (Km/people); s_{brt} is average travel distance by BRT passenger, (Km/people); D is the number of days in a year.

The first and third terms on the right side of Equation (2) are total toll charge revenue of URT and BRT respectively. The second and fourth terms of equation are value of saved travel time of URT and BRT compared with ordinary bus during the life time T. The fifth term is URT air pollution treatment cost saving, namely if using URT, government will save air pollution treatment cost.

3. **Model.** Based on the cost and benefit analysis in Section 2, the optimal length model is established with the assumption that all of URT and BRT travellers have the same value of time.

3.1. **Passenger predictive model.** The passenger-flow spatial distribution model of URT [10] can be used to predict passenger flow. The spatial distribution model is discretized, which is as follows.

$$P_{ub} = P_0 \cdot \left\{1 - \exp[-(\alpha/d)]^{\beta}\right\} \quad (3)$$

where P_{ub} is passenger flow of every station of URT or BRT (people/day); P_0 is passenger flow of city center station (people/day); d is the distance between the station and city center (Km); α is the city scale factor (Km); β is the shape factor. In Figure 1, impact of α, β on the passenger is given. From Figure 1, we can see that given α, the smaller β is, the faster passenger flow will decrease as d increases. Given β, the smaller α is, the slower passenger flow will decrease as d increases. Note that α, β can be determined by

