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LOW SENSITIVITY CONTROL FOR MINIMUM-PHASE SYSTEMS USING DOUBLE FEEDBACK CONTROL

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Abstract. *In this paper, we examine a design method for low-sensitivity control with robust stability using two-degree-of-freedom control and double feedback control. When the system has low-sensitivity characteristic, the steady state error for the reference input and influence of the disturbance for the output become small. It is said that the low-sensitivity control has bad robust stability characteristic. However, according to Yamada, for the certain class of uncertainty, low-sensitivity control makes the system robustly stable. In this paper, we expand the result by Yamada and propose a design method of low-sensitivity control system using two-degree-of-freedom control and double feedback control.*
Keywords: Minimum-phase system, Low-sensitivity control, Sensitivity function, Robust stability

1. **Introduction.** In this paper, we examine a design method that provides low-sensitivity control systems with robust stability for single-input and single-output continuous time-invariant systems. Several studies have been conducted on the robust stabilization problem [1, 2, 3, 4, 5, 6, 7]. Doyle and Stein derive the basic solution for this problem [1], and the necessary and sufficient conditions for the multiplicative uncertainty and the additive uncertainty are shown. Chen and Desoer derive the complete proof of the solution presented by Doyle and Stein [2].

Kimura considers the robust stabilizability problem for single-input and single-output systems [8]. Vidyasagar and Kimura expand the findings of Kimura [8] for multiple-input and multiple-output systems [9].

According to [1, 2, 3], in order to keep the stability for the large uncertainty, the complementary sensitivity function must be small value. To make the complementary sensitivity function small, brings the control systems lower performance in the meaning of disturbance attenuation property and so on. Therefore, we must make the sensitivity function small to produce the control system with high disturbance attenuation property. Since the sum of the sensitivity function and the complementary function is equal to 1, we cannot obtain either low sensitivity or high robust stability characteristics, although low sensitivity does not always make the system unstable. Maeda and Vidyasagar consider this problem as an infinite gain margin one [10, 11]. Nogami et al. clarify the condition that high-gain controller does not make the system unstable and also propose a design method [12]. Doyle et al. consider this low sensitivity control problem from another view point; there exists a class of uncertainty that has the property low sensitivity making the system robustly stable [14]. Therefore, if the uncertainty is described in [14], we can construct low sensitivity characteristics with robust stability. In this meaning, the uncertainty in [14] is suitable for high-performance robust control system design. However, this uncertainty cannot be applied to a system having a varying number of open right half plane poles. There exist applications such that the number of unstable poles changes. For example, the number of right half plane poles of a large flexible spacecraft changes when the configuration of the spacecraft is changed [9]. The problem of obtaining the robust stability condition for the system having a varying number of the closed right half plane poles is difficult because the problem does not reduce to the small gain theorem. The reference in [19] tackles and solves this problem using phase information [19].

In order to design the low-sensitivity control system, the control structure is important. The internal model control structure [20] is known as an effective control structure for low-sensitivity control. However, internal model control cannot apply for unstable systems. Zhou and Ren overcome this problem and propose a new control structure named generalized internal model control (GIMC) [21]. GIMC control structures are applied to several applications [22, 23, 24, 25]. References in [22, 23, 24, 25] considers the fault-tolerant control using GIMC structure. However, no paper considers the low-sensitivity control.

In this paper, we expand the result in [19] and propose a design method of two-degree-of-freedom control system for the minimum-phase systems having varying number of unstable poles such that the influence of the uncertainty to the output is reduced. That system can be build using low-sensitivity controller. In addition, we present a design method of double two-degree-of-freedom control system such that the influence of the uncertainty to the output is reduced compared with conventional two-degree of freedom control system.

Notations

R	the set of real numbers.
C	the set of complex numbers.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real rational functions.
$\|\cdot\|_\infty$	H_∞ norm.
$\Re\{\cdot\}$	real part of $\{\cdot\}$.

