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MODEL PREDICTIVE CONTROL FOR A 3DOF LABORATORY HELICOPTER BASED ON DISTURBANCE PREDICTION

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Abstract. *Unmanned helicopters may encounter disturbances and this will affect flight attitude when they are carrying out aggressive missions. In this paper, a model predictive control strategy for a three degree freedom (3DOF) laboratory helicopter is studied to resist measurable disturbance. A linearized state space model with disturbance item is first derived to describe the attitude during flighting process. Model predictive controller based on disturbance prediction for a class of smooth and measurable disturbance is then designed. Experimental results show that the adopted model predictive control based on disturbance prediction resists disturbance effectively.*
Keywords: Model predictive control, Disturbance, 3DOF laboratory helicopter, State space model

1. **Introduction.** Unmanned helicopters have the advantages of flying at low altitude and low speed, taking-off and landing vertically in a small area of ground and reposing in the air. These characteristics make them have wide applications and development prospects for both civil and military fields. However, their flight attitude control is a great challenge due to their special dynamic properties, such as nonlinearity, coupling, parametric uncertainties and disturbance [1].

The trajectory tracking controller is an important part of an unmanned helicopter and attracts much attention of the researchers to propose many flight control schemes, such as adaptive sliding mode control [2], robust linear quadratic regulator (LQR) control [3] and nonlinear adaptive control [4]. In recent years, model predictive control (MPC) has been widely used in industrial process control due to its ability to deal with multivariable systems and transport delays in the optimization problem [5]. Some researchers utilized MPC to control helicopter systems [6,7]. In practical applications, the attitude of the system may deviate from the expected due to the presence of disturbances, which possibly lead to influence system instability. Thus, it is necessary to consider external disturbances. Due to the principle of MPC, the future behavior of the system needs to be predicted. Future disturbances are not known at current sampling time. Usually assume that it is in accordance with the current sampling time [5] which will produce predictive error largely and affect control performance. Thus, if we can estimate and predict the future disturbances online effectively, it is possible to eliminate disturbance influences and the system can still be in accordance with the desired attitude. In this work, for a class of smooth and measurable disturbance, we design a model predictive controller based on disturbance prediction to predict future disturbances. The performance of applied control technique is illustrated and compared to the model predictive controller without predicting disturbances.

The paper is organized as follows. In Section 2, the linear time-invariant model of the laboratory helicopter dynamic is described and a state space representation with disturbance item is derived based on the model. In Section 3, the prediction method of smooth disturbances is described and a model predictive controller of the laboratory helicopter dynamic is designed. In Section 4, several experiments are carried out to verify the method is effective. Finally, conclusions are drawn in Section 5.

2. **Problem Statement and Preliminaries.** This work is based on the model of a 3DOF laboratory helicopter system from Quanser Consulting, Inc. The 3DOF helicopter experiment provides a bench top model of a tandem rotor helicopter, which is used for transport, search and rescue missions. The 3DOF helicopter consists of a base upon which an arm is mounted. The arm carries the helicopter body on one end and a counter weight on the other end. The arm can pitch about an elevation axis as well as swivel about a travel axis. Encoders that are mounted on these axes allow measuring the elevation, pitch and travel of the arm. The effective position resolution is 0.0879 degrees about the elevation and pitch axis and 0.0439 degrees about the travel axis. Two DC motors with propellers mounted on the helicopter body can generate a force proportional to the voltages applied to the DC motors approximatively [8]. The helicopter experimental system is shown in Figure 1.

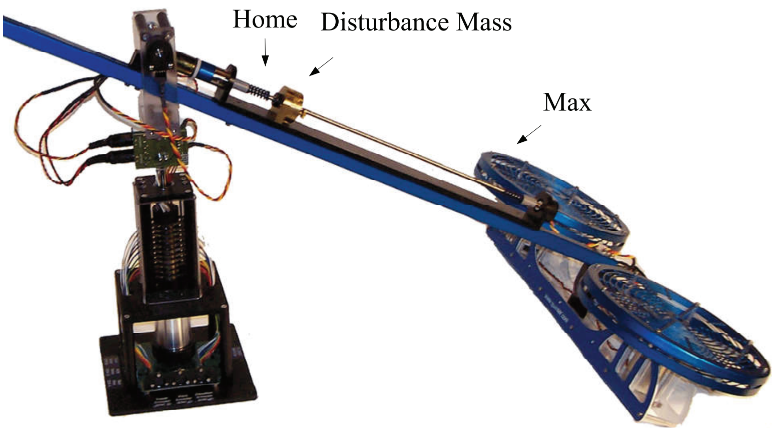


Figure 1. Laboratory helicopter model

The system dynamics can be described by a highly nonlinear state model [4]. We can linearize the nonlinear equations under following assumptions.

