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A DECOMPOSITION APPROACH FOR SCHEDULING OF ELECTRIC VEHICLE CHARGING UNDER PREEMPTIVE CHARGING SCHEME

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Abstract. *In this paper, we deal with electric vehicle (EV) charging scheduling problem from the aggregator perspective. In particular, we solve the problem of an aggregator optimizing charging schedules of EVs with the objective of minimizing the charging cost. Moreover, we formulate the problem under the preemptive charging scheme in which interruptions are allowed during the charging process to increase charging flexibility. To solve the resulting optimization problem, we propose a decomposition approach based on column generation technique. Our computational experiments show that our approach performs well to the extent that it can be used in practice to solve large-scale instances.*
Keywords: Electric vehicle, Charging, Scheduling, Decomposition approach

1. **Introduction.** Electric vehicles (EVs) have the significant potential to facilitate the ongoing transformation of modern energy system towards a low-carbon future. On the other hand, the large-scale integration of electric vehicles comes with challenges for the operation of the existing power system and the charging infrastructure, such as voltage fluctuation, power loss, and network congestion [1,2]. Coordination and control of charging load of EVs can mitigate EV's negative impact on the power system. Such coordination can be performed by an aggregator, which is usually a central entity, e.g., existing utility, acting as an interface between EVs and the power system operator. In this paper, the aggregator is assumed to be a for-profit entity that tries to minimize charging costs by redistributing aggregated charging load in such a way that the charging load increases as much as possible when the electricity is relatively cheap. The aggregator could also be a non-profit entity that provides the power grid with regulation services by increasing or decreasing charging rates in response to grid conditions, thereby helping to maintain stability [3]. We investigate optimal scheduling of EV charging from the perspective of the aggregator.

The EV charging scheduling problem can be classified by the charging scheme. Most works in the literature consider a continuous charging scheme under which the charging rate can change continuously between zero and the maximum rate [2,5,6]. In practice, EVs are usually charged at discrete or constant charging rate. Such a charging scheme has also been considered in several works [7-9]. Han et al. [7] and Beaude et al. [8] assumed that the EV is charged at a constant charging rate. Binetti et al. [9] considered the case where the charging process can be interrupted.

In this paper, we focus on realistic charging scenario that takes account of the typical charging profile of the lithium-ion battery that most EVs use. Specifically, under such a charging profile, EV is charged at constant charging rate until the SOC reaches a certain level, after which the charging rate decreases to zero as the SOC reaches full charge. Moreover, to increase the charging flexibility, interruptions are allowed during the charging process. We present the mathematical formulation based on network flow for the resulting charging scheduling problem. However, the proposed formulation can be large and impractical when the number of EVs is large. To overcome this, we propose a new formulation based on the variable representing a path generated from network flow constraints. To solve this formulation efficiently, we propose a decomposition algorithm based on the column generation technique, which is known to be suitable for solving formulations with a large number of variables.

This paper is structured as follows. Section 2 describes the charging scenario assumed in the paper. The mathematical formulation and solution method are given in Sections 3 and 4, respectively. The computational results are discussed in Section 5, and our conclusions are given in Section 6.

2. **Charging Scenario.** Consider a scenario where a single aggregator coordinates the charging behavior of all EVs that subscribe for charging. The aggregator can be assumed to operate a charging station. The charging horizon is discretized into finite time periods ($t \in T = \{1, \ldots, \tau\}$) within which each EV ($v \in V$) plugs in and charges a certain amount of electricity. The electricity price (M^t) is assumed to vary depending on time, which motivates the aggregator to minimize total charging costs by distributing charging load optimally over a planning period. We denote as L^t the maximum allowable charging load at time t. We assume that it represents the amount of power the aggregator procured at the day-ahead market. The aggregator collects information on each EV's charging requirement before proceeding to the optimization process. It is assumed that EV v informs the aggregator of its arrival time τ_v^S, initial battery level π_v^S, departure time τ_v^D, and desired battery level π_v^D. We denote as $T(v) = \left\{\tau_v^S, \ldots, \tau_v^D - 1\right\}$ the set of admissible time periods of EV v.

