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AN ACTUATOR FAILURE RELIABLE TRACKING CONTROL FOR OMNIDIRECTIONAL REHABILITATIVE TRAINING WALKER

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Abstract. *In this study, a new nonlinear reliable control method with a redundant degree of freedom against actuator fault is proposed for an omnidirectional walker. The redundant input model with a uniform actuator fault is constructed by separating the corresponding columns of the control matrix. The robust adaptive control method is presented to deal with the separated term related to the extrinsic bounded interference of the system. A controller with fixed gains and variable gains is designed to maintain the stability of the tracking error system. An adjustable control law is designed and a sufficient condition of system stability is attained. Simulation results confirm the effectiveness of the proposed method.*
Keywords: Omnidirectional walker, Reliable control, Redundant degree of freedom, Actuator failure

1. **Introduction.** Rehabilitative robot control has attracted much attention recently. However, actuator failures can suddenly occur during motion and increase the difficulty of tracking control. In fact, control systems during actuator failure can produce severe performance deterioration or even catastrophic effects [1]. To ensure system reliability, many modern control applications, particularly in robotic systems, have redundant actuators [2]. In our previous study, we discussed a control method using a redundant degree of freedom for the problem of fault tolerance when an actuator of a rehabilitation robot has an outage failure [3]. Unfortunately, no related research has been reported on designing a reliable controller that compensates for an actuator outage, stuck faults, and loss of effectiveness by using a redundant degree of freedom.

On the other hand, robust adaptive techniques have been widely used to design controllers for nonlinear systems [4,5]. It should be stressed that these techniques use the same controller with fixed gain throughout normal and fault cases. As the number of possible failures and the degree of system redundancy increase, the controller with fixed gain becomes more conservative and the attainable control performance indexes might not be satisfactory [6]. In [7], a fixed-gain robust controller was designed for an omnidirectional rehabilitative training walker, but only limited faults could be handled. Hence, an adjustable control law is desirable because it can maintain a more satisfactory and robust performance than controllers with fixed gain [8,9]. Failed actuators do not provide the specified control input, so the design of a controller with adaptive variable gains is very important.

This paper presents a new fault-tolerant control scheme for rehabilitative robots. Considering theoretical studies and engineering applications, there are a number of problems to be a worthwhile endeavor. In the present paper, we will investigate an omnidirectional rehabilitative training walker (ODW) [10]. The main contributions of this paper are summarized as follows.

(i) A redundant input model of the ODW with a uniform actuator fault is constructed by separating corresponding columns of the control matrix.
(ii) A robust adaptive control method is proposed to deal with the separated term related to the extrinsic bounded interference. A controller with fixed gains and variable gains is designed to maintain the stability of the tracking error system.
(iii) As an application, reliable tracking control with a redundant degree of freedom for the ODW is considered. The efficiency of the proposed scheme is demonstrated.

The remainder of this paper is organized as follows. In Section 2, a redundant input system with uniform actuator fault is formulated. The main results that provide a solution to the reliable robust adaptive tracking control problem are presented in Section 3. Simulation results are given in Section 4, and concluding remarks are provided in Section 5.

2. **Dynamic Model of the Omnidirectional Walker.** Figure 1 shows the structure of the ODW with four omniwheels, and Figure 2 shows the coordinate system and parameters used to develop the ODW model.