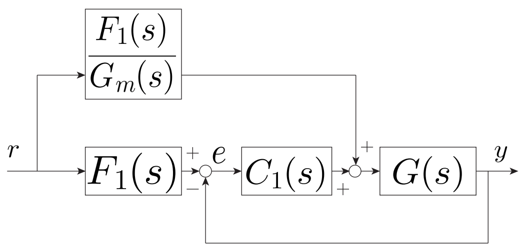

Figure 1. Two-degree-of-freedom control system

2. **Problem Formulation.** Consider the two-degree-of-freedom control system in Figure 1. Here, $G(s) \in R(s)$ is a strictly proper plant. $G(s)$ is assumed to have no zero in the closed right half plane. $C_1(s) \in R(s)$ is the feedback controller, $F_1(s) \in RH_\infty$ is the feed forward controller, r is the reference input and y is the output. The nominal plant of $G(s)$ denotes $G_m(s) \in R(s)$. Here, $G_m(s)$ is also assumed to have no zero in the closed right half plane. Note that the assumption that both $G(s)$ and $G_m(s)$ have no zero in the closed right half plane is required in order to achieve low-sensitivity characteristics in arbitrary frequency range [14, 15]. Using $G_m(s)$, $G(s)$ is assumed to be written by the form in

$$G(s) = G_m(s)\left(1 + \Delta(s)\right), \tag{1}$$

where $\Delta(s)$ is the uncertainty and all rational functions satisfy

$$\left|\frac{\Delta(j\omega)}{1+\Delta(j\omega)}\right| < |W(j\omega)| \quad (\forall\omega \in R), \tag{2}$$

and $W(s) \in RH_\infty$.

Note 1. *Sometimes, the reader misunderstands the assumption that the plant of minimum-phase is quite severe. There exist minimum-phase plants, such as multi-degree-of-freedom structure* [26] *and DC motor* [27]*. If the plant is of non-minimum-phase, using the parallel compensation technique* [28, 29, 30, 31, 32]*, the augmented plant including the plant and the parallel compensator will be of minimum-phase.*

Note 2. *Note that the class of uncertainty $\Delta(s)$ includes the plant having varying number of unstable poles, that is, we examine the robust control system design for the plant having varying number of unstable poles.*

Note 3. *Note that, the uncertainty $\Delta(s)$ satisfying (2) is different from the normal description of multiplicative uncertainty in* [1, 2, 3, 4]*. In order to clarify the class of $\Delta(s)$ satisfying (2), we show two numerical examples of the region of $\Delta(s)$. The first example is the region of $\Delta(s)$ satisfying*

$$\left|\frac{\Delta(j\omega)}{1+\Delta(j\omega)}\right| < 1 \quad (\forall\omega \in R). \tag{3}$$

The region of $\Delta(s)$ satisfying (3) is shown in Figure 2. Here the region of $\Delta(s)$ is located at the right hand side of $\Re\{\Delta(j\omega)\} = -1/2$, but the border is not included.

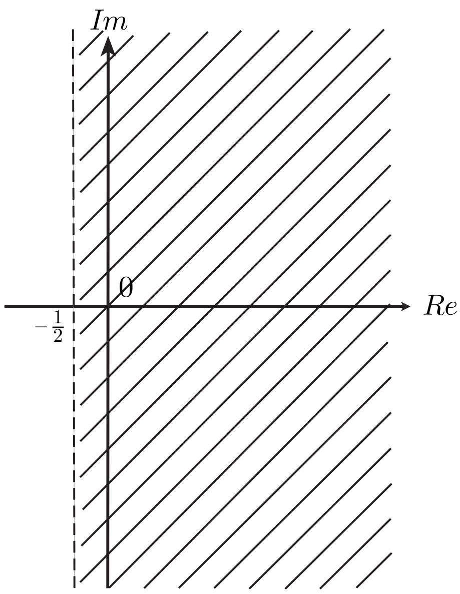

FIGURE 2. The region of $\Delta(s)$ satisfying (3)

Next we show the region of $\Delta(s)$ satisfying

$$\left|\frac{\Delta(j\omega)}{1+\Delta(j\omega)}\right| < 0.9 \quad (\forall\omega \in R). \tag{4}$$

The region of $\Delta(s)$ satisfying (4) is shown in Figure 3. Here the region of $\Delta(s)$ is located inside of the circle of which the center is $(4.26, 0)$ and the radius is 4.74, but the border is not included. Figure 3 shows that the region of $\Delta(s)$ satisfying (4) is circle. This point is similar to $\Delta(s)$ in [1, 2, 3, 4]*. However, from Figure 2, this expression holds only in the case of $|W(j\omega)| < 1$.*

