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STUDY OF POWER OUTPUT MODEL OF GRID-CONNECTED PV GENERATION SYSTEM

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Abstract. *For achieving active and reactive power decoupling in the process, this paper has established an integrated photovoltaic (PV) system static model according to three phase single stage grid-connected PV system divided into DC part and AC part. For DC part, we adopted way of derivative extreme value in the mathematical field and considered the MPPT control. Active power output model of PV power system was established on the basis of in depth study and analysis of the classical mathematical model of PV cells. For the AC part, reactive power output model of PV system was established by using state space averaging method and considering power transmission characteristics of inductance and capacitance, as well as switching characteristics of inverters. The effectiveness and feasibility of model have been verified by building classic three phase single stage grid-connected PV system and comparing output curve of the simulation model with the calculated data of the model.*
Keywords: Grid-connected PV generation system, PV model, MPPT, State space averaging method

1. **Introduction.** With the depletion of fossil energy and the aggravation of environmental pollution, it is imperative to develop and utilize new energy resources. The solar energy is widely used as an important part of new energy sources. More importantly, model of grid-connected PV generation system is the basis of power system planning, stability analysis and control [1].

The modeling method of grid-connected PV system can be usually divided into two types. One is the mechanism modeling method [2-6] by which inner mechanism based on the elements of PV system is reflected and mathematical model is established, then the overall mathematical models of PV system, as small signal model, equivalent circuit model, state space average model and detailed time domain model, etc., are integrated by using physical laws. Although this modeling method can reflect the composition and structure of PV system clearly and directly, model of PV system becomes much more complicated as general models of all devices become more complex and go against the solution of state variables and the formation of model generality. The other one is hybrid modeling method [7-9] by which external characteristics of modeling objects are kept and internal structures are simplified, that is to say, based on PV system whose core is inverter, the elements out of the inverter are simplified to equivalent circuit models as topological structure of inverter, such as switch function model, controlled source model and equivalent two-port model. This modeling method has contributed to the lack of overall coordination and is unable to reflect dynamic characteristics of every part of PV system comprehensively.

This paper proposes a mathematical model, aiming at the single-stage three-phase grid-connected PV system, based on mathematical models of each part of PV system. To obtain comprehensive mathematical expression of input and output of grid-connected PV system, system is divided into DC part and AC part and mathematical equations for each part are established. In order to decouple P-Q and obtain the comprehensive static model of PV system, output power model of PV system is established by two threads, DC part and AC part. At last, the effectiveness and feasibility of model have been verified by simulation.

2. **Division of DC and AC Part of Grid-Connected PV System.** In order to facilitate research on power flow of PV system for three-phase single-stage grid-connected PV system, model of output power shown as in Figure 1 is established through two main threads, DC part and AC part. DC part mainly includes PV arrays and MPPT that is front of inverter. Active power output of PV system can be assured at circumstance of satisfying the maximum power transmission, assuming conversion efficiency of inverter is 100%. AC part mainly includes inverter and its back end part that connects grid, which decides reactive power output of PV system.

