## OPTIMIZATION MODELS APPLIED TO CROPS IN THE AGRICULTURAL PRODUCTION OF MÉXICO

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ABSTRACT. This paper shows two optimization models applied to crops in the agricultural production, which are: 1) to maximize the utility; 2) to minimize the cost. Constraint functions for the two models are formulated: constraint on land availability; constraint on internal consumer demand; constraint on external consumer demand. Numerical examples are presented to validate the two models: 1) to obtain the maximum utility; 2) to find the minimum cost considering the constraint equal to or less than the land availability, and taking account of the constraint equal to the land availability. The results show that optimization techniques will significantly improve the net utility with optimal crop areas allocation, plus the minimum cost for internal consumption and also the minimum cost using the available surface providing the optimal crop areas is presented. Keywords: Optimization models applied to crops, Agricultural production, Maximum utility, Minimum cost

1. Introduction. Successful management in modern conditions of economy requires among other things maximal adjustment of the business policy of an agricultural enterprise to the demands of the general business conditions.

Crop area planning is essential for agricultural production systems management and in agricultural systems one of major challenges is crop for selection. Given a farmland, a water resource and a list of crops, the objective is to determine the optimal (or near optimal) cropping patterns [1-3].

The methodology of weighted goal programming has been successfully implemented on real agricultural systems. By example, the most important papers applying optimization techniques are: "An MCDM approach to production analysis: An application to irrigated farms in southern Spain" [4], "Irrigation water pricing: Differential impacts on irrigated farms" [5], "The regional impact of irrigation water pricing in Greece under alternative scenarios of European policy: A multicriteria analysis" [6], "Fertilizer price policy, the environment and farms behavior" [7], "Evaluation of tobacco cultivation alternatives under the EU common agricultural policy (CAP)" [8], "A multicriteria model for planning agricultural regions within a context of groundwater rational management" [9], "Optimization of irrigation water utilization for agricultural production" [10], "Optimization model of agricultural production system in grain farms under risk, in Sorriso, Brazil" [11], "Organophosphated fertilizers production in humifert process" [12], "Optimization of chelates production process for agricultural administration of inorganic micronutrients" [13], "Agricultural production structure optimization: A case study of major grain producing areas, China" [14].

This paper shows two optimization models applied to crops in the agricultural production, which are: 1) to maximize the utility; 2) to minimize the cost. Constraint functions for the two models are formulated: constraint on land availability; constraint on internal consumer demand; constraint on external consumer demand. Numerical examples to validate the two models are presented: Model 1 the maximum utility is obtained; Model 2 the minimum cost is found. Model 1 considers the constraint equal to or less than the land availability, and Model 2 takes account of two cases: 1) the constraint is equal to or less than the land availability; 2) the constraint is equal to the land availability.

The paper is organized as follows. Methodology (Section 2) describes the formulation of the models to obtain the maximum utility and the minimum cost of the crops in the agricultural production. In Section 3, numerical examples are presented to validate the new model to obtain the areas of each one of the crops in order to attain the maximum utility and/or the minimum cost. Results and discussion are presented in Section 4. Conclusions (Section 5) complete the paper.

2. Methodology. Crops production planning is modeled with constraints for solving optimization based on genetic algorithm.

2.1. **Objective functions.** The goal is to maximize the utility, and minimize the cost from the crops production.

2.1.1. Objective function to maximize the utility. The total revenue " $R_t$ " on all the crops is obtained:

$$R_t = \sum_{i=1}^n S_i X_i P_i \tag{1}$$

where  $S_i$  is the extent of surface to be cultivated for the *i*-th crop, where i = 1, 2, 3, ..., n (hectares);  $X_i$  is the yield in *ton* for the *i*-th crop, per unit surface (per hectare);  $P_i$  is the price of the product in dollars for the *i*-th crop, per *ton*.

The total investment cost " $C_t$ " for all crops for the entire extent surface is found:

$$C_t = \sum_{i=1}^n S_i X_i C_i \tag{2}$$

where  $C_i$  is the investment cost in dollars per unit (per ton) of the *i*-th crop.

