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DUAL-AXIS ROTATIONAL MODULATION BASED ON FOUR-GYRO SYMMETRICAL NON-ORTHOGONAL REDUNDANT CONFIGURATION

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Abstract. *A new dual-axis rotational modulation method based on four-gyro symmetrical non-orthogonal redundant configuration is designed to improve the accuracy and reliability of inertial navigation systems (INS). After the comparison of reliability between different kinds of configurations consisting of four gyros, it is demonstrated that the symmetrical non-orthogonal redundant configuration has higher reliability than orthogonal configuration does. Then the equivalent error equation was derived in this paper to explore the influence to INS accuracy brought by error of IMU with symmetrical non-orthogonal redundant configuration. Based on the equation, dual-axis rotational modulation is proposed to four-gyro symmetrical non-orthogonal redundant configuration system so that accuracy and reliability of system can be improved at the same time. Experiment and analysis results show that the proposed modulation and configurations method can improve long-term velocity output accuracy from 0.126m/s and 0.1285m/s to less than 0.015m/s and 0.001m/s during 8 hours, and improve latitude and longitude output accuracy from* $0.27'$ *and* $1.75'$ *to less than* $0.048'$ *and* $0.055'$ *during 8 hours.*
Keywords: INS, Redundant configuration, Dual-axis rotational modulation

1. **Introduction.** Inertial navigation system (INS) has been widely used in military and civil field to fulfill the navigation task in more and more complex environment, because of its high accuracy and reliability. There are two schemes to meet more requirements. One is device-level [1], a method improving the reliability and precision of single inertial device to achieve better system performance, however with a long life cycle. The other is system-level, such as redundancy configuration to improve reliability or rotational modulation technology to restrain inertial measurement unit (IMU) drift [2].

In recent years, the redundant configuration, fault detection, isolation and reconstruction have made a real breakthrough. By giving an optimal condition for redundancy configuration, [3] has proposed a four-gyro symmetrical non-orthogonal redundant configuration which can provide a good navigation performance, but has no effect on accuracy. [4] puts forwards a fault detection and isolation method which is more suitable for oblique configuration, but the accuracy cannot get a boost from it. Rotation modulation as one of hot topics is an effective method to enhance the accuracy of an INS. Low frequency gyroscope error along the three sensitive axes can be modulated, and the accuracy of INS will be improved. A dual-axis rotational modulation and an accurate calibration method are proposed to improve the precision of INS [5]. [6] has proposed an algorithm to improve the attitude precision with rotating modulation. However, the researches above cannot improve the reliability at the same time.

In this paper, a new dual-axis rotational modulation mothed based on four-gyro symmetrical non-orthogonal redundant configuration is proposed. The main contribution of this method is to provide a more accurate and reliable navigation scheme to INS to improve navigation accuracy and reliability. The complicated mechanical structure of dual-axis rotating inertial navigation system will decrease the reliability, but because of the redundant configuration the proposed method can overcome this disadvantage. Section 2 presents the reliability of different kinds of redundant configurations. Section 3 analyzes the error characteristic of redundant configuration systems, and obtains a new modulation method based on four-gyro symmetrical non-orthogonal redundant configuration by proposing the dual-axis rotational modulation. Section 4 presents the experiment results, followed by the conclusions in Section 5.

2. **Comparison and Analysis of Navigation Reliability.** Generally, the malfunction of gyroscopes is subject to Poisson distribution. The reliability expression of a single gyroscope can be described as [1]:

$$R(t) = \varepsilon^{-\lambda t} \tag{1}$$

The mean time between failures (MTBF) of a single gyroscope can be written as:

$$MTBF = \int_0^\infty R(t)dt = \int_0^\infty e^{-\lambda t}dt = \frac{1}{\lambda} \tag{2}$$

where λ is the failure rate of a single gyroscope, and t is the working time of device.

