# OPTIMIZATION MODELS APPLIED TO CROPS IN THE AGRICULTURAL PRODUCTION OF MÉXICO 

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Received October 2016; accepted January 2017


#### Abstract

This paper shows two optimization models applied to crops in the agricultural production, which are: 1) to maximize the utility; 2) to minimize the cost. Constraint functions for the two models are formulated: constraint on land availability; constraint on internal consumer demand; constraint on external consumer demand. Numerical examples are presented to validate the two models: 1) to obtain the maximum utility; 2) to find the minimum cost considering the constraint equal to or less than the land availability, and taking account of the constraint equal to the land availability. The results show that optimization techniques will significantly improve the net utility with optimal crop areas allocation, plus the minimum cost for internal consumption and also the minimum cost using the available surface providing the optimal crop areas is presented.


Keywords: Optimization models applied to crops, Agricultural production, Maximum utility, Minimum cost

1. Introduction. Successful management in modern conditions of economy requires among other things maximal adjustment of the business policy of an agricultural enterprise to the demands of the general business conditions.

Crop area planning is essential for agricultural production systems management and in agricultural systems one of major challenges is crop for selection. Given a farmland, a water resource and a list of crops, the objective is to determine the optimal (or near optimal) cropping patterns [1-3].

The methodology of weighted goal programming has been successfully implemented on real agricultural systems. By example, the most important papers applying optimization techniques are: "An MCDM approach to production analysis: An application to irrigated farms in southern Spain" [4], "Irrigation water pricing: Differential impacts on irrigated farms" [5], "The regional impact of irrigation water pricing in Greece under alternative scenarios of European policy: A multicriteria analysis" [6], "Fertilizer price policy, the environment and farms behavior" [7], "Evaluation of tobacco cultivation alternatives under the EU common agricultural policy (CAP)" [8], "A multicriteria model for planning agricultural regions within a context of groundwater rational management" [9], "Optimization of irrigation water utilization for agricultural production" [10], "Optimization model of agricultural production system in grain farms under risk, in Sorriso, Brazil" [11], "Organophosphated fertilizers production in humifert process" [12], "Optimization of chelates production process for agricultural administration of inorganic micronutrients" [13], "Agricultural production structure optimization: A case study of major grain producing areas, China" [14].

This paper shows two optimization models applied to crops in the agricultural production, which are: 1) to maximize the utility; 2) to minimize the cost. Constraint functions
for the two models are formulated: constraint on land availability; constraint on internal consumer demand; constraint on external consumer demand. Numerical examples to validate the two models are presented: Model 1 the maximum utility is obtained; Model 2 the minimum cost is found. Model 1 considers the constraint equal to or less than the land availability, and Model 2 takes account of two cases: 1) the constraint is equal to or less than the land availability; 2) the constraint is equal to the land availability.

The paper is organized as follows. Methodology (Section 2) describes the formulation of the models to obtain the maximum utility and the minimum cost of the crops in the agricultural production. In Section 3, numerical examples are presented to validate the new model to obtain the areas of each one of the crops in order to attain the maximum utility and/or the minimum cost. Results and discussion are presented in Section 4. Conclusions (Section 5) complete the paper.
2. Methodology. Crops production planning is modeled with constraints for solving optimization based on genetic algorithm.
2.1. Objective functions. The goal is to maximize the utility, and minimize the cost from the crops production.
2.1.1. Objective function to maximize the utility. The total revenue " $R_{t}$ " on all the crops is obtained:

$$
\begin{equation*}
R_{t}=\sum_{i=1}^{n} S_{i} X_{i} P_{i} \tag{1}
\end{equation*}
$$

where $S_{i}$ is the extent of surface to be cultivated for the $i$-th crop, where $i=1,2,3, \ldots, n$ (hectares); $X_{i}$ is the yield in ton for the $i$-th crop, per unit surface (per hectare); $P_{i}$ is the price of the product in dollars for the $i$-th crop, per ton.

The total investment cost " $C_{t}$ " for all crops for the entire extent surface is found:

$$
\begin{equation*}
C_{t}=\sum_{i=1}^{n} S_{i} X_{i} C_{i} \tag{2}
\end{equation*}
$$

where $C_{i}$ is the investment cost in dollars per unit (per ton) of the $i$-th crop.
Now, the net utility (net revenue) " $U_{n}$ " is obtained:

$$
\begin{equation*}
U_{n}=R_{t}-C_{t} \tag{3}
\end{equation*}
$$

Substituting Equations (1) and (2) into Equation (3) obtains:

$$
\begin{equation*}
U_{n}=\sum_{i=1}^{n} S_{i} X_{i} P_{i}-\sum_{i=1}^{n} S_{i} X_{i} C_{i}=\sum_{i=1}^{n} S_{i} X_{i}\left(P_{i}-C_{i}\right) \tag{4}
\end{equation*}
$$