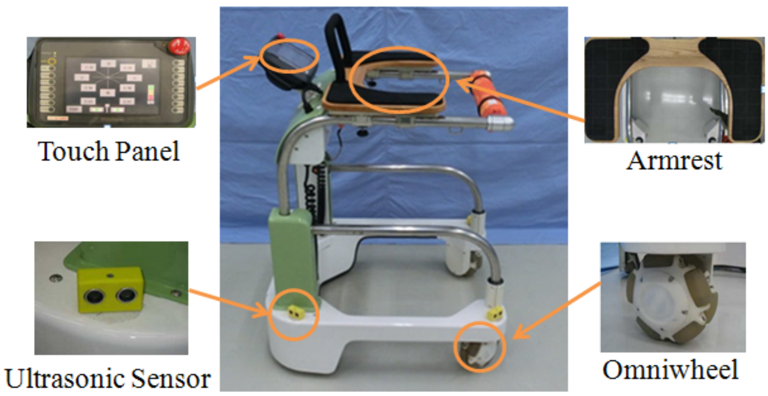

FIGURE 1. Structure of the ODW

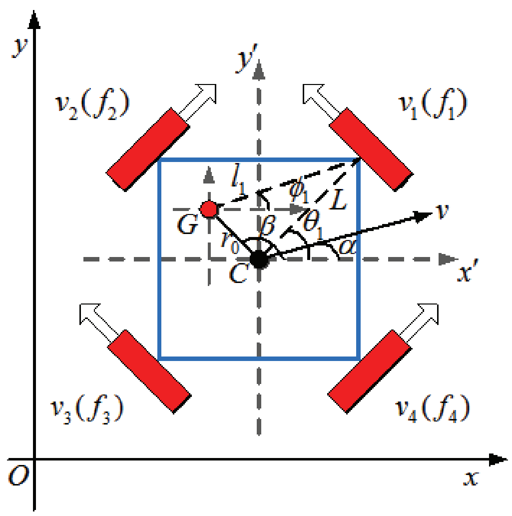

FIGURE 2. ODW coordinate system

The nomenclature used in Figure 2 is as follows.
$\Sigma(x, O, y)$: Absolute coordinate system
$\Sigma(x', G, y')$: Translation coordinate system
v: Speed of the ODW
v_i: Speed of an omnidirectional wheel, $i = 1, 2, 3, 4$
f_i: Force on each omnidirectional wheel
L: Distance from the center of gravity of the walker to each omnidirectional wheel
α: Angle between the x' axis and the direction of v
β: Angle between the x' axis and r_0
θ_i: Angle between the x' axis and the position of each omnidirectional wheel
l_i: Distance from the center of gravity to the middle of each omnidirectional wheel
ϕ_i: Angle between the x' axis and l_i
G: Center of gravity of the walker
r_0: Distance between G and the center of gravity due to the load

Based on [11], the dynamic model is expressed as follows:

$$M_0 K(\theta)\ddot{X}(t) + M_0 \dot{K}\left(\theta, \dot{\theta}\right)\dot{X}(t) = B(\theta)u(t) \tag{1}$$

where

$$M_0 = \begin{bmatrix} M+m & 0 & 0 \\ 0 & M+m & 0 \\ 0 & 0 & I_0 + mr_0^2 \end{bmatrix}, \quad K(\theta) = \begin{bmatrix} 1 & 0 & p \\ 0 & 1 & q \\ 0 & 0 & 1 \end{bmatrix}, \quad X(t) = \begin{bmatrix} x(t) \\ y(t) \\ \theta(t) \end{bmatrix},$$

$$B(\theta) = \begin{bmatrix} -\sin\theta_1 & \sin\theta_2 & \sin\theta_3 & -\sin\theta_4 \\ \cos\theta_1 & -\cos\theta_2 & \cos\theta_3 & \cos\theta_4 \\ \lambda_1 & -\lambda_2 & -\lambda_3 & \lambda_4 \end{bmatrix}, \quad \begin{array}{c} \lambda_1 = l_1\cos(\theta_1 - \phi_1) \\ \lambda_2 = l_2\cos(\theta_2 - \phi_2) \\ \lambda_3 = l_3\cos(\theta_3 - \phi_3) \\ \lambda_4 = l_4\cos(\theta_4 - \phi_4) \end{array},$$

$$\begin{array}{c} p = \dfrac{1}{2}[(\lambda_1 - \lambda_3)\sin\theta + (\lambda_2 - \lambda_4)\cos\theta] \\ q = \dfrac{1}{2}[(\lambda_2 - \lambda_4)\sin\theta - (\lambda_1 - \lambda_3)\cos\theta] \end{array}, \quad u(t) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}.$$