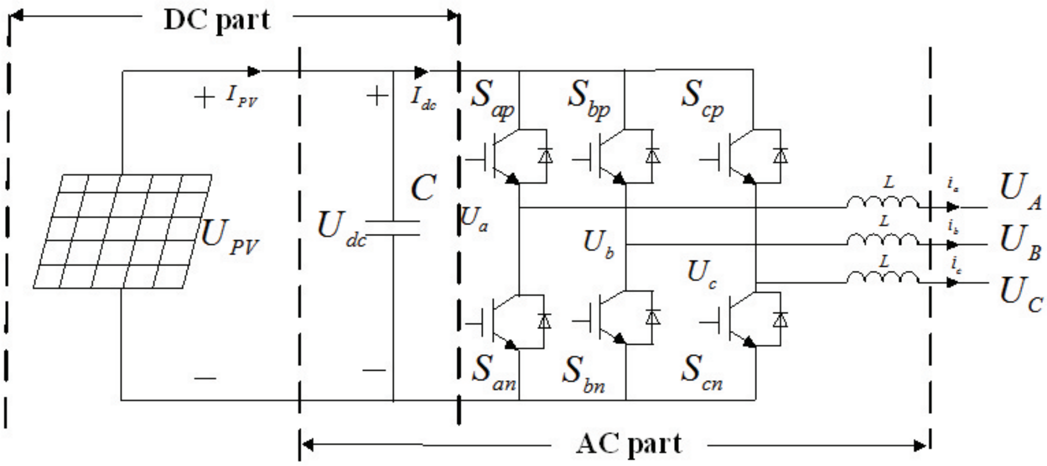

FIGURE 1. Structure of signal-stage three-phase grid-connected PV system

I_{PV} is current corresponding to port voltage of PV array. U_{PV} is port voltage of PV array. As is shown in Figure 1, S_{ip} and S_{in}, $(i = a, b, c)$ represent states of switch devices and switch function can be described as $S_i = S_{ip} = 1 - S_{in}$ $(i = a, b, c)$.

2.1. **Active power output model of DC part.** For DC part, output of active power transmitted from PV system to grid is equal to the power output by inverter under the control of MPPT strategy from PV array. Active power output model of grid-connected PV system based on active power model of PV array and the MPPT strategy of Incremental Conductance Algorithm (ICA) will be established as shown in Equation (1).

$$\begin{cases} P_{PV} = I_{PV}U_{PV} = N_P I_{SC} U_{PV} - N_P I_0 U_{PV} \left[\exp\left(\dfrac{U_{PV} + I_{PV} R_S}{n N_S U_T} \right) - 1 \right] \\ \dfrac{dP_{PV}}{dU_{PV}} = 0 \end{cases} \tag{1}$$

where I_{SC} is short current of PV cell. N_P and N_S are numbers of parallel and series PV modules. I_0 is negative saturation current when PV cell is approximately equivalent to diode. R_S is series resistance on PV cell. Ideal factor when PV cell is approximately equivalent to diode can be represented as n. U_T can be expressed as m[kT/q] whose k, q, T and m respectively represent the Boltzmann constant, electric charge, operating temperature and number of PV cells connected in series into PV modules.

2.2. **Reactive power output model of AC part.** At first, for three-phase voltage type inverter circuit, state space model of circuit based on basic law of circuit in three-phase coordinate system can be established according to its topological structure. Secondly, state space average model which utilizes average value of state variables in switching period instead of state variables in state space model is established [10]. The relation between three-phase line voltage of AC side and DC voltage is shown in Equation (2).

$$\begin{bmatrix} u_{ab} \\ u_{bc} \\ u_{ca} \end{bmatrix} = \begin{bmatrix} S_a - S_b \\ S_b - S_c \\ S_c - S_a \end{bmatrix} u_{dc} = \begin{bmatrix} S_{ab} \\ S_{bc} \\ S_{ca} \end{bmatrix} u_{dc} \tag{2}$$

AC side state equation of three-phase voltage inverter circuit is shown.

$$\begin{cases} u_{AB} = L\dfrac{di_b}{dt} - L\dfrac{di_a}{dt} + u_a - u_b \\ u_{BC} = L\dfrac{di_c}{dt} - L\dfrac{di_b}{dt} + u_b - u_c \\ u_{CA} = L\dfrac{di_a}{dt} - L\dfrac{di_c}{dt} + u_c - u_a \end{cases} \tag{3}$$

So Equation (3) can be simplified into Equation (4):

$$\begin{bmatrix} u_{AB} \\ u_{BC} \\ u_{CA} \end{bmatrix} = L\frac{d}{dt}\begin{bmatrix} i_b - i_a \\ i_c - i_b \\ i_a - i_c \end{bmatrix} + \begin{bmatrix} u_a - u_b \\ u_b - u_c \\ u_c - u_a \end{bmatrix} = -3L\frac{d}{dt}\begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{bmatrix} + \begin{bmatrix} u_{ab} \\ u_{bc} \\ u_{ca} \end{bmatrix} \tag{4}$$