2.1.2. Objective function to minimize the cost. The total investment cost is:

$$
\begin{equation*}
C_{t}=\sum_{i=1}^{n} S_{i} X_{i} C_{i} \tag{5}
\end{equation*}
$$

and $C_{i}$ is found by the equation as follows:

$$
\begin{equation*}
C_{i}=C P_{i}+C S_{i}+C F_{i}+C H_{i}+C I W_{i}+C I C P D_{i}+C H_{i}+C M_{i} \tag{6}
\end{equation*}
$$

where $C P_{i}$ is the cost on preparation per unit of land (per hectare) for the $i$-th crop; $C S_{i}$ is the cost on seeds per unit of land (per hectare) for the $i$-th crop; $C F_{i}$ is the cost on fertilizers per unit of land (per hectare) for the $i$-th crop; $C H_{i}$ is the cost on human power per unit of land (per hectare) for the $i$-th crop; $C I W_{i}$ is the cost on irrigation water per unit of land (per hectare) for the $i$-th crop; CICPD $_{i}$ is the cost on integral control of pests and diseases per unit of land (per hectare) for the $i$-th crop; $C H_{i}$ is the cost on harvest per unit of land (per hectare) for the $i$-th crop; $C M_{i}$ is the cost on miscellaneous per unit of land (per hectare) for the $i$-th crop.

### 2.2. Constraint functions.

2.2.1. Constraint on land availability.

$$
\begin{equation*}
\sum_{i=1}^{n} S_{i} \leq S_{t} \tag{7}
\end{equation*}
$$

where $S_{t}$ is available total surface.
2.2.2. Constraint on internal consumer demand.

$$
\begin{equation*}
X_{i} S_{i} \geq D_{i} \tag{8}
\end{equation*}
$$

where $D_{i}$ is internal consumer demand for the $i$-th crop.
2.2.3. Constraint on external consumer demand.

$$
\begin{equation*}
X_{i} S_{i} \leq E_{i} \tag{9}
\end{equation*}
$$

where $E_{i}$ is external consumer demand for the $i$-th crop.
3. Numerical Examples. Crops company has 150 hectares which produce six items: green forage maize, green forage safflower, width dried chili, alluvium bean, ear of corn, and potato. The company's products are for consumption of its members and foreign sales. The company is organized in such a way that you must first meet the demands of its members before making foreign sales. Any production surplus is sold at market price. Table 1 presents for each product during growing season, the information following: the projected yield (tons/hectare), investment cost (dollars/hectare), investment cost (dollars/ton), quantity requested by members (tons), maximum demand of the market (tons), and the price of the product (dollars/ton).

Objective function to maximize the utility " $U_{\max }$ " by Equation (4) is obtained:

$$
\begin{aligned}
U_{\max }= & S_{1} X_{1}\left(P_{1}-C_{1}\right)+S_{2} X_{2}\left(P_{2}-C_{2}\right)+S_{3} X_{3}\left(P_{3}-C_{3}\right)+S_{4} X_{4}\left(P_{4}-C_{4}\right) \\
& +S_{5} X_{5}\left(P_{5}-C_{5}\right)+S_{6} X_{6}\left(P_{6}-C_{6}\right)
\end{aligned}
$$

where $S_{1}, X_{1}, P_{1}$ and $C_{1}$ are the green forage maize, $S_{2}, X_{2}, P_{2}$ and $C_{2}$ are the green forage safflower, $S_{3}, X_{3}, P_{3}$ and $C_{3}$ are the width dried chili, $S_{4}, X_{4}, P_{4}$ and $C_{4}$ are the alluvium bean, $S_{5}, X_{5}, P_{5}$ and $C_{5}$ are the ear of corn, and $S_{6}, X_{6}, P_{6}$ and $C_{6}$ are the potato.

Substituting the values of Table 1 provided by the Department of Mexico Agriculture finds:

$$
U_{n}=281.05 S_{1}+181.00 S_{2}+2554.00 S_{3}+225.70 S_{4}+662.69 S_{5}+1427.40 S_{6}
$$

Objective function to minimize the cost " $C_{\text {min }}$ " by Equation (5) is obtained:

$$
C_{\min }=S_{1} X_{1} C_{1}+S_{2} X_{2} C_{2}+S_{3} X_{3} C_{3}+S_{4} X_{4} C_{4}+S_{5} X_{5} C_{5}+S_{6} X_{6} C_{6}
$$

Now, substituting the values of Table 1 provided by the Department of Mexico Agriculture finds:

$$
C_{\min }=873.95 S_{1}+699 S_{2}+2650.99 S_{3}+519.00 S_{4}+874.01 S_{5}+4445.00 S_{6}
$$