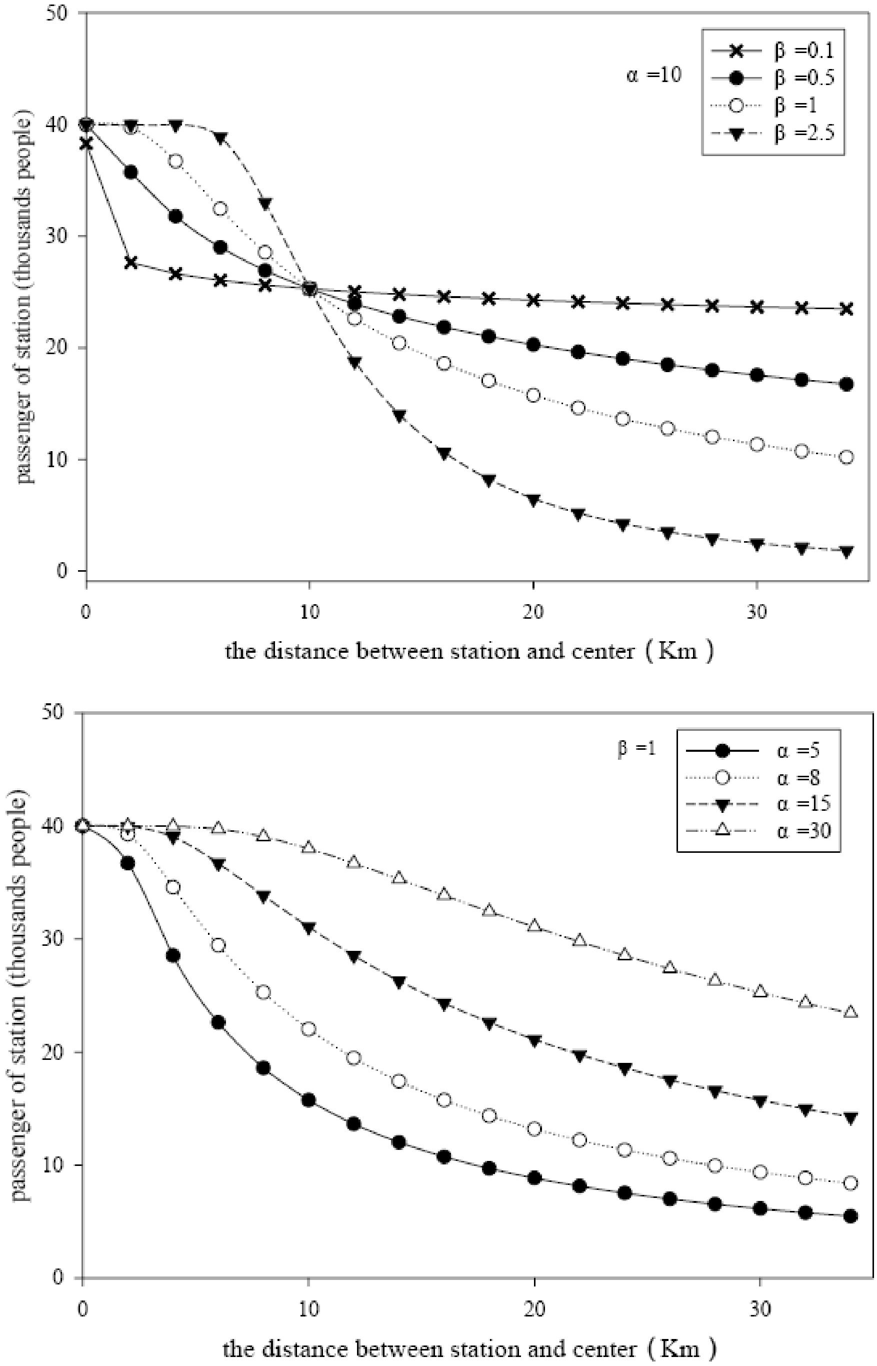

Figure 1. The impact of α, β on the passenger

traffic survey on ordinary bus networks around the urban rapid transit system to be built [10].

The total passenger flow of URT and BRT can be computed as follows.

$$P_{urt} = \sum_{0}^{m} P_0 \cdot \left\{1 - \exp[-(\alpha/d)]^{\beta}\right\} \tag{4}$$

$$P_{brt} = \sum_{m+1}^{n} P_0 \cdot \left\{1 - \exp[-(\alpha/d)]^{\beta}\right\} \tag{5}$$

where P_{urt}, P_{brt} are the passenger of each station of URT and BRT line respectively (people/day); $m = x/l_s$ is number of URT stations; $n = L/l_s$ is total number of rapid stations; l_s is the average distance between two stations (Km). Based on the above equations, optimal model can be built.

3.2. **Optimal model of rapid transit length.** Through the above analysis, passenger flow in the basic urban rail transit model was calculated by the spatial distribution model. Nonlinear optimization model was built as follows.

Objective function:

$$\max Z(x, y) = M_{to} - C_{to} \tag{6}$$

subject to

$$x + y = L \tag{7}$$

$$C_1 x + C_3 y \leq M \tag{8}$$

$$x, y \geq 0 \tag{9}$$

where L is the length of URT, (Km); M is the total investment of urban rapid transit system (billion RMB Yuan). Other notations are the same as above. L, α are given, city size parameter $\alpha \leq L \leq r$ (r is city radius which is the distance between city center and city borderline). We only use the length and construction fund as model constraints. In practice, other constraints can be added.

4. **A Numerical Example.** In order to explain above optimal model of rapid transit system, a numerical example is given.

Example 4.1. *Assume that a city builds rapid transit system to meet the traffic demand. The rapid transit system construction mode includes URT and BRT, the total length L of rapid transit system is 50 Km (less than the city radius). The largest transit program investment capital from government is 20 billion RMB Yuan. Passenger flow variation trend satisfies the spatial distribution model. Passenger flow of city center is up to 60 thousand (people/day), city scale parameter α equals 30 Km, and shape factor β equals 0.8. Other basic data are listed in Table 2.*

Table 2. Basic data of model

Basic value				URT data		BRT data	
P_{urt}	according	T	50 years	C_1	0.6 billion RMB Yuan	C_3	0.2 billion RMB Yuan
P_{bus}	to formula	l_s	0.9 Km	C_2	18 million RMB Yuan/Km	C_4	8 million RMB Yuan/Km
t_{urt}	3.303 min	S_{urt}	16.5 Km	B_1	4 RMB Yuan	C_5	6.01 million/Km
t_{brt}	4.45 min	S_{brt}	12.5 Km	B_2	16 RMB Yuan/h	B_3	3 RMB Yuan
t_{orb}	5.485 min	D	365 days	B_5	6.01 million RMB Yuan/Km	B_4	16 RMB Yuan/h

Note that: symbol is same as above.

According to the basic conditions listed in Table 2, URT and BRT initial value were set to the length of 20 Km and 30 Km respectively. With Matlab (Figure 2), based on nonlinear optimal model (6), the optimal construction length of URT and BRT is 20.7 Km and 29.3 Km respectively under the condition of a total rapid transit system construction length of 50 Km and 20 billion RMB Yuan construction fund. The operation benefit within 50 years is up to 496.342 billion RMB Yuan. Then rapid transit system will not only effectively solve traffic problems, but also save investment.