In practice, EVs are usually charged following pre-determined charging profiles. In this paper, we adopt a typical lithium-ion battery charging profile that can be approximated as a piecewise linear function of the elapsed charging time as shown in Figure 1 [1]. Note that, under this charging profile, it is assumed that the EV is charged at constant rate until the SOC reaches a certain level, after which the charging rate decreases to zero as the SOC approaches its maximum. Most of the works in the literature rely on the assumption that charging power can change between zero and the maximum at any time for reasons of simplicity. However, such an assumption can yield sub-optimal or even infeasible charging schedules.

Under the piecewise linear charging profile, the EV is charged without interruptions once the charging starts. An example of discretized charging schedule under such a charging profile is shown in Figure 2(a), where the EV is charged at constant power for

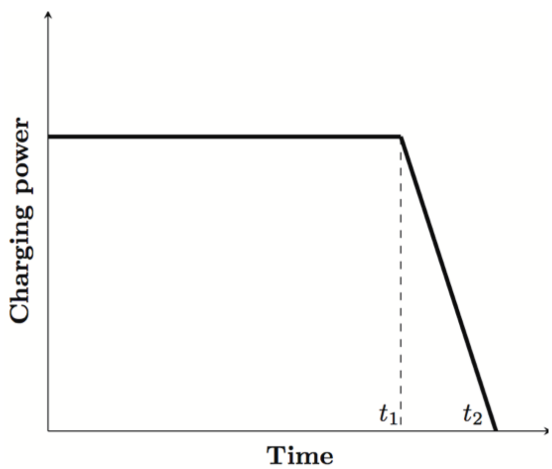
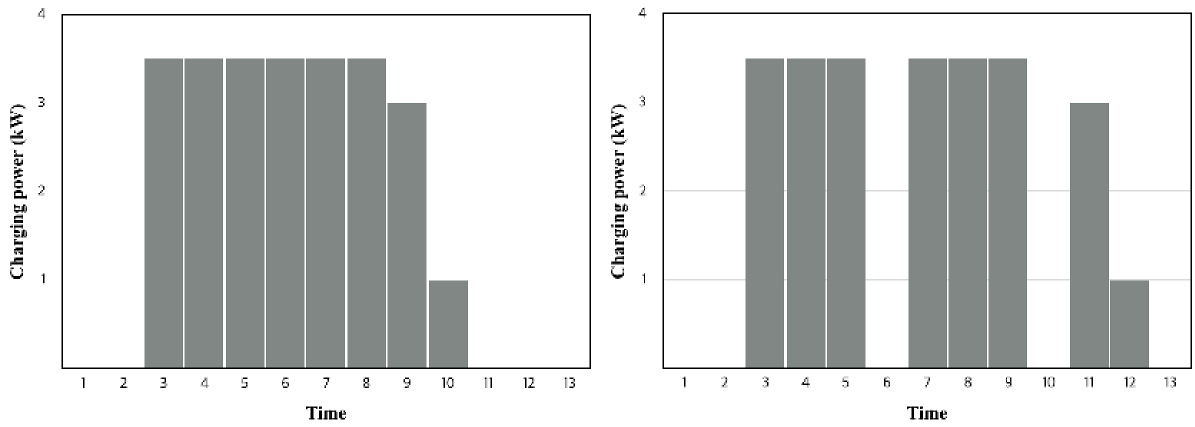


FIGURE 1. Piecewise linear charging profile



(a) (b)

Figure 2. Examples of discretized charging schedule: (a) non-preemptive charging, (b) preemptive charging

six successive hours after which, the charging power is decreased for the next 2 hours. The aggregator determines only the charging start time, as the charging schedule can be fixed once the charging start time is given, which restricts flexibility in charging control. In order to enhance the flexibility in charging control, therefore, we additionally allow interruptions in the middle of the charging process. We call this scheme the *preemptive charging scheme* [3]. Figure 2(b) shows an example of preemptive charging schedule, which incorporates two interruptions into the non-preemptive charging schedule of Figure 2(a). Han et al. [7] also considered a charging scheme similar to ours. However, they assumed a constant charging power.