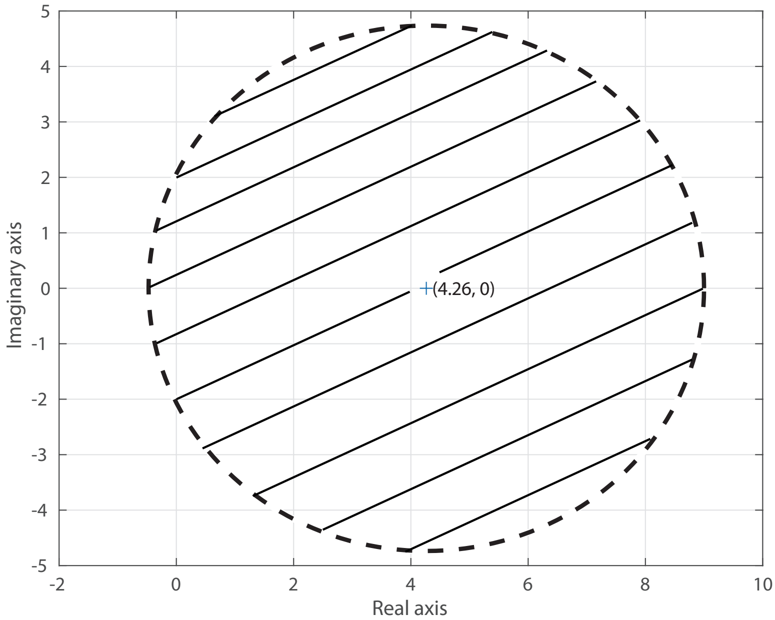

FIGURE 3. The region of $\Delta(s)$ satisfying (4)

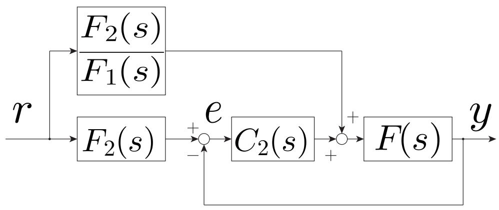

FIGURE 4. Double two-degree-of-freedom feedback control system

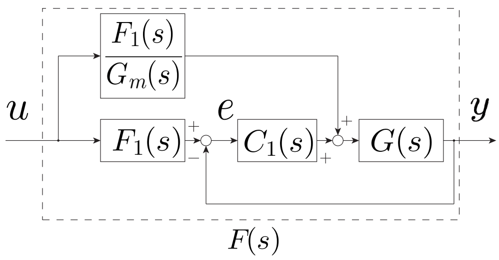

FIGURE 5. Local two-degree-of-feedback control system

The first problem is to consider a design method of two-degree-of-freedom control system in Figure 1 to reduce the influence of the uncertainty $\Delta(s)$ to the output y.

In order to reduce the influence of the uncertainty $\Delta(s)$ to the output y compared with that in Figure 1, we consider double two-degree-of-freedom control system in Figure 4. Here, $C_2(s) \in R(s)$ is the feedback controller, $F_2(s) \in RH_\infty$ is the feed forward controller, and the block diagram of $F(s)$ is shown in Figure 5. From Figure 1, Figure 4 and Figure 5, the structure in Figure 4 includes the two-degree-of-freedom control system in Figure 5. This is the reason why we call the control system in Figure 4 the double two-degree-of-freedom feedback control system. The second problem is to consider a design method of double two-degree-of-freedom control system in Figure 4 to reduce the influence of the uncertainty $\Delta(s)$ to the output y.

3. **A Design Method of Two-Degree-of-Freedom Control System.** In this section, we propose a design method of two-degree-of-freedom control system in Figure 1 to reduce the influence of the uncertainty $\Delta(s)$ to the output y.