According to [10], virtual line currents are defined as i_{ab}, i_{bc} and i_{ca}, which met the relation that $i_a = i_{ab} - i_{bc}$, $i_b = i_{bc} - i_{ab}$, $i_c = i_{ca} - i_{bc}$ and $i_{ab} + i_{bc} + i_{ca} = 0$. So i_{ab}, i_{bc} and i_{ca} can be represented as $i_{ab} - i_{bc}$, $i_{bc} - i_{ab}$ and $i_{ca} - i_{bc}$. The relation between three-phase current of AC side and DC current is presented as follows:

$$i_{dc} = [S_a \quad S_b \quad S_c]\ [i_a \quad i_b \quad i_c]^T = [S_{ab} \quad S_{bc} \quad S_{ca}]\ [i_{ab} \quad i_{bc} \quad i_{ca}]^T \tag{5}$$

State equation of DC side of three-phase voltage type inverter circuit is as follows:

$$i_{dc} = -C\frac{dU_{dc}}{dt} + i_{PV} \tag{6}$$

Summed up Equations (4) and (6), so we can obtain Equation (7).

$$\begin{cases} \dfrac{d\mathbf{i}_{l-l}}{dt} = -\dfrac{1}{3L}\mathbf{U}_{L-L} + \dfrac{1}{3L}\mathbf{U}_{l-l} \\ \dfrac{dU_{dc}}{dt} = -\dfrac{1}{C}i_{dc} + \dfrac{i_{PV}}{C} \end{cases} \tag{7}$$

In order to simplify calculation, it is supposed that $\mathbf{U}_{L-L}$ is $[\ u_{AB} \quad u_{BC} \quad u_{CA}\ ]^T$, $\mathbf{U}_{l-l}$ is $[\ u_{ab} \quad u_{bc} \quad u_{ca}\ ]^T$, $\mathbf{i}_{l-l}$ is $[\ i_{ab} \quad i_{bc} \quad i_{ca}\ ]^T$, $\mathbf{s}_{l-l}$ is $[\ S_{ab} \quad S_{bc} \quad S_{ca}\ ]^T$.

According to switch characteristics of DC/AC inverter, the relation between switch function and value of voltage and current of AC and DC circuit of inverter is shown.

$$\begin{cases} \mathbf{U}_{l-l} = \mathbf{s}_{l-l}U_{dc} \\ i_{dc} = \mathbf{s}_{l-l}^T\mathbf{i}_{l-l} \end{cases} \tag{8}$$

Equations (6), (7) and (8) can be combined as Equation (9):

$$\begin{cases} \dfrac{d\mathbf{i}_{l-l}}{dt} = -\dfrac{1}{3L}\mathbf{U}_{L\text{-}L} + \dfrac{1}{3L}\mathbf{s}_{l-l}U_{dc} \\ \dfrac{dU_{dc}}{dt} = -\dfrac{1}{C}\mathbf{s}_{l-l}^{T}\mathbf{i}_{l-l} + \dfrac{i_{PV}}{C} \end{cases} \tag{9}$$

In the equation above, $\mathbf{s}_{l-l}$ is discontinuous, so there is a discontinuity in Equation (9). For state equation, state average need to be calculated in a switch period:

$$\begin{cases} \dfrac{d\left\langle \mathbf{i}_{l-l}\right\rangle_{T_S}}{dt} = -\dfrac{1}{3L}\left\langle \mathbf{U}_{L-L}\right\rangle_{T_S} + \dfrac{1}{3L}\left\langle \mathbf{s}_{l-l}U_{dc}\right\rangle_{T_S} \\ \dfrac{d\left\langle U_{dc}\right\rangle_{T_S}}{dt} = -\dfrac{1}{C}\left\langle \mathbf{s}_{l\text{-}l}^{T}\mathbf{i}_{l-l}\right\rangle_{T_S} + \dfrac{\left\langle i_{PV}\right\rangle_{T_S}}{C} \end{cases} \tag{10}$$

State variables of inductor current $\mathbf{i}_{l-l}$ are continuous with small variation after average value is obtained in a switch period. State variables of capacitance voltage $\mathbf{U}_{L-L}$ is continuous with small variation in a switch period. So the approximate relation can be got:

$$\begin{aligned} \left\langle \mathbf{s}_{l-l}^{T}\mathbf{i}_{l-l}\right\rangle_{T_S} &= \left\langle \mathbf{s}_{l-l}\right\rangle_{T_S}\left\langle \mathbf{i}_{l-l}\right\rangle_{T_S} = \mathbf{D}_{l-l}\left\langle \mathbf{i}_{l-l}\right\rangle_{T_S} \\ \left\langle \mathbf{s}_{l-l}U_{dc}\right\rangle_{T_S} &= \left\langle \mathbf{s}_{l-l}\right\rangle_{T_S}\left\langle U_{dc}\right\rangle_{T_S} = \mathbf{D}_{l-l}\left\langle U_{dc}\right\rangle_{T_S} \end{aligned} \tag{11}$$