3. **Mathematical Formulation.** In this section, we propose mathematical formulation of the EV charging scheduling problem under the preemptive charging scheme. Prior to building the mathematical formulation, we first associate a transition network to each EV by which we can visualize the charging schedule of the EV. We define nodes in the network as a pair (t, k) where t represents the time period and k represents the SOC level ranging from 1 to κ_v, the number of different SOC levels. The transitions between nodes correspond to one of the following two cases. The transition from a node (t, k) to a node $(t + 1, k + 1)$ corresponds to the event that EV v with SOC level k at the beginning of time t is charged during time t. The transition from a node (t, k) to a node $(t + 1, k)$ corresponds to the event that EV v with SOC level k at the beginning of time t is not charged during time t. Figure 3 illustrates all possible transitions associated with node (t, k).

In this transition network, a feasible charging schedule of EV v can be described by a path from node $\left(\tau_v^S, 1\right)$ to node $\left(\tau_v^D, \kappa_v\right)$.

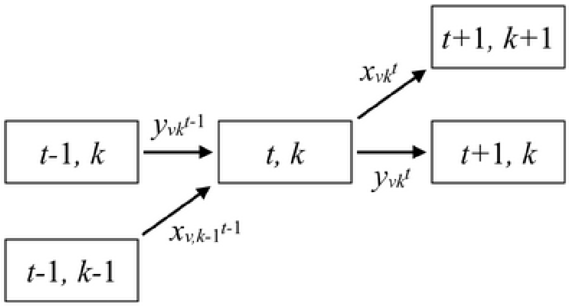


Figure 3. Transitions from and to node (t, k)

We now present the formulation of the charging scheduling problem under the preemptive charging scheme based on the network flow model on the transition networks [9]. In the formulation, the binary variable x_{vk}^t takes value 1 if EV v with SOC level k at the beginning of time t is charged during time t, and 0 otherwise. Similarly, the binary variable y_{vk}^t takes value 1 if EV v with SOC level k at the beginning of time t is not charged during time t, and 0 otherwise. The formulation (P) can be developed as follows:

$$\text{(P)} \ \min \sum_{v \in V} \sum_{t \in T(v)} \sum_{k=1}^{\kappa_v - 1} M^t P_{vk} x_{vk}^t \tag{1}$$

$$\text{s.t.} \ \sum_{v \in V} \sum_{k=1}^{\kappa_v - 1} P_{vk} x_{vk}^t \leq L^t, \ \forall t \in T, \tag{2}$$

$$x_{vk}^t + y_{vk}^t = y_{vk}^{t-1} + x_{v(k-1)}^{k-1}, \ \forall v \in V, \ t \in T(v) \setminus \left\{ \tau_v^S, \tau_v^D \right\}, \ k = 1, \ldots, \kappa_v, \tag{3}$$

$$x_{v(k=1)}^{\left(t=\tau_v^S\right)} + y_{v(k=1)}^{\left(t=\tau_v^S\right)} = 1, \ \forall v \in V, \tag{4}$$

$$x_{v(k=\kappa_v)}^{\left(t=\tau_v^D - 1\right)} + y_{v(k=\kappa_v - 1)}^{\left(t=\tau_v^D - 1\right)} = 1, \ \forall v \in V, \tag{5}$$

$$x_{vk}^t, \ y_{vk}^t \in \{0, 1\} \tag{6}$$

Objective (1) minimizes the charging costs over a planning horizon, where P_{vk} represents the charging power of EV v when it is charged from SOC level k. Constraints (2) enforce maximum limits on the total charging power at each time period. Constraints (3)-(5) describe flow conservation constraints at each node in the transition networks. In particular, constraints (4) and (5) are flow conservation constraints defined at the source and sink nodes, respectively. Finally, constraints (6) enforce the integrality of variables.