Assumption 1: ignore all friction.

Assumption 2: ignore the length of pendulum for the elevation axis and the length of pendulum for pitch axis.

Assumption 3: helicopter is axisymmetric about the pendulum for pitch axis (i.e., mass of the front section of the helicopter is equal to mass of the rear section).

Then the following three differential equations can be obtained

$$\begin{cases} J_\varepsilon \ddot{\varepsilon} = k_f V_s l_a - F_g l_a \\ J_\theta \ddot{\theta} = k_f V_d l_h \\ J_\phi \ddot{\phi} = -K_g \theta \end{cases} \tag{1}$$

where V_s, V_d are sum and difference of control voltages of the front and back motors, respectively. ε, θ, ϕ are elevation angle, pitch angle and travel angle. J_ε, J_θ, J_ϕ are moment of inertia about the elevation, pitch and travel axes. And $J_\varepsilon = 2m_f {l_a}^2 + m_w {l_w}^2$, $J_\theta = 2m_f {l_h}^2$, $J_\phi = m_w {l_w}^2 + 2m_f {l_h}^2 + 2m_f {l_a}^2$. $F_g = m_g g$ is the effective force of helicopter. $K_g = (l_w m_w - 2l_a m_f)\, g$. The symbols used in the above model are described in Table 1.

TABLE 1. Main parameters description of the 3DOF laboratory helicopter

Symbol	Description	Value	Units
m_f, m_b	Mass of front and back propeller assembly	0.713	kg
m_w	Mass of the counterweight	1.87	kg
m_g	Effective mass of helicopter	0.17	kg
k_f	Propeller force-thrust constant	0.1188	N/V
l_a	Distance between travel axis to helicopter body	0.66	m
l_h	Distance between pitch axis to each motor	0.178	m
l_w	Distance between travel axis to counterweight	0.47	m
g	Gravitational constant	9.81	m/s^2

In order to study the ability of resisting disturbance, we introduce the active disturbance system (ADS) on elevation axis which can produce a class of smooth and measurable disturbance. This kind of disturbance could simulate a class of real disturbance. The ADS as shown in Figure 1 consists of a lead screw driven by a motor. Attached to the lead screw is a mass that can be made to travel along the arm. The position of disturbance mass on the arm can be measured by an encoder. We introduce constant matrix B_{cd}, and interference item

$$d(t) = \frac{(m_g + m_\delta(t))\, g l_a}{J_\varepsilon} \tag{2}$$

where m_g is effective mass of helicopter and $m_\delta(t)$ is effective disturbance mass. Moving the mass results in changing $m_\delta(t)$. Let state $x = \left[\varepsilon, \theta, \phi, \dot{\varepsilon}, \dot{\theta}, \dot{\phi}\right]^T$, input $u = [V_f, V_b]^T$, where V_f, V_b are the control voltages of the front and back motors, respectively. Elevation angle ε and pitch angle θ are chosen as the controlled variables, i.e., $y = [\varepsilon, \theta]^T$, Then, we can obtain the following state space equations

$$\begin{cases} \dot{x}(t) = A_c x(t) + B_{cu} u(t) + B_{cd} d(t) \\ y(t) = Cx(t) \end{cases} \tag{3}$$

where

$$A_c = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-K_g}{J_\phi} & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_{cu} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{k_f l_a}{J_\varepsilon} & \frac{k_f l_a}{J_\varepsilon} \\ \frac{k_f l_h}{J_\theta} & -\frac{k_f l_h}{J_\theta} \\ 0 & 0 \end{bmatrix},$$

$$B_{cd} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 2 shows the effective disturbance that is attained at various positions along the arm away from the home position to max position [9]. As can be seen, it approximates linear relationship between mass position and effective disturbance.