Take gyroscope with single degree freedom as an example. Considering that a sensor failure occurs equally and independently, the reliability expression of a non-redundant IMU can be described as:

$$R_3(t) = R^3 = e^{-3\lambda t} \tag{3}$$

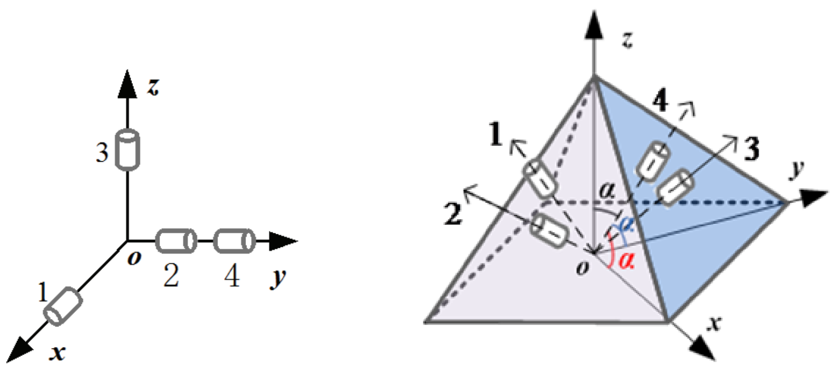
The corresponding MTBF is:

$$MTBF_3 = \int_0^\infty R_3(t)dt = \frac{1}{3\lambda} \tag{4}$$

The common four-gyro redundant IMU configurations are shown in Figure 1.

According to Figure 1(a), with a single-gyro failure condition of gyroscope No.2 or gyroscope No.4, the IMU can also have a good navigation performance, of which the reliability expression is obtained as follows:

$$R_a(t) = C_2^1 R(1-R)R^2 + R^4 = 2e^{-3\lambda t} - e^{-4\lambda t} \tag{5}$$



(a) The orthogonal configuration

(b) The symmetrical non-orthogonal configuration

FIGURE 1. The four-gyro redundant configuration

The corresponding MTBF is:

$$MTBF_a = \int_0^{\infty} R_a(t)dt = \frac{5}{12\lambda} \tag{6}$$

According to Figure 1(b), $\alpha = 54.73°$, and no matter which gyroscope has a single-gyro failure, the IMU also has a good navigation performance. The reliability expression is obtained as follows:

$$R_b(t) = C_4^3 R^3(1-R) + R^4 = 4e^{-3\lambda t} - 3e^{-4\lambda t} \tag{7}$$

The corresponding MTBF is:

$$MTBF_b = \int_0^{\infty} R_b(t)dt = \frac{7}{12\lambda} \tag{8}$$

According to Equations (3)∼(8), with a redundant gyro in INS, the MTBF of INS increases and the reliability improves. Considering the size and high precision of inertial component, it is not wise to improve the reliability of INS simply by increasing the number of inertial devices. We should turn to the study of increasing the reliability with the same component count. Comparing orthogonal configuration and diagonal configuration, with the same number of gyros, the reliability of the symmetrical non-orthogonal redundant configuration is higher than that of the orthogonal configuration.

3. Rotational Modulation Based on Redundant Configuration.

3.1. Accuracy of symmetrical non-orthogonal redundant configuration. With the same number of gyroscopes, the four-gyro symmetrical non-orthogonal redundant configuration can provide higher reliability to INS than orthogonal configuration. However, in the non-orthogonal redundant scheme, the measurement accuracy of axes in body coordinate frame does not coincide with the inertial sensor, so it is necessary to resolve the evaluating of gyro drifts in the body coordinate frame.

The proposed system involves three essential coordinate frames. One is the inertial coordinate frame (I-frame); the second is the body coordinate frame (B-frame); the last is the solidified frame (S-frame), which coincides with B-frame at the beginning temporarily, but relatively static to the IMU.

For the redundant scheme with four gyros, the measurement equation of four gyros can be described as follows:

$$\boldsymbol{\omega}_{\boldsymbol{im}}^{\boldsymbol{m}} = \boldsymbol{H}\boldsymbol{\omega} = \begin{bmatrix} -\cos\alpha & -\cos\alpha & \cos\alpha \\ \cos\alpha & -\cos\alpha & \cos\alpha \\ \cos\alpha & \cos\alpha & \cos\alpha \\ -\cos\alpha & \cos\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{9}$$

where $\boldsymbol{\omega}_{\boldsymbol{im}}^{\boldsymbol{m}} = [\ \omega_{i1}^1 \quad \omega_{i2}^2 \quad \omega_{i3}^3 \quad \omega_{i4}^4\ ]^T$ is the output of the four gyros; $\boldsymbol{\omega} = [\ \omega_x \quad \omega_y \quad \omega_z\ ]^T$ is three angular rates along axes of the body coordinate frame; $\boldsymbol{H}$ is the measurement matrix.