Here, M is the mass of the ODW, m is the user's equivalent mass, I_0 is the inertia of the walker's mass, and mr_0^2 is the inertia of mass caused by m. Moreover, f_1, f_2, f_3, and f_4 are the input forces of the system, and θ is the angle between the x' axis and the position of the first omniwheel, $\theta = \theta_1$. We then have $\theta_2 = \theta + \frac{\pi}{2}$, $\theta_3 = \theta + \pi$, and $\theta_4 = \theta + \frac{3\pi}{2}$. As can be seen from the system of differential Equation (1), although four control input forces are found (f_1, f_2, f_3, and f_4), only three are independent. This implies that the walker has a redundant degree of freedom.

The problem of tracking control in the case of actuator failure that might occur in the control channels is considered. To formulate the reliable control problem, the following actuator fault model is adopted.

$$u^F(t) = \left(I - \rho^F\right)u(t) = \left[\begin{array}{cc} u_0(t) & (I-\rho)u_i(t) \end{array}\right]^T \tag{2}$$

where $\rho^F = diag\left\{\rho_1^j, \rho_2^j, \ldots, \rho_m^j\right\}$. When $\rho_i^j = 0$, there is no fault for the ith actuator in the jth fault mode; when $\rho_i^j = 1$, the ith actuator is outage in the jth fault mode; when $0 < \rho_i^j < 1$, in the jth fault mode the type of actuator faults is loss of effectiveness; and ρ is a diagonal matrix that is described by $\rho_i^j \neq 0$, $i = 1, 2, \ldots, m$.

As reported in [12], the input of the fault actuator is separated from Equation (1). Thus, the walker redundant input model is described as follows:

$$M_0 K(\theta)\ddot{X}(t) + M_0 \dot{K}\left(\theta, \dot{\theta}\right)\dot{X}(t) = B_0(\theta)u_0(t) + B_i(\theta)(I-\rho)u_i(t) \tag{3}$$

3. **Robust Adaptive Controller Design.** The adaptive controller with fixed gains and variable gains of the system (3) is designed as

$$u_0(t) = B_0^{-1}(\theta)\left[(M_0K(\theta))\left(\ddot{X}_d(t) + \left(K_d + \widehat{\rho}(t)K_{di}\right)\dot{e}(t) + \left(K_p + \widehat{\rho}(t)K_{pi}\right)e(t)\right) + M_0\dot{K}\left(\theta,\dot{\theta}\right)\dot{X}(t)\right] \tag{4}$$

where $\widehat{\rho}(t)$ is the estimation of ρ, and $\tilde{\rho}(t) = \widehat{\rho}(t) - \rho$. Combining (3) with (4), the error state equation is

$$\ddot{e}(t) + \left(K_d + \widehat{\rho}(t)K_{di}\right)\dot{e}(t) + \left(K_p + \widehat{\rho}(t)K_{pi}\right)e(t) = D\omega(t) \tag{5}$$

where $D = (M_0K)^{-1}B_i(\theta)$, $\omega(t) = -(I-\rho)u_i(t)$.

Before solving the robust adaptive controller, some preparations are needed. A separating method for D is given as follows.