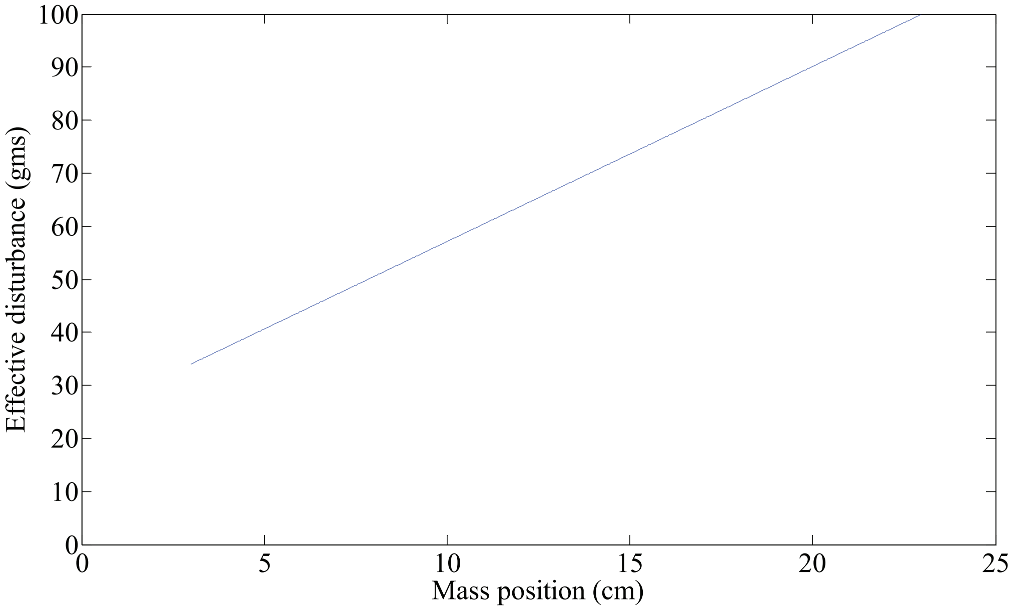


Figure 2. Effective disturbance vs mass position

We discrete continuous-time system (3) and select the discretization method as zero-order holder (ZOH). Thus, a corresponding discrete-time system has been gained as

$$\begin{cases} x(k+1) = Ax(k) + B_u u(k) + B_d d(k) \\ y(k) = Cx(k) \end{cases} \tag{4}$$

where A, B_u and B_d are coefficient matrices corresponding to A_c, B_{cu} and B_{cd} after discretization.

3. Design and Analysis of Control System.

3.1. Prediction algorithm of disturbance.

MPC strongly depends on plant model. If we do not correct disturbance promptly, it will cause model mismatch and the tracking performance can be decreased. Therefore, we design a disturbance predictor (DP) described below in detail to predict future disturbances.

This work assumes that the disturbances satisfy the following conditions.

Assumption 4: disturbances can be measured.

Assumption 5: disturbances are smooth.

The measured value $m_\delta(t)$ of effective disturbance is obtained at the sampling time k. Then interference item d can be calculated according to (2). Sampling interval is T_s. The rate of change of disturbance item is

$$q(k) = \frac{d(k) - d(k-1)}{T_s} \tag{5}$$

We use linear function to predict future disturbances. Namely future disturbances increase or decrease in accordance with the rate of change of the disturbance at current sampling time. Predictions for the future disturbances are as follow.

$$\begin{aligned} &d(k+1|k) = d(k) + qT_{\mathrm{s}} \\ &d(k+2|k) = d(k+1|k) + qT_{\mathrm{s}} \\ &\quad\vdots \\ &d(k+p-1|k) = d(k+p-2|k) + qT_{\mathrm{s}} \end{aligned} \tag{6}$$

Then disturbance increment is as follows:

$$\begin{aligned} &\Delta d(k) = qT_{\rm s} \\ &\Delta d(k+1|k) = qT_{\rm s} \\ &\quad\vdots \\ &\Delta d(k+p-1|k) = qT_{\rm s} \end{aligned} \tag{7}$$

where $\Delta d(k) = d(k) - d(k-1)$.