Now, the net utility (net revenue) " $U_n$ " is obtained:

$$U_n = R_t - C_t \tag{3}$$

Substituting Equations (1) and (2) into Equation (3) obtains:

$$U_n = \sum_{i=1}^n S_i X_i P_i - \sum_{i=1}^n S_i X_i C_i = \sum_{i=1}^n S_i X_i (P_i - C_i)$$
(4)

2.1.2. Objective function to minimize the cost. The total investment cost is:

$$C_t = \sum_{i=1}^n S_i X_i C_i \tag{5}$$

and  $C_i$  is found by the equation as follows:

$$C_i = CP_i + CS_i + CF_i + CH_i + CIW_i + CICPD_i + CH_i + CM_i$$
(6)

where  $CP_i$  is the cost on preparation per unit of land (per hectare) for the *i*-th crop;  $CS_i$  is the cost on seeds per unit of land (per hectare) for the *i*-th crop;  $CF_i$  is the cost on fertilizers per unit of land (per hectare) for the *i*-th crop;  $CH_i$  is the cost on human power per unit of land (per hectare) for the *i*-th crop;  $CIW_i$  is the cost on irrigation water per unit of land (per hectare) for the *i*-th crop;  $CIPD_i$  is the cost on integral control of pests and diseases per unit of land (per hectare) for the *i*-th crop;  $CH_i$  is the cost on harvest per unit of land (per hectare) for the *i*-th crop;  $CM_i$  is the cost on miscellaneous per unit of land (per hectare) for the *i*-th crop;  $CM_i$  is the cost on miscellaneous per unit of land (per hectare) for the *i*-th crop;  $CM_i$  is the cost on miscellaneous per unit of land (per hectare) for the *i*-th crop;  $CM_i$  is the cost on miscellaneous per unit of land (per hectare) for the *i*-th crop;  $CM_i$  is the cost on miscellaneous per unit of land (per hectare) for the *i*-th crop;  $CM_i$  is the cost on miscellaneous per unit of land (per hectare) for the *i*-th crop;  $CM_i$  is the cost on miscellaneous per unit of land (per hectare) for the *i*-th crop;  $CM_i$  is the cost on miscellaneous per unit of land (per hectare) for the *i*-th crop.

### 2.2. Constraint functions.

2.2.1. Constraint on land availability.

$$\sum_{i=1}^{n} S_i \le S_t \tag{7}$$

where  $S_t$  is available total surface.

2.2.2. Constraint on internal consumer demand.

$$X_i S_i \ge D_i \tag{8}$$

where  $D_i$  is internal consumer demand for the *i*-th crop.

2.2.3. Constraint on external consumer demand.

$$X_i S_i \le E_i \tag{9}$$

where  $E_i$  is external consumer demand for the *i*-th crop.

3. Numerical Examples. Crops company has 150 hectares which produce six items: green forage maize, green forage safflower, width dried chili, alluvium bean, ear of corn, and potato. The company's products are for consumption of its members and foreign sales. The company is organized in such a way that you must first meet the demands of its members before making foreign sales. Any production surplus is sold at market price. Table 1 presents for each product during growing season, the information following: the projected yield (tons/hectare), investment cost (dollars/hectare), investment cost (dollars/hectare), investment cost (dollars/hectare), investment cost (tons), and the price of the product (dollars/ton).

Objective function to maximize the utility " $U_{\text{max}}$ " by Equation (4) is obtained:

$$U_{\max} = S_1 X_1 (P_1 - C_1) + S_2 X_2 (P_2 - C_2) + S_3 X_3 (P_3 - C_3) + S_4 X_4 (P_4 - C_4) + S_5 X_5 (P_5 - C_5) + S_6 X_6 (P_6 - C_6)$$

where  $S_1$ ,  $X_1$ ,  $P_1$  and  $C_1$  are the green forage maize,  $S_2$ ,  $X_2$ ,  $P_2$  and  $C_2$  are the green forage safflower,  $S_3$ ,  $X_3$ ,  $P_3$  and  $C_3$  are the width dried chili,  $S_4$ ,  $X_4$ ,  $P_4$  and  $C_4$  are the alluvium bean,  $S_5$ ,  $X_5$ ,  $P_5$  and  $C_5$  are the ear of corn, and  $S_6$ ,  $X_6$ ,  $P_6$  and  $C_6$  are the potato.