$$D = (M_0K)^{-1}B_i(\theta) = \begin{bmatrix} \dfrac{b_{i1}}{M+m} - \dfrac{pb_{i3}}{I_0+mr_0^2} \\ \dfrac{b_{i2}}{M+m} - \dfrac{qb_{i3}}{I_0+mr_0^2} \\ \dfrac{b_{i3}}{I_0+mr_0^2} \end{bmatrix} = D_1 - D_2 \tag{6}$$

where

$$B_i(\theta) = \begin{bmatrix} b_{i1} & b_{i2} & b_{i3} \end{bmatrix}^T,\ D_1 = \begin{bmatrix} \dfrac{b_{i1}}{M+m} & \dfrac{b_{i2}}{M+m} & 0 \end{bmatrix}^T,$$

and

$$D_2 = \begin{bmatrix} \dfrac{pb_{i3}}{I_0+mr_0^2} & \dfrac{qb_{i3}}{I_0+mr_0^2} & -\dfrac{b_{i3}}{I_0+mr_0^2} \end{bmatrix}^T.$$

Also,

$$D_1 = H_1F_1(t)E_1 \tag{7}$$

$$D_2 = H_2F_2(t)E_2 \tag{8}$$

where $F_1^T(t)F_1(t) \le I$ and $F_2^T(t)F_2(t) \le I$; $E_1^TE_1 = c_1$ and $E_2^TE_2 = c_2$, with c_1, c_2 known constants such that $c_1 < c_2$; H_1, H_2 are known constant matrices.

Theorem 3.1. *Considering the error state Equation (5), suppose that symmetric matrices $P > 0$, $T > 0$ exist such that the following linear matrix inequalities (LMIs) hold:*

$$\begin{bmatrix} -TK_d & P - TK_p \\ 0 & 0 \end{bmatrix} \le 0 \tag{9}$$

$$\begin{bmatrix} -\rho TK_{di} & TH_1 & -\rho TK_{pi} \\ H_1^TT & -2I & 0 \\ 0 & 0 & 0 \end{bmatrix} \le 0 \tag{10}$$

Then, the control input (4) and adaptive law $\dot{\widehat{\rho}}(t)$ defined by

$$\dot{\widehat{\rho}}(t) = l\left(\dot{e}^T(t)TK_{di}\dot{e}(t) + \dot{e}^T(t)TK_{pi}e(t)\right) \tag{11}$$

solve the problem of asymptotically stable and reliable robust adaptive tracking for an actuator failure.

Proof: Define the Lyapunov function

$$V(t) = \frac{1}{2}e^T(t)Pe(t) + \frac{1}{2}\dot{e}^T(t)T\dot{e}(t) + \frac{1}{2l}\tilde{\rho}^2(t)$$

The time derivative of $V(t)$ along the trajectory of system (5) is given by

$$\begin{aligned}\dot{V}(t) &= \dot{e}^T(t)Pe(t) + \dot{e}^T(t)T\ddot{e}(t) + \frac{\tilde{\rho}(t)\dot{\tilde{\rho}}(t)}{l} \\ &= \dot{e}^T(t)Pe(t) - \dot{e}^T(t)TK_d\dot{e}(t) - \dot{e}^T(t)TK_pe(t) - \rho\dot{e}^T(t)TK_{di}\dot{e}(t) - \rho\dot{e}^T(t)TK_{pi}e(t) \\ &\quad -\tilde{\rho}(t)\dot{e}^T(t)TK_{di}\dot{e}(t) - \tilde{\rho}(t)\dot{e}^T(t)TK_{pi}e(t) + \dot{e}^T(t)TD\omega(t) + \frac{\tilde{\rho}(t)\dot{\widehat{\rho}}(t)}{l}\end{aligned}$$

Choosing the adaptive law as (11), we have

$$\begin{aligned}\dot{V}(t) \leq& \dot{e}^T(t)Pe(t) - \dot{e}^T(t)TK_d\dot{e}(t) - \dot{e}^T(t)TK_pe(t) - \rho\dot{e}^T(t)TK_{di}\dot{e}(t) \\ &-\rho\dot{e}^T(t)TK_{pi}e(t) + \dot{e}^T(t)TD\omega(t)\end{aligned}$$