According to the theory of linear weighted minimum variance, the least square method can express the estimated value of $\boldsymbol{\omega}$ as:

$$\hat{\boldsymbol{\omega}} = \left(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{m} \tag{10}$$

The output signal of the gyroscopes is described as follows:

$$\tilde{\boldsymbol{\omega}}_{\boldsymbol{im}}^{\boldsymbol{m}} = \boldsymbol{\omega}_{\boldsymbol{im}}^{\boldsymbol{m}} + \boldsymbol{\delta\omega}_{\boldsymbol{im}}^{\boldsymbol{m}} \tag{11}$$

$\boldsymbol{A} = \left(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^{\mathrm{T}}$, in S-frame the estimated angular velocity $\hat{\boldsymbol{\omega}}_{\boldsymbol{is}}^{\boldsymbol{s}}$ can be described as:

$$\hat{\boldsymbol{\omega}}_{\boldsymbol{is}}^{\boldsymbol{s}} = \boldsymbol{A}(\boldsymbol{\omega}_{\boldsymbol{im}}^{\boldsymbol{m}} + \boldsymbol{\delta\omega}_{\boldsymbol{im}}^{\boldsymbol{m}}) = \boldsymbol{A}\boldsymbol{\omega}_{\boldsymbol{im}}^{\boldsymbol{m}} + \boldsymbol{A}\boldsymbol{\delta\omega}_{\boldsymbol{im}}^{\boldsymbol{m}} \tag{12}$$

where $\boldsymbol{\delta\omega_{im}^{m}}$ is measurement error of gyros described as follows:

$$\boldsymbol{\delta\omega_{im}^{m}} = (\boldsymbol{K_g} + \boldsymbol{C_g})\boldsymbol{\omega_{im}^{m}} + \boldsymbol{\varepsilon_g} + \boldsymbol{n_g} \tag{13}$$

where $\boldsymbol{K_g}$ is factor error of gyros, $\boldsymbol{C_g}$ is installation error of gyros, $\boldsymbol{\varepsilon_g}$ is constant drift of gyros, and $\boldsymbol{n_g}$ is white-noise of gyros.

In S-frame the equivalent error of gyroscope drift derived and expressed in the vector form is:

$$\boldsymbol{\delta\hat{\omega}_{is}^{s}} = \boldsymbol{A\delta\omega_{im}^{m}} = \boldsymbol{A}(\boldsymbol{K_g} + \boldsymbol{C_g})\boldsymbol{\omega_{im}^{m}} + \boldsymbol{A\varepsilon_g} + \boldsymbol{An_g} \tag{14}$$

In S-frame the equivalent error of gyroscope constant drift $\boldsymbol{\hat{\varepsilon}}$ can be simplified as

$$\boldsymbol{\hat{\varepsilon}} = \boldsymbol{A\varepsilon_g} = \begin{bmatrix} \hat{\varepsilon}_x & \hat{\varepsilon}_y & \hat{\varepsilon}_z \end{bmatrix} \tag{15}$$

Coinciding with S-frame, in B-frame $\boldsymbol{\hat{\omega}_{ib}^{b}}$ and $\boldsymbol{\delta\hat{\omega}_{ib}^{b}}$ can be obtained:

$$\boldsymbol{\hat{\omega}_{ib}^{b}} = \boldsymbol{\hat{\omega}_{is}^{s}} \tag{16}$$

$$\boldsymbol{\delta\hat{\omega}_{ib}^{b}} = \boldsymbol{\delta\hat{\omega}_{is}^{s}} \tag{17}$$

Equations (11)∼(15) demonstrate that with the four-gyro symmetrical non-orthogonal redundant configuration, in B-frame the equivalent error of gyroscope drift caused by low frequency gyroscope error along the sensitive axis is still low-frequency, which means though the reliability of INS has been improved, the accumulated errors have not been modulated.

3.2. **Rotational modulation based on redundant configuration.** Considering that the effect of four-gyro symmetrical non-orthogonal redundant configuration only presents on reliability, the dual-axis rotational modulation is proposed to improve the precision of INS.