3.2. **Design of a model predictive controller.** The control system structure adopted in this work is shown in Figure 3. The model (2) is rewritten as incremental model [9]

$$\begin{cases} \Delta x(k+1) = A\Delta x(k) + B_u\Delta u(k) + B_d\Delta d(k) \\ y(k) = C\Delta x(k) + y(k-1) \end{cases} \tag{8}$$

where

$$\begin{aligned} \Delta x(k) = x(k) - x(k-1) \\ \Delta u(k) = u(k) - u(k-1) \end{aligned} \tag{9}$$

Control horizon is m, prediction horizon is p, and $m \le p$. In order to derive prediction equation system, make the assumption: control increment is unchanged when exceeding control horizon.

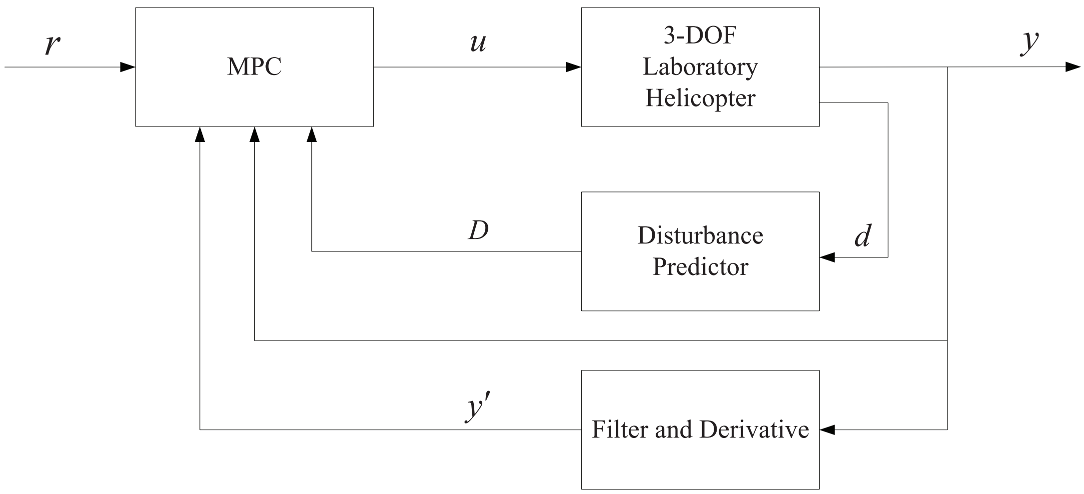


FIGURE 3. Structure of control system

The control vector, system predictive output, and disturbance increment are defined as below, respectively.

$$\Delta U(k) = \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+m-1) \end{bmatrix}_{(m\times n_u)\times 1}, \quad Y(k+1|k) = \begin{bmatrix} y(k+1|k) \\ y(k+2|k) \\ \vdots \\ y(k+p|k) \end{bmatrix}_{(p\times n_c)\times 1},$$

$$\Delta D(k) = \begin{bmatrix} \Delta d(k) \\ \Delta d(k+1|k) \\ \vdots \\ \Delta d(k+p-1|k) \end{bmatrix}_{p\times 1}$$

The prediction for the plant output during future sampling instant is

$$Y(k+1|k) = S_x\Delta x(k) + \mathrm{I}y(k) + S_d\Delta D(k) + S_u\Delta U(k) \tag{10}$$

where

$$S_x = \begin{bmatrix} CA \\ \sum_{i=1}^{2} CA^i \\ \vdots \\ \sum_{i=1}^{p} CA^i \end{bmatrix}_{(p\times n_c)\times n_x}, \; S_d = \begin{bmatrix} CB_d & 0 & 0 & \cdots & 0 \\ \sum_{i=1}^{2} CA^{i-1}B_d & CB_d & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{p} CA^{i-1}B_d & \sum_{i=1}^{p-1} CA^{i-1}B_d & \cdots & \cdots & CB_d \end{bmatrix}_{(p\times n_c)\times p},$$

$$\mathrm{I} = \begin{bmatrix} I_{n_c\times n_c} \\ I_{n_c\times n_c} \\ \vdots \\ I_{n_c\times n_c} \end{bmatrix}_{(p\times n_c)\times n_c},$$

$$S_u = \begin{bmatrix} CB_u & 0 & 0 & \cdots & 0 \\ \sum_{i=1}^{2} CA^{i-1}B_u & CB_u & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{m} CA^{i-1}B_u & \sum_{i=1}^{m-1} CA^{i-1}B_u & \cdots & \cdots & CB_u \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{p} CA^{i-1}B_u & \sum_{i=1}^{p-1} CA^{i-1}B_u & \cdots & \cdots & \sum_{i=1}^{p-m+1} CA^{i-1}B_u \end{bmatrix}_{(p\times n_c)\times(m\times n_c)}$$