Substituting the values of Table 1 provided by the Department of Mexico Agriculture finds:

$$U_n = 281.05S_1 + 181.00S_2 + 2554.00S_3 + 225.70S_4 + 662.69S_5 + 1427.40S_6$$

Objective function to minimize the cost " $C_{\min}$ " by Equation (5) is obtained:

$$C_{\min} = S_1 X_1 C_1 + S_2 X_2 C_2 + S_3 X_3 C_3 + S_4 X_4 C_4 + S_5 X_5 C_5 + S_6 X_6 C_6$$

Now, substituting the values of Table 1 provided by the Department of Mexico Agriculture finds:

$$C_{\min} = 873.95S_1 + 699S_2 + 2650.99S_3 + 519.00S_4 + 874.01S_5 + 4445.00S_6$$

Constraint on land availability by Equation (7) is obtained:

$$S_1 + S_2 + S_3 + S_4 + S_5 + S_6 \le 150$$

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Crop	Yield	Investment	Investment	Quantity	Maximum	Price of
	X.	cost	cost C	requested	demand of	the product
	(tong/hostore)	(dollars/hectare)	(dollarg/top)	by members	the market	$P_i$
	(tons/nectare)		(donars/ton)	(tons)	(tons)	(dollars/ton)
Green						
forage	$35.00^{*}$	874.00*	24.97	100	1000	$33.00^{*}$
maize						
Green						
forage	20.00*	699.00*	34.95	70	700	$44.00^{*}$
safflower						
Width						
dried	$1.50^{*}$	2651.00*	1767.33	3	50	$3470.00^{*}$
chili						
Alluvium	1 10*	510.00*	471.99	20	200	677.00*
bean	1.10	519.00	471.02	20	300	077.00*
Ear of	19.70*	874.00*	60 00	20	500	191.00*
corn	12.70	014.00	00.02	50	500	121.00
Potato	21.20*	4445.00*	209.67	40	700	277.00*

TABLE 1. Information about each crop that must be produced by the company

\*Data provided by the Department of Mexico Agriculture

Constraint on internal consumer demand by Equation (8) is found:

$$\begin{array}{l} 35S_1 \geq 100 \\ 20S_2 \geq 70 \\ 1.5S_3 \geq 3 \\ 1.1S_4 \geq 20 \\ 12.7S_5 \geq 30 \\ 21.2S_6 \geq 40 \end{array}$$

Constraint on external consumer demand by Equation (9) is obtained:

$$\begin{array}{l} 35S_1 \leq 1000 \\ 20S_2 \leq 700 \\ 1.5S_3 \leq 50 \\ 1.1S_4 \leq 300 \\ 12.7S_5 \leq 500 \\ 21.2S_6 \leq 700 \end{array}$$

Table 2 shows the results obtained by the MAPLE-15 software.

TABLE 2. Results obtained for the crop	ps produced by the company
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Crop	Conditions of land area cultivated	Objective function (dollars)	Green forage maize (hectare)	Green forage safflower (hectare)	Width dried chili (hectare)	Alluvium bean (hectare)	Ear of corn (hectare)	Potato (hectare)
To maximize	Land surface	169442.34	22.596	3.500	33.333	18.182	39.370	33.019
the utility	$\leq 150$							
To minimize the cost	$\begin{array}{c} \text{Land} \\ \text{surface} \\ \leq 150 \end{array}$	30133.23	2.857	3.500	2.00	18.182	2.362	1.887
	$ \begin{array}{r} \text{Land}\\ \text{surface}\\ = 150 \end{array} $	92004.28	2.857	3.500	2.00	137.394	2.362	1.887

4. **Results and Discussion.** Data provided by the Department of Mexico Agriculture shows the current crops pattern in research zone (Table 1).

Observing the results, the allocated land for objective function (maximize the utility): the width dried chili, the ear of corn and the potato are cultivated up to the maximum level (maximum demand of the market); the green forage safflower and the alluvium bean are cultivated up to the minimum level (internal consumer demand). For objective function (minimize the cost) for land surface  $\leq 150$ : all the crops are cultivated for the minimum level (internal consumer demand). For objective function surface = 150: also all the crops are cultivated for the minimum level (internal consumer demand) excluding the alluvium bean that is grown up to 137.394 hectare whose internal consumer demand is of 18.182 hectare.

5. **Conclusions.** This paper presents two models to obtain the maximum utility and the minimum cost for planning and the optimization of the crops in the agricultural production. The model is used in order to achieve better procedures, better market policy and the simulation of the most realistic decision process.

Real examples to obtain the maximum utility and the minimum cost have been presented to demonstrate the efficiency of the optimization techniques.

The proposed models can be further used to evaluate different products, industrial type and different regions in agriculture.

The suggestions for future research may be, if the constraint functions consider the following: the cost on preparation per unit of land (per hectare) for each crop; the cost and quantity on seeds per unit of land (per hectare) for each crop; the cost and quantity on fertilizers per unit of land (per hectare) for each crop; the cost and quantity on human power per unit of land (per hectare) for each crop; the cost and quantity on irrigation water per unit of land (per hectare) for each crop; the cost on integral control of pests and diseases per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop;

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