Along the z-axis, IMU rotating with a certain angular velocity, the direction cosine matrix from S-frame to B-frame can be expressed as follows:

$$\boldsymbol{T_s^b} = \begin{bmatrix} \cos\omega t & -\sin\omega t & 0 \\ \sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 1 \end{bmatrix} = (\boldsymbol{T_b^s})^{\boldsymbol{T}} \tag{18}$$

Then the angular velocity in B-frame can be described as:

$$\boldsymbol{\hat{\omega}_{ib}^{b}} = \boldsymbol{T_s^b\hat{\omega}_{is}^{s}} + \boldsymbol{\omega_{sb}^{b}} = \boldsymbol{\omega_{ib}^{b}} + \boldsymbol{T_s^bA\delta\omega_{is}^{s}} \tag{19}$$

Thus, $\boldsymbol{\hat{\omega}_{ib}^{b}}$ can be obtained by integrating Equation (14) as follows:

$$\boldsymbol{\hat{\omega}_{ib}^{b}} = \boldsymbol{\omega_{ib}^{b}} + \boldsymbol{T_s^bA}(\boldsymbol{K_g} + \boldsymbol{C_g})\boldsymbol{T_b^s\omega_{ib}^{b}} + \boldsymbol{T_s^bA}(\boldsymbol{K_g} + \boldsymbol{A_g})\boldsymbol{\omega_{bs}^{s}} + \boldsymbol{T_s^bA\varepsilon_g} + \boldsymbol{T_s^bAn_g} \tag{20}$$

Assuming that constant drift is $\boldsymbol{\varepsilon_g^b}$, it can be expressed as follows:

$$\boldsymbol{\varepsilon_g^b} = \boldsymbol{T_s^bA\varepsilon_g} = \begin{bmatrix} \hat{\varepsilon}_x\cos\omega t - \hat{\varepsilon}_y\sin\omega t \\ \hat{\varepsilon}_x\sin\omega t + \hat{\varepsilon}_y\cos\omega t \\ \hat{\varepsilon}_z \end{bmatrix} \tag{21}$$

According to Equation (21), horizontal constant drift is modulated from constant form into periodical high frequency form, whose average value is zero in a rotation period. Navigation errors caused by horizontal gyro drifts are no longer accumulated with time.

In order to modulate drifts of three gyros, IMU rotates along horizontal and vertical axes in turn, and the accuracy of INS can be improved.

4. **Experimentation.** An experimentation system is constructed to verify the effect brought by dual-axis rotational modulation on the accuracy of INS with four-gyro symmetrical non-orthogonal redundant configuration.

The drifts of gyros are 0.001°/h, 0.0015°/h, 0.002°/h, 0.003°/h, and the accuracy of accelerometers are 20μg. Ignoring the factor errors and installation error, the experimental result is shown in these figures.

As seen in these figures, rotational modulation system has higher accuracy than non-rotational modulation system, and dual-axis rotational modulation has better effect than single-axis rotational modulation. With dual-axis rotational modulation, velocity error during 8 hours in four-gyro symmetrical non-orthogonal redundant configuration system declined from 0.126m/s and 0.1285m/s to less than 0.015m/s and 0.001m/s, and latitude and longitude steady-state error during 8 hours declined from 0.27′ and 1.75′ to less than 0.048′ and 0.055′. The dual-axis rotational modulation based on four-gyro symmetrical