Duty ratio between line to line is $\mathbf{D}_{l-l} = [D_{ab}\ \ D_{bc}\ \ D_{ca}]^T$

$$\begin{cases} D_{ab} = \left\langle s_{ab}(t)\right\rangle_{T_S} = \dfrac{1}{T_S}\displaystyle\int_{t}^{t+T_S} s_{ab}(\tau)d\tau = D_a - D_b \\ D_{bc} = \left\langle s_{bc}(t)\right\rangle_{T_S} = \dfrac{1}{T_S}\displaystyle\int_{t}^{t+T_S} s_{bc}(\tau)d\tau = D_b - D_c \\ D_{ca} = \left\langle s_{ca}(t)\right\rangle_{T_S} = \dfrac{1}{T_S}\displaystyle\int_{t}^{t+T_S} s_{ca}(\tau)d\tau = D_c - D_a \end{cases} \tag{12}$$

According to equations above, average model of three-phase voltage type DC/AC inverter in the switch period is obtained.

$$\begin{cases} \dfrac{d\left\langle \mathbf{i}_{l-l}\right\rangle_{T_S}}{dt} = -\dfrac{1}{3L}\left\langle \mathbf{U}_{L-L}\right\rangle_{T_S} + \dfrac{1}{3L}\mathbf{D}_{l-l}\left\langle U_{dc}\right\rangle_{T_S} \\ \dfrac{d\left\langle U_{dc}\right\rangle_{T_S}}{dt} = -\dfrac{1}{C}\mathbf{D}_{l-l}^{T}\left\langle \mathbf{i}_{l-l}\right\rangle_{T_S} + \dfrac{\left\langle i_{PV}\right\rangle_{T_S}}{C} \end{cases} \tag{13}$$

The model obtained is of four order system. If the input is DC and duty ratio function is of a three-phase symmetrical sine wave, output of voltage and current are sine waves in steady state. So model can be described as equation as follows under ideal conditions.

$$\begin{cases} \dfrac{d\mathbf{i}_{l-l}}{dt} = -\dfrac{1}{3L}\mathbf{U}_{L-L} + \dfrac{1}{3L}\mathbf{D}_{l-l}U_{dc} \\ \dfrac{dU_{dc}}{dt} = -\dfrac{1}{C}\mathbf{D}_{l-l}^{T}\mathbf{i}_{l-l} + \dfrac{I_{PV}}{C} \end{cases} \tag{14}$$

Function form of $\mathbf{U}_{L-L}$ is trigonometric and U_{dc} is constant. The elements in duty ratio function are corresponding to effective value of elements in switch function $\mathbf{s}_{l-l}$.

$$\mathbf{s}_{l-l} = \frac{\mathbf{U}_{l-l}}{U_{dc}} \tag{15}$$

Reactive power injected to the grid is as follows:

$$\begin{aligned} Q &= U_{dc}\left(-\mathbf{D}_{l-l}{}^{T} I_{l-l} + I_{pv}\right) - \frac{1}{3}\left(U_{dc}\mathbf{D}_{l-l}{}^{T} - \mathbf{U}_{L-L}{}^{T}\right) I_{l-l} \\ &= \left(\frac{1}{3}\mathbf{U}_{L-L}{}^{T} - \frac{4}{3}U_{dc}\mathbf{D}_{l-l}{}^{T}\right) I_{l-l} + U_{dc}I_{pv} \end{aligned} \tag{16}$$

According to practical situation of operation, in order to ensure stable operation of PV system, solution in the model should satisfy some constraints are as follows. Value of the operating limit of PV system is expressed by constraints below. The physical characteristics of PV system are as follows: $0 \leq P \leq P_{\max}$; $0 \leq Q \leq Q_{\max}$; $D_{l\min} \leq D_l \leq D_{l\max}$.