In order to clarify the influence of the uncertainty $\Delta(s)$ to the output y in Figure 1, the transfer function from the reference input r to the output y in Figure 1 is written as the form in

$$y = F_1(s)\left(1 + H_1(s)\right) r, \tag{5}$$

where $F_1(s)$ is the transfer function from r to y in the case of $\Delta(s) = 0$ and $H_1(s)$ is the transfer function that indicates the influence of $\Delta(s)$ to the output y. That is, $H_1(s)$ is the index function that indicates the influence of $\Delta(s)$ for the output y in Figure 1.

The transfer function from r to y in Figure 1 is written by

$$\begin{aligned} y &= \frac{F_1(s)}{1 - \dfrac{1}{1 + C_1(s)G_m(s)} \dfrac{\Delta(s)}{1 + \Delta(s)}} r \\ &= F_1(s) \left(1 + \frac{S_1(s)\dfrac{\Delta(s)}{1+\Delta(s)}}{1 - S_1(s)\dfrac{\Delta(s)}{1+\Delta(s)}} \right) r \\ &= F_1(s)\left(1 + H_1(s)\right) r, \end{aligned} \tag{6}$$

where $S_1(s)$ is the sensitivity function in Figure 1 and written by

$$S_1(s) = \frac{1}{1 + G_m(s)C_1(s)} \tag{7}$$

and

$$H_1(s) = \frac{S_1(s)\dfrac{\Delta(s)}{1+\Delta(s)}}{1 - S_1(s)\dfrac{\Delta(s)}{1+\Delta(s)}}. \tag{8}$$

From (8), in order to reduce the influence of $\Delta(s)$ for y, the feedback controller $C_1(s)$ must be designed to minimize $\|S_1(s)W(s)\|_\infty$, because the upper bound of $\Delta(s)/(1 + \Delta(s))$ is $W(s)$ as (2).

Robust stability condition of the control system in Figure 1 is summarized in the next theorem.

Theorem 3.1. *Assume that $C_1(s)$ stabilizes the nominal plant $G_m(s)$ and $F_1(s)/G_m(s) \in RH_\infty$. Two-degree-of-freedom control system in Figure 1 is stable if and only if*

$$\|S_1(s)W(s)\|_\infty < 1 \tag{9}$$

holds true.

Proof of Theorem 3.1 is omitted on account of space limitation.

In this way, the result in this section is summarized as follows.

(1) In order to design the low-sensitivity control system in Figure 1, the feedback controller $C_1(s)$ needs to minimize $\|S_1(s)W(s)\|_\infty$, at worst $C_1(s)$ must satisfy

$$\|S_1(s)W(s)\|_\infty < 1.$$

(2) The number of the closed right half plane poles of $G_m(s)$ is not necessarily equal to that of $G(s)$.

4. **A Design Method of Double Two-Degree-of-Freedom Control System.** In this section, we present a design method of double two-degree-of-freedom control system in Figure 4 to reduce the influence of the uncertainty $\Delta(s)$ to the output y.

In order to clarify the influence of the uncertainty $\Delta(s)$ to the output y in Figure 4, the transfer function from the reference input r to the output y in Figure 4 is written as the form in

$$y = F_2(s)\left(1 + H_2(s)\right) r, \tag{10}$$

where $F_2(s)$ is the transfer function from r to y in Figure 4 in the case of $\Delta(s) = 0$ and $H_2(s)$ is the transfer function that indicates the influence of $\Delta(s)$ to the output y. That is, $H_2(s)$ is the index function that indicates the influence of $\Delta(s)$ for the output y in Figure 4.

The transfer function from r to y in Figure 4 is written by

$$\begin{aligned} y &= \frac{F_2(s)}{1 - S_1(s)\dfrac{\Delta(s)}{1+\Delta(s)}}\left(1 - \frac{F_1(s)C_2(s)S_1(s)\dfrac{\Delta(s)}{1+\Delta(s)}}{1 - S_1(s)\dfrac{\Delta(s)}{1+\Delta(s)} + F_1(s)C_2(s)}\right) r \\ &= F_2(s)\left(1 + \frac{S_1(s)S_2(s)\dfrac{\Delta(s)}{1+\Delta(s)}}{1 - S_1(s)S_2(s)\dfrac{\Delta(s)}{1+\Delta(s)}}\right) \\ &= F_2(s)\left(1 + H_2(s)\right) r, \end{aligned} \tag{11}$$

where $S_2(s)$ is the sensitivity function in Figure 4 and written by

$$S_2(s) = \frac{1}{1 + F_1(s)C_2(s)}, \tag{12}$$

and

$$H_2(s) = \frac{S_1(s)S_2(s)\dfrac{\Delta(s)}{1+\Delta(s)}}{1 - S_1(s)S_2(s)\dfrac{\Delta(s)}{1+\Delta(s)}}. \tag{13}$$