The formulation (P) can be large and impractical when the number of EVs is large. To overcome this, we propose the formulation (P′) based on the variable representing a path in the transition networks. In the next section, we will develop an algorithm for solving the problem via the formulation (P′). Recall that a feasible solution of the set of constraints (3)-(6) for EV v corresponds to a path from a node $\left(\tau_v^S, 1\right)$ to a node $\left(\tau_v^D, \kappa_v\right)$ in the transition network of EV v. Let K(v) denote the set of all feasible paths of EV v. Then (P) can be reformulated using path variables as the following problem.

$$\text{(P}'\text{)} \ \min \sum_{v \in V} \sum_{k \in K(v)} C_v^k r_v^k \tag{7}$$

$$\text{s.t.} \ \sum_{v \in V} \sum_{k \in K(v)} P_v^{kt} r_v^k \leq L^t, \ \forall t \in T, \tag{8}$$

$$\sum_{k \in K(v)} r_v^k \geq 1, \ \forall v \in V, \tag{9}$$

$$r_v^k \in \{0, 1\}, \tag{10}$$

where the binary variable r_v^k takes value 1 if feasible path k of EV v is selected, 0 otherwise. In the objective (7), C_v^k is the total cost needed to charge EV v along the path k. In constraints (8), P_v^{kt} is the charging power of EV v at time period t when the charging schedule follows path k. Constraints (9) ensure that at least one path must be selected for each EV v. Finally, integrality of path variables is imposed by constraints (10).

4. **Solution Method.** In this section, we propose the algorithm for solving the EV charging scheduling problem based on the formulation (P′). Note that the formulation (P) can be solved using MIP solvers. However, the formulation remains computationally intractable for large-scale instances as the number of variables and constraints depends on the size of transition networks. The formulation (P′) can also have too many variables

because each variable corresponds to a feasible path in the transition network and the number of paths can be very large. However, (P′) is more suitable to apply decomposition approach as the formulation can be easily decomposed into the problem for path generation and the problem for coordinating generated paths. In the following part, we first show how to solve the LP relaxation of (P′) using column generation. Then, we introduce a simple heuristic algorithm to find an integer solution based on the variables generated during the column generation procedure.

The idea of the column generation algorithm that we propose is, instead of solving the LP relaxation of (P′) directly, to repeatedly solve a restricted master problem that includes only a subset of the columns and a pricing problem that generates new columns to be added to the restricted master problem. Let $K'(v)$ be the set of feasible paths of EV v generated up to a certain iteration of the algorithm. The restricted master problem (RM) can then be expressed as:

$$\text{(RM)} \ \min \sum_{v\in V}\sum_{k\in K'(v)} C_v^k r_v^k \tag{11}$$

$$\text{s.t.} \ \sum_{v\in V}\sum_{k\in K'(v)} P_v^{kt} r_v^k \le L^t, \ \forall t\in T, \tag{12}$$

$$\sum_{k\in K'(v)} r_v^k \ge 1, \ \forall v\in V, \tag{13}$$

$$0 \le r_v^k \le 1. \tag{14}$$

Let γ_t and μ_v be the dual variables associated with constraints (12) and (13) respectively. After solving the restricted master problem, we check whether there are any paths with a negative reduced cost that are not included in the restricted master problem. If such paths exist, they are added to (RM), and the process is repeated; otherwise, the current optimal solution to (RM) is optimal for the LP relaxation of (P′) and therefore the algorithm terminates.

In order to determine paths with negative reduced cost, denote as $(\bar{\gamma}_t, \bar{\mu}_v)$ the optimal dual values for the current optimal solution of (RM). The reduced cost of a candidate path k of EV v is:

$$C_v^k - \sum_{t=1}^{T} P_v^{kt}\bar{\gamma}_t - \bar{\mu}_v.$$

We therefore solve the following pricing problem for EV v in order to generate new paths with negative reduced cost:

$$\min \sum_{e\in X_k} c_e^X$$
$$\text{s.t.} \ \ k\in K(v),$$

where X_k is the set of charging transitions contained in the path k of EV v, and $c_e^X = (M^t - \bar{\gamma}_t)\,P_{vl}^t$, where l and t are uniquely defined from transition $e:(t,l)\to(t+1,l+1)$. For each EV v, this pricing problem is the shortest path problem from the source node $\left(\tau_v^S, 1\right)$ to the sink node $\left(\tau_v^D, \kappa_v\right)$ over the transition network of EV v with positive costs on transitions; therefore, it can be solved rapidly using dynamic programming algorithm. If the optimal cost is less than $\bar{\mu}_v$, a new path can be added to the restricted master. We have solved the LP relaxation of (P′); hence the solution obtained can have fractional values. However, we must find an integer solution to obtain a feasible charging schedule of all EVs. To do this, we solve the final restricted master problem with binary restrictions on path variable r_v^k, after the LP relaxation has been solved using column generation. Note that this is equivalent to solving (P′) with only a subset of path variables that was generated during the column generation process. This heuristic looks simple, but works very well, which will be demonstrated via computational experiments in the next section.

5. **Computational Results.** In this section, we present the results of our computational experiments. All tests were performed on a 2.4-Ghz Intel Core i5 processor with 8GB RAM. We solved the restricted master problems using Cplex 12.6 with default parameter settings.

Simulation settings are as follows. A planning horizon of 17 hours is considered, and each time period is given as 1 hour. The hourly electricity price is randomly chosen in the range of $[5, 15]$. We assume that a battery of 25kWh is used for all EVs and it is charged with a rate of 3.5kW for the first 6 hours, 3kW and 1kW for the next 2 hours. The initial battery level is randomly chosen in the range of $[1, 16]$ kW, and all EVs are fully charged once they start charging. We assume that the arrival time of each EV follows a normal distribution with a mean of 2h and a standard deviation of 1h, and the departing time follows a normal distribution with a mean of 15h and a standard deviation of 1h. We solved the problem for different number of EVs (from 100 to 1,000) and the maximum hourly charging load is given as 1.5 times the number of EVs.

We first present the computational performance of our column generation based heuristic algorithm for solving the preemptive charging problem. Table 1 shows computational times needed to solve the problem. We can see that our algorithm performs well to the extent that the problem with 1,000 EVs can be solved within 10 seconds. Note that the column generation procedure runs very fast compared to the MIP heuristic (solving the final restricted master problem with binary variables), and MIP heuristic becomes a bottleneck as the number of EVs increases.

Table 1. Computational times (in seconds)

#EV	MIP heuristic	cg
100	0.14	0.24
300	1.02	0.32
500	2.53	0.35
1,000	8.8	0.97

We next compare total charging costs of two charging schemes (preemptive charging and non-preemptive charging). The total charging cost of preemptive charging is obtained from our heuristic algorithm and that of non-preemptive charging can be obtained by solving a simple mixed integer programming problem [8]. Table 2 shows the results. As expected, the preemptive charging yields less charging cost than non-preemptive charging with the help of interruptions during the charging process. The cost reduction ranges from 5.5% to 8.9%. We can benefit from such interruptions as they enable charging load to be distributed in such a way that we increase charging amount as much as possible when electricity price is low.

Table 2. Comparison of charging costs (non-preemptive vs. preemptive)

#EV	non-preemptive	preemptive
100	15,961	14,655
300	45,765	41,662
500	77,937	73,478
1,000	155,013	146,442

6. **Conclusions.** In this paper, we considered the scheduling of EV charging under preemptive charging scheme, which can increase the flexibility in charging control since it allows interruptions during the charging process. We proposed an efficient decomposition algorithm for solving a mathematical formulation of the preemptive charging scheduling problem. Our computational experiments demonstrated that the proposed algorithm performed well on large-sized instance and therefore can be used in practice. Moreover, we showed that the charging costs can be reduced significantly by introducing preemptive charging scheme. Future research may include the investigation of decentralized optimization of EV charging schedules. The centralized optimization performed by the aggregator considered in this paper may have limitations since it can be difficult to obtain all the private information of the EVs. In this case, EVs can collaborate with the aggregator and decide their charging schedule in a decentralized fashion. Such a collaborative optimization strategy is worth investigating.

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