From (6), (7), and (8), it follows that

$$\begin{aligned}\dot{V}(t) &= \dot{e}^T(t)Pe(t) - \dot{e}^TTK_d\dot{e}(t) - \dot{e}^T(t)TK_pe(t) - \rho\dot{e}^T(t)TK_{di}\dot{e}(t) - \rho\dot{e}^TTK_{pi}e(t) \\ &\quad +\frac{1}{2}\dot{e}^TTH_1H_1^TT\dot{e}(t) - \frac{1}{2}\dot{e}^T(t)TH_2H_2T\dot{e}(t) + \frac{c_1-c_2}{2}\omega^T(t)\omega(t) \\ &= \begin{bmatrix} \dot{e}^T(t) & e^T(t) \end{bmatrix} \begin{bmatrix} -TK_d & P-TK_p \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{e}(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} \dot{e}^T(t) & e^T(t) \end{bmatrix} \Theta_0 \begin{bmatrix} \dot{e}(t) \\ e(t) \end{bmatrix} \\ &\quad -\frac{c_2-c_1}{2}\omega^T(t)\omega(t)\end{aligned}$$

where

$$\Theta_0 = \begin{bmatrix} -\rho TK_{di} + \frac{1}{2}TH_1H_1^TT & -\rho TK_{pi} \\ 0 & 0 \end{bmatrix}$$

According to (9), (10), and the Schur complement lemma, which further implies that $\dot{V}(t) < 0$, the error state Equation (5) is asymptotically stable.

Remark 3.1. *For system (3), when $\rho = 0$, which is a normal case, the controller is chosen as*

$$u_0(t) = B^T(\theta)\left[B(\theta)B^T(\theta)\right]^{-1}\left[(M_0K)\left(\ddot{X}_d + K_d\dot{e}(t) + K_pe(t)\right) + M_0\dot{K}\dot{X}\right]$$

and the error state equation is

$$\ddot{e}(t) + K_d\dot{e}(t) + K_pe(t) = 0 \tag{12}$$

The proper choice of controller parameters K_d and K_p implies that $e(t)$ and $\dot{e}(t)$ of system (12) go to zero asymptotically. The existence of K_d and K_p can be examined according to the LMI of (9). It is easy to see that the design of controller (4) includes the normal and the fault cases. The aforementioned fact shows that the design condition for the reliable tracking controller in Theorem 3.1 is more relaxed than the condition for the traditional fault-tolerant controller design with fixed gains.

4. **Simulation Results.** Consider actuators that could fail during the course of system operation. Suppose that for any t, only one actuator fails. Without loss of generality, we consider the fourth actuator to be the fault. Therefore, $X_d(t)$ is described as follows:

$$\begin{aligned}x_d(t) &= 20\left(1-e^{-0.2t}\right) \\ y_d(t) &= 20\left(1-e^{-0.2t}\right) \\ \theta_d(t) &= \frac{\pi}{2}\end{aligned}$$

The physical parameters of the ODW used in the simulation are $M = 58$kg, $L = 0.4$m, $I_0 = 27.7$kg· m^2, trainer load $m = 60$kg, center of gravity shift $r_0 = 0.1$m and $\beta = \frac{\pi}{4}$rad.

We suppose that all of the actuators are normal at the beginning 100s, and then the fourth actuator is stuck, i.e., input force $f_4 = 10.0$N. The controller parameters are

$K_p = diag\{100, 80, 100\}$ and $K_d = diag\{180, 190, 180\}$. After 100s, the added control matrices are $K_{pi} = diag\{20, 0.2, 5\}$ and $K_{di} = diag\{50, 50, 100\}$. The simulation results are given in Figures 3-8.

Figures 3, 4, and 5 plot the trajectories of the x and y positions and the orientation angle, respectively. The path tracking of the line is shown in Figure 6. The error state equation realizes asymptotic stability, although the fourth actuator becomes a fault after 100s. The input force of the fourth actuator is shown in Figure 7. The ODW can achieve safe continuous smooth tracking, and the tracking error converges near zero within a small period of time, as shown in Figure 8.