Constraint on land availability by Equation (7) is obtained:

$$
S_{1}+S_{2}+S_{3}+S_{4}+S_{5}+S_{6} \leq 150
$$

TABLE 1. Information about each crop that must be produced by the company

| Crop | Yield <br> $X_{i}$ <br> (tons/hectare) | Investment <br> (dollars/hectare) | Investment <br> cost $C_{i}$ <br> dollars/ton) | Quantity <br> requested <br> by <br> members <br> (tons) | Maximum market <br> demand of <br> (tons) | Price of <br> the product <br> $P_{i}$ <br> (dollars/ton) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Green <br> forage <br> maize | $35.00^{*}$ | $874.00^{*}$ | 24.97 | 100 | 1000 | $33.00^{*}$ |
| Green <br> forage <br> safflower | $20.00^{*}$ | $699.00^{*}$ | 34.95 | 70 | 700 | $44.00^{*}$ |
| Width <br> dried <br> chili | $1.50^{*}$ | $2651.00^{*}$ | 1767.33 | 3 | 50 | $3470.00^{*}$ |
| Alluvium <br> bean | $1.10^{*}$ | $519.00^{*}$ | 471.82 | 20 | 300 | $677.00^{*}$ |
| Ear of <br> corn | $12.70^{*}$ | $874.00^{*}$ | 68.82 | 30 | 500 | $121.00^{*}$ |
| Potato | $21.20^{*}$ | $4445.00^{*}$ | 209.67 | 40 | 700 | $277.00^{*}$ |

*Data provided by the Department of Mexico Agriculture

Constraint on internal consumer demand by Equation (8) is found:

$$
\begin{aligned}
& 35 S_{1} \geq 100 \\
& 20 S_{2} \geq 70 \\
& 1.5 S_{3} \geq 3 \\
& 1.1 S_{4} \geq 20 \\
& 12.7 S_{5} \geq 30 \\
& 21.2 S_{6} \geq 40
\end{aligned}
$$

Constraint on external consumer demand by Equation (9) is obtained:

$$
\begin{aligned}
& 35 S_{1} \leq 1000 \\
& 20 S_{2} \leq 700 \\
& 1.5 S_{3} \leq 50 \\
& 1.1 S_{4} \leq 300 \\
& 12.7 S_{5} \leq 500 \\
& 21.2 S_{6} \leq 700
\end{aligned}
$$

Table 2 shows the results obtained by the MAPLE-15 software.
TABLE 2. Results obtained for the crops produced by the company

| Crop | Conditions <br> of land <br> area <br> cultivated | Objective <br> function <br> (dollars) | Green <br> forage <br> maize <br> (hectare) | Green <br> forage <br> saffower <br> (hectare) | Width <br> dried <br> chili <br> (hectare) | Alluvium <br> bean <br> (hectare) | Ear of <br> corn <br> (hectare) | Potato <br> (hectare) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| To <br> maximize <br> the utility | Land <br> surface <br> $\leq 150$ | 169442.34 | 22.596 | 3.500 | 33.333 | 18.182 | 39.370 | 33.019 |
| To <br> minimize <br> the cost | Land <br> surface <br> $\leq 150$ | 30133.23 | 2.857 | 3.500 | 2.00 | 18.182 | 2.362 | 1.887 |
|  | Land <br> surface <br> $=150$ | 92004.28 | 2.857 | 3.500 | 2.00 | 137.394 | 2.362 | 1.887 |

4. Results and Discussion. Data provided by the Department of Mexico Agriculture shows the current crops pattern in research zone (Table 1).

Observing the results, the allocated land for objective function (maximize the utility): the width dried chili, the ear of corn and the potato are cultivated up to the maximum level (maximum demand of the market); the green forage safflower and the alluvium bean are cultivated up to the minimum level (internal consumer demand). For objective function (minimize the cost) for land surface $\leq 150$ : all the crops are cultivated for the minimum level (internal consumer demand). For objective function (minimize the cost) for land surface $=150$ : also all the crops are cultivated for the minimum level (internal consumer demand) excluding the alluvium bean that is grown up to 137.394 hectare whose internal consumer demand is of 18.182 hectare.
5. Conclusions. This paper presents two models to obtain the maximum utility and the minimum cost for planning and the optimization of the crops in the agricultural production. The model is used in order to achieve better procedures, better market policy and the simulation of the most realistic decision process.

Real examples to obtain the maximum utility and the minimum cost have been presented to demonstrate the efficiency of the optimization techniques.

The proposed models can be further used to evaluate different products, industrial type and different regions in agriculture.

The suggestions for future research may be, if the constraint functions consider the following: the cost on preparation per unit of land (per hectare) for each crop; the cost and quantity on seeds per unit of land (per hectare) for each crop; the cost and quantity on fertilizers per unit of land (per hectare) for each crop; the cost and quantity on human power per unit of land (per hectare) for each crop; the cost and quantity on irrigation water per unit of land (per hectare) for each crop; the cost on integral control of pests and diseases per unit of land (per hectare) for each crop; the cost on harvest per unit of land (per hectare) for each crop.

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