From this equation, when the gain of $C_2(s)$ tends to bigger, (11) is closer to

$$y = F_2(s)r. \tag{14}$$

In addition, there exists the possibility to reduce the influence of $\Delta(s)$ for the output y because that is related to not only $C_1(s)$ but also $C_2(s)$.

Robust stability condition of the control system in Figure 4 is summarized in the next theorem.

Theorem 4.1. *Assume that $C_2(s)$ stabilize $F_1(s)$ and $F_2(s)/F_1(s) \in RH_\infty$. The double feedback control system in Figure 4 is stable if and only if*

$$\|S_1(s)S_2(s)W(s)\|_\infty < 1. \tag{15}$$

Proof of Theorem 4.1 is omitted on account of space limitation.

5. **Comparison of the Influence of $\Delta(s)$.** In this section, we compare the influence of $\Delta(s)$ for the output y between the two-degree-of-freedom control system in Figure 1 and the double two-degree-of-freedom control system in Figure 4.

In order to compare the influence of $\Delta(s)$ for the output y between the two-degree-of-freedom control system in Figure 1 and the double two-degree-of-freedom control system in Figure 4, $F_1(s) = F_2(s)$ is needed. If the gain of $H_2(s)/H_1(s)$ is less than 1, then the influence of $\Delta(s)$ for the output y in Figure 4 is reduced compared with that in Figure 1. From (8) and (13), $H_2(s)/H_1(s)$ is given by

$$\begin{aligned}\frac{H_2(s)}{H_1(s)} &= \frac{S_2\left(1-S_1(s)\right)\dfrac{\Delta(s)}{1+\Delta(s)}}{1-S_1(s)S_2(s)\dfrac{\Delta(s)}{1+\Delta(s)}} \\ &= 1-\frac{1-S_2(s)}{1-S_1(s)S_2(s)\dfrac{\Delta(s)}{1+\Delta(s)}} \\ &= 1-K(s), \end{aligned} \tag{16}$$

where

$$K(s) = \frac{1-S_2(s)}{1-S_1(s)S_2(s)\dfrac{\Delta(s)}{1+\Delta(s)}}. \tag{17}$$

For the frequency range ω to satisfy $|S_2(j\omega)| \simeq 0$, $K(s)$ is close to 1 and from (16) we have

$$\left|\frac{H_2(j\omega)}{H_1(j\omega)}\right| \simeq 0. \tag{18}$$

That is, for the frequency range ω to satisfy $|S_2(j\omega)| \simeq 0$, the influence of $\Delta(s)$ for the output y in Figure 4 is reduced compared with that in Figure 1. For the frequency range $\bar{\omega}$ to satisfy $|S_2(j\bar{\omega})| \simeq 1$, $K(s)$ is close to 0 and from (16) we have

$$\left|\frac{H_2(j\bar{\omega})}{H_1(j\bar{\omega})}\right| \simeq 1. \tag{19}$$

That is, for the frequency range $\bar{\omega}$ to satisfy $|S_2(j\bar{\omega})| \simeq 1$, the influence of $\Delta(s)$ for the output y in Figure 4 is equal to that in Figure 1.

6. **Conclusion.** In this paper, we expanded the result in [19] and proposed a design method of two-degree-of-freedom control system such that the influence of the uncertainty to the output is reduced. We find that using double two-degree-of-freedom control system we can design control system such that the influence of the uncertainty to the output is reduced compared with conventional two-degree-of-freedom control system. Since the numerical examples and a design procedure are omitted on account of space limitation, we will show numerical examples and a design procedure in another article. In addition, we will expand the result in this paper and examine multiplex feedback control system to design low-sensitivity control and will clarify the control limitation of multiplex feedback control system in the future.