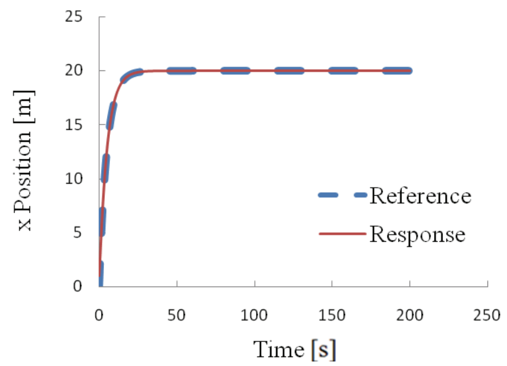

FIGURE 3. Tracking performance of x position

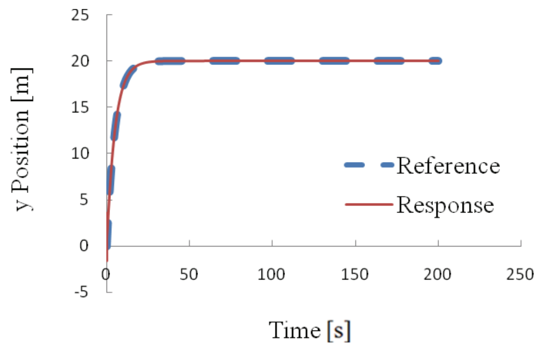

FIGURE 4. Tracking performance of y position

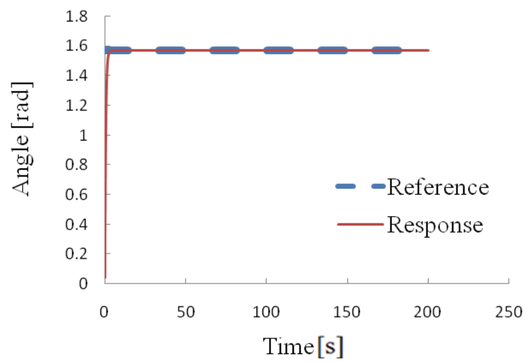

FIGURE 5. Tracking performance of angle

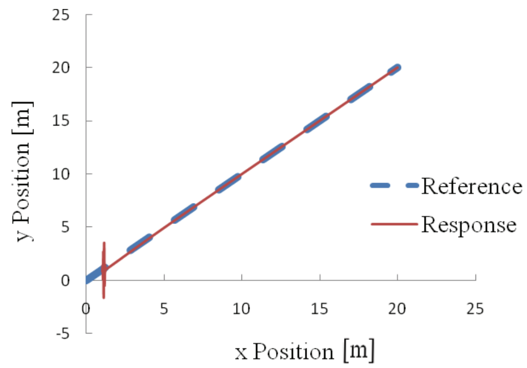

FIGURE 6. Path tracking of line

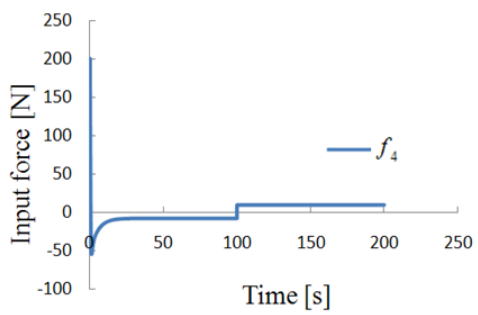

FIGURE 7. Input force

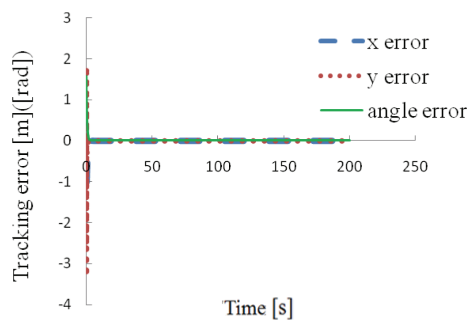

FIGURE 8. Trajectory tracking error

To verify the effectiveness of reliable control, we provide figures with the fourth actuator outage fault. The controller parameters are chosen as $K_{pi} = diag\{10, 0.1, 10\}$ and $K_{di} = diag\{10, 10, 10\}$ on the basis of K_p and K_d. The simulation results are given in the following.