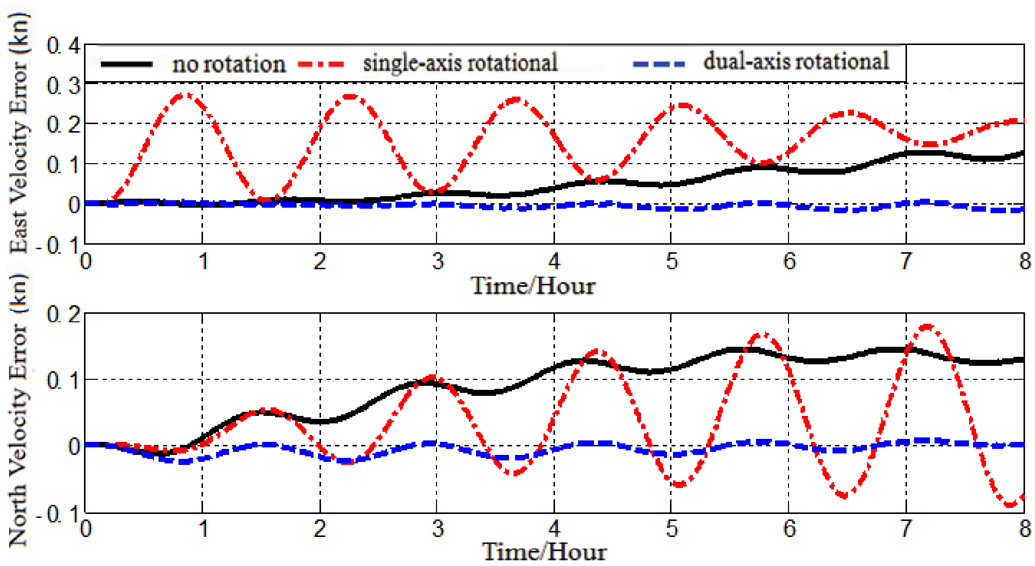


FIGURE 2. Comparison of horizontal velocity error

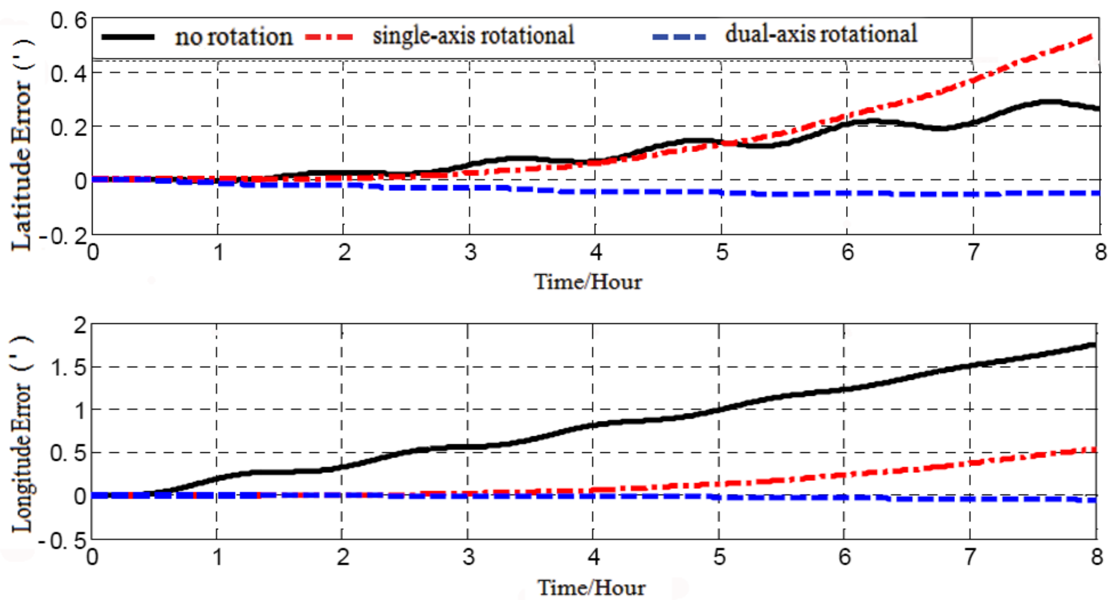


FIGURE 3. Comparison of horizontal position error

non-orthogonal redundant configuration can improve accuracy and reliability of INS at the same time.

5. **Conclusions.** In this paper, dual-axis rotational modulation based on four-gyro symmetrical non-orthogonal redundant configuration is proposed to overcome the disadvantage of typical rotation INS or redundant INS which cannot improve accuracy and reliability at the same time. With the analysis of four-gyro redundant system's reliability, by involving the solidified frame (S-frame), the equivalent error of gyro drift equation is firstly derived in symmetrical non-orthogonal configuration. Based on the error equation, dual-axis rotational modulation is applied to improving the accuracy of four-gyro symmetrical non-orthogonal redundant configuration system. The simulation experiment results have demonstrated that precision and reliability can be improved both. While the best way to combine redundant configuration with dual-axis rotational modulation was not taken into account in this study and will be studied as one of future tasks.

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