Based on the original MPC problem, the following open-loop optimal control problem can be formulated:

$$\min_{\Delta U(k)} \quad J = ||\Gamma_y(Y_p(k+i|k) - R(k+i))||^2 + ||\Gamma_u \Delta U(k)||^2 \tag{11}$$

where weighting matrix and reference input are defined as below, respectively.

$$\begin{cases} \Gamma_y = diag(\Gamma_{y,1}, \Gamma_{y,1}, \cdots, \Gamma_{y,p}) \\ \Gamma_u = diag(\Gamma_{u,1}, \Gamma_{u,1}, \cdots, \Gamma_{u,m}) \end{cases}, \quad R(k+1) = \begin{bmatrix} r(k+1) \\ r(k+2) \\ \vdots \\ r(k+p) \end{bmatrix}_{(p\times n_c)\times 1}$$

In order to solve the optimization problem conveniently, define the auxiliary variable

$$\rho = \begin{bmatrix} \Gamma_y(Y_p(k+1|k) - R(k+1)) \\ \Gamma_u \Delta U(k) \end{bmatrix} \tag{12}$$

Then, the objective function (7) is changed into

$$J = \rho^T \rho \tag{13}$$

Take (10) into (12)

$$\rho = \begin{bmatrix} \Gamma_y S_u \\ \Gamma_u \end{bmatrix} \Delta U(k) - \begin{bmatrix} \Gamma_y E_p(k+1|k) \\ 0 \end{bmatrix} = Az - b \tag{14}$$

where

$$z = \Delta U(k), \quad A = \begin{bmatrix} \Gamma_y S_u \\ \Gamma_u \end{bmatrix}, \quad b = \begin{bmatrix} \Gamma_y E_p(k+1|k) \\ 0 \end{bmatrix}, \tag{15}$$

$$E_p(k+1|k) = R(k+1) - S_x \Delta x(k) - \mathrm{I} y_c(k) - S_d \Delta D(k) \tag{16}$$

Therefore, the unconstrained predictive control open-loop optimal control problem (11) is changed into

$$\min_z \rho^T \rho, \quad \text{where} \;\; \rho = Az - b \tag{17}$$

When $\frac{d\rho^T\rho}{dz} = 0$, the optimal solution of the parameter vector is

$$z = \left(A^T A\right)^{-1} A^T b \tag{18}$$

Taking (15) into (18), obtain optimized sequence of optimal control problem

$$\Delta U(k) = \left(S_u^T \Gamma_y^T \Gamma_y S_u + \Gamma_u^T \Gamma_u\right)^{-1} S_u^T \Gamma_y^T \Gamma_y E_p(k+1|k) \tag{19}$$

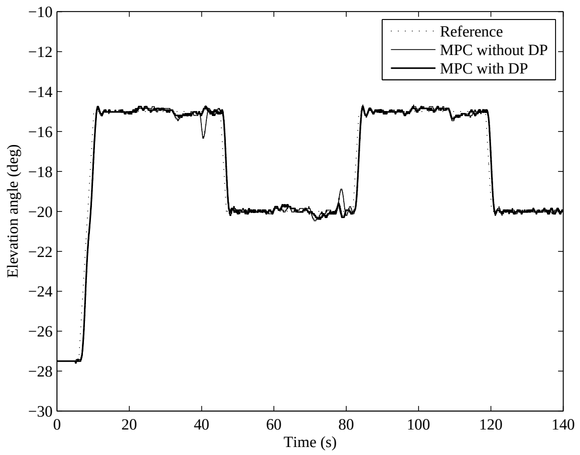
According to the receding horizon control law, the control variable at sampling instant k is