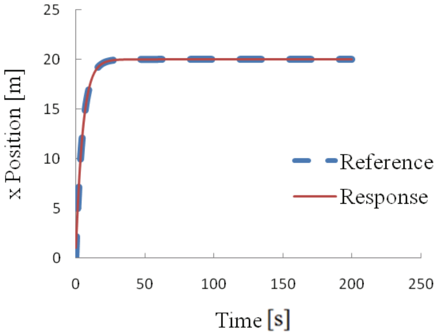

FIGURE 9. Trajectory tracking of x position

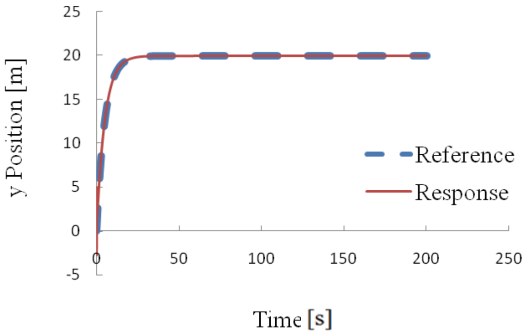

FIGURE 10. Trajectory tracking of y position

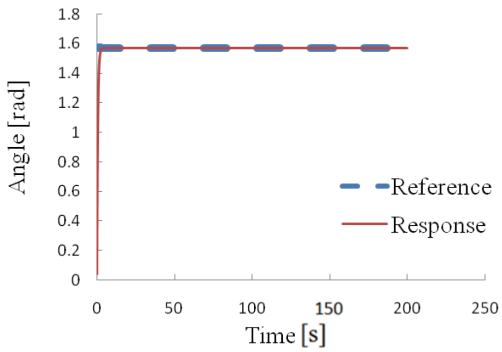

FIGURE 11. Trajectory tracking of angle

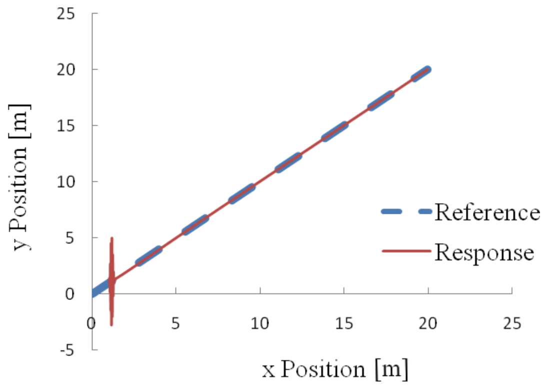

FIGURE 12. Path tracking of line

Figures 9, 10, and 11 plot the tracking performance of the ODW for the x position, y position, and orientation angle, respectively. Figure 12 plots the path tracking of line. It is evident that the y position has some tracking errors in initial time when the fourth actuator is outage, which results in the path tracking of line emerging oscillation at the beginning time as in Figure 12. After a little adjusting time, the ODW can realize asymptotic trajectory tracking. These simulation results demonstrate that the controller (4) is effective when one actuator is at fault. Thus, three non-fault functioning actuators can maintain the programmed sequential motion of the ODW that normally operates with four actuators.

Next, the comparative simulation is conducted with [13]. We use the feedback linearization method in [13] when the fourth actuator is loss of effectiveness of $\rho = 0.8$. The results of the simulation are presented in Figures 13-16.