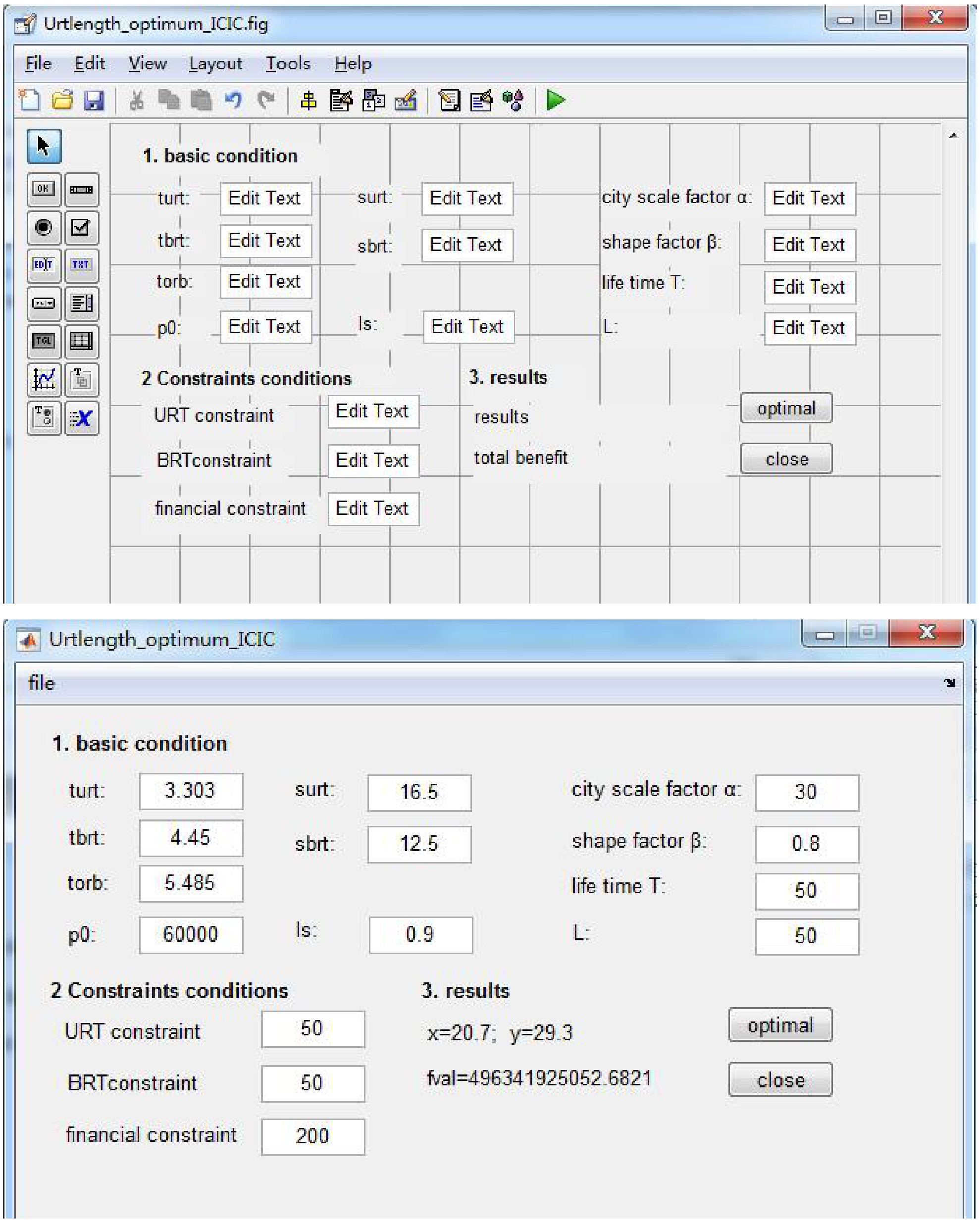

FIGURE 2. Optimal programming design

5. **Conclusions.** Several factors, such as construction cost, operating cost and revenue of URT and BRT, have important impact on urban rapid transit planning and construction. With the goal of maximum benefit, a rapid transit length optimal model is established under the constraints of a total urban rapid transit construction length and construction fund. Based on above example, a numerical example is given. The results show that, under the constraints of total length 50 Km of urban rapid transit, 20 billion RMB Yuan of construction fund, the optimal length of URT and BRT are 20.7 Km and 29.3 Km respectively, with the assumption that construction costs of URT and BRT are 0.6 billion (RMB Yuan/Km) and 0.2 billion (RMB Yuan/Km) respectively; its total maximal benefit within 50 years is up to 496.342 billion RMB Yuan. This optimal model not only optimizes the length of urban rapid system, but also guarantees the passenger flow of rapid system based on the spatial distribution model.

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