$$\Delta u(k) = K_{mpc} E_p(k+1|k) \tag{20}$$

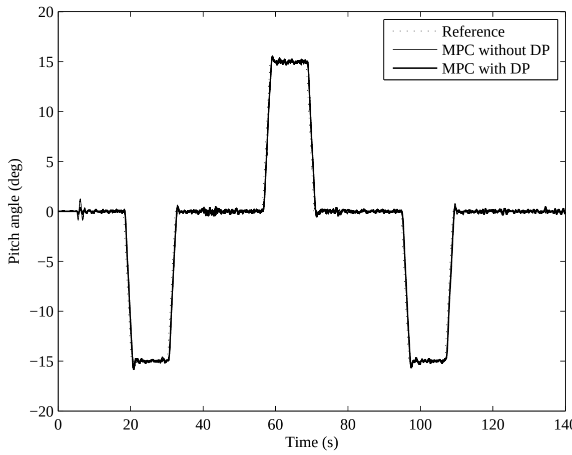
where $K_{mpc} = [I_{n_u \times n_u} \ \ 0 \ \ \cdots \ \ 0]_{n_u \times (n_u \times m)} \left(S_u^T \Gamma_y^T \Gamma_y S_u + \Gamma_u^T \Gamma_u\right)^{-1} S_u^T \Gamma_y^T \Gamma_y$.

It is necessary to develop a way to estimate the system states. By taking low pass filter, the angular velocities states of system states $\dot{\varepsilon}$, $\dot{\theta}$, $\dot{\phi}$ are estimated.

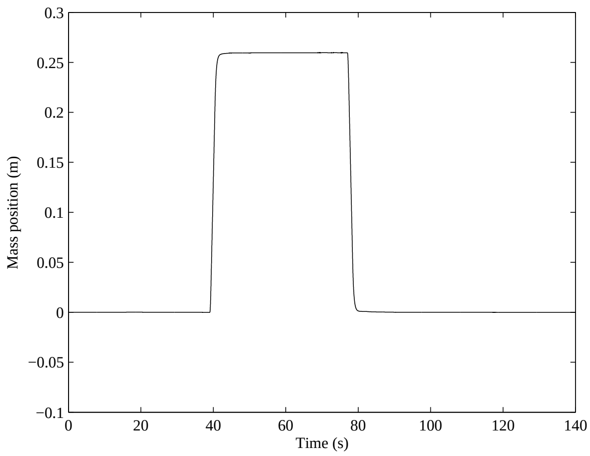
4. **Experimental Results.** The experiment setup is shown in Figure 1. A 1.86GHz computer with Quanser Q4 data acquisition board is used to process feedback signals and derive the control input for the system. The mechanical system parameters used are shown in Table 1. The initial condition is $x[0] = [-27.5 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]^T$. The following parameters are chosen by experience. The prediction horizon and control horizon are $p = 100$ and $m = 20$, respectively. Sampling time is $T_s = 0.008$(s). $\Gamma_{y,a} = 10$, where



(a) Response of elevation angle



(b) Response of pitch angle



(c) Position of disturbance mass during experiment

FIGURE 4. Tracking results with external disturbance

$a = 1, 2, \ldots, p$ and $\Gamma_{u,b} = 10$, where $b = 1, 2, \ldots, m$. Considering the characteristics of front and back motors, inputs of the two motors are limited between +20(V) and −20(V). The low pass filter $15791\text{s}/s^2 + 226.1947\text{s} + 15791$ are adopted for $\dot{\varepsilon}$, $\dot{\theta}$, $\dot{\phi}$ estimation.

Figure 4(a) and Figure 4(b) show a comparison of elevation and pitch responses under disturbance between the MPC with DP and without DP. The latter assumes that the disturbances are not changed after current sampling time. Figure 4(c) shows position of disturbance mass during experiment. It can be seen from the result of the experiments, at the stage of external disturbance changing about $t = 40$(s) and $t = 80$(s), elevation response of the MPC without DP has larger shocks. On the contrary, the MPC with DP can efficiently compensate for external disturbance and have a better tracking performance.

5. **Conclusions.** In this paper, a model predictive controller of a 3DOF laboratory helicopter which is based on disturbance prediction is designed. System design with the proposed controller is carried out and the improved control performance compared to the controller without disturbance prediction is illustrated via experiments. The experimental results show that our proposed controller provides a better 3DOF laboratory helicopter attitude control.

In further research, we will put more emphasis on modifying the values of parameters to perfect the mathematical models and consider other issues that influence the flight performance.

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