As shown in Figures 13 and 14, the ODW cannot track training trajectory of x and y positions. Orientation angle can be realized trajectory tracking as in Figure 15. The ODW shows trajectory tracking error as in Figure 16. All of the actuators are normal, and the trajectory tracking can be finished using the method in [13]. However, the trajectory tracking cannot be accomplished when one of actuators is failure. Thus, the reliable control method can guarantee the walker's continuous safe motion when one actuator fails. The ODW can rely on the remaining three functioning health actuators to maintain the training sequence.

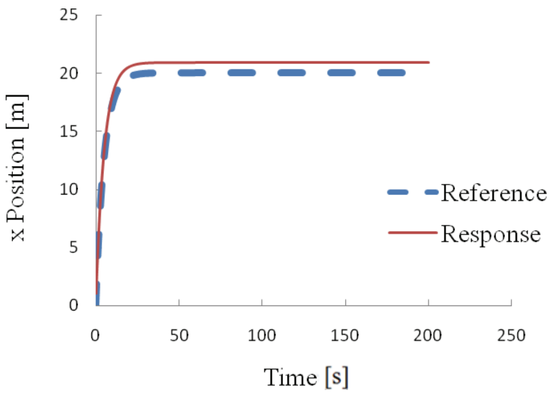

Figure 13. Trajectory tracking of x position

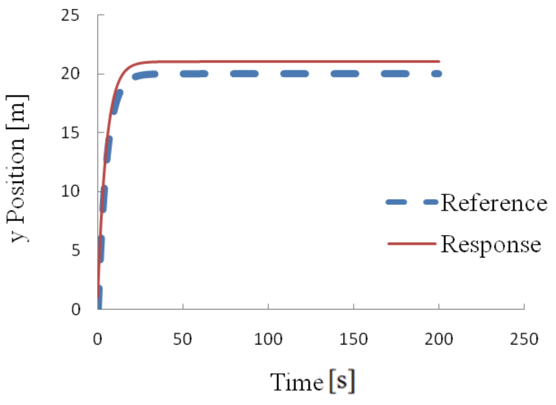

Figure 14. Trajectory tracking of y position

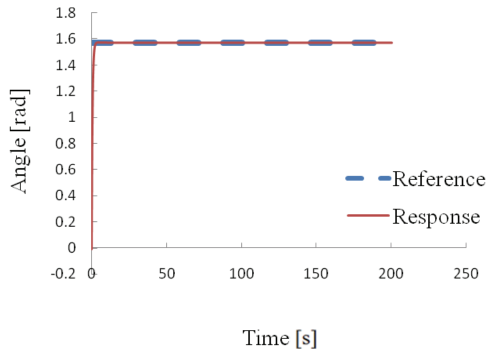

Figure 15. Trajectory tracking of angle

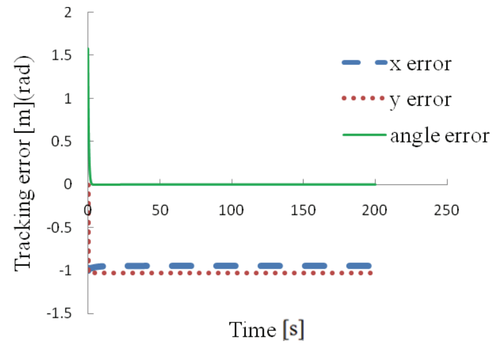

Figure 16. Trajectory tracking error

5. Conclusion.

Reliable control with a nonlinear redundant degree of freedom against actuator fault is proposed for the ODW. By using the common Lyapunov function, the obtained controller can stabilize the walker. The tracking results are consistent with a pre-programmed training path designed by a medical professional. The simulation results for a new synthesis design to resolve the safety issues associated with actuator failures demonstrate the effectiveness of the proposed method. It is probable that, in addition to the ODW, the proposed method can also be applied to other wheeled rehabilitation robots. In the future, we will use the proposed reliable controller to deal with velocity constraints for rehabilitation robots in order to guarantee user safety.

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