(1) Natural conditions, such as intensity of sunshine, temperature of environment and time and space distribution, will limit power output and efficiency of PV system.
(2) Temperature rise of inverter switching tube. Temperature rise of switching tube of inverter is mainly determined by current and switching frequency of switching tube. Therefore, switching tube current and switching frequency will be limited.
(3) Maximum operating voltage of switch tube is limited. When operating voltage at both ends of switch tube exceeds a certain limit, it will break down.
(4) Limit of inverter's own rated capacity. Output of active and reactive power of PV system cannot exceed rated capacity of inverter.
(5) Other constraints, such as limit of capacitor voltage level at the front-end of inverter, temperature rise constraint of back-end of inverter and the constraint of PV array temperature rise.

Table 1. Photovoltaic cells output power under different light intensity value

Light intensity (W/m^2)	1000	800	600
P (W)	5398	4344.6	3126.825
Q (var)	1271	1708	1654

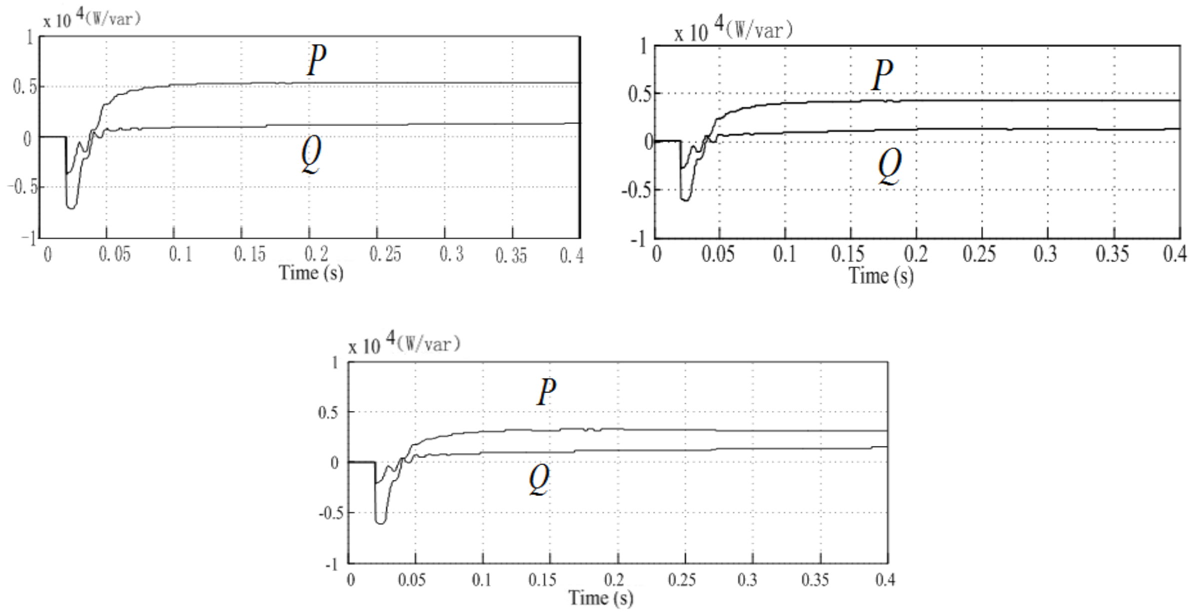

Figure 2. Output power curves under light intensity of 1000 W/m^2, 800 W/m^2 and 600 W/m^2

3. **Simulation of Active and Reactive Power Output of Grid-Connected PV System.** It is assumed that operating voltage of system is 220V, working frequency is 50Hz, external temperature is 25°C, light intensity is 1000W/m^2, 800W/m^2 and 600W/m^2. Firstly, mathematical model of PV system is used to calculate output power of different light according to conditions mentioned above, see Table 1.

Using MATLAB to simulate power output curve under corresponding light conditionsas is shown in Figure 2. From Figure 2, when light intensity is input to a given system, PV system output will eventually become stable after going through a series of fluctuations.

From comparison of Table 1 and Figure 2, model can correctly reflect actual output power (active power, reactive power) of PV system.

4. **Conclusions.** This paper presents a new modeling thinking of PV system, and a method of establishing a comprehensive model of simple PV system. According to proposed method, system circuit is divided as part of DC and AC with PQ decoupling. By comparing output curve of simulation model and model data, effectiveness and feasibility of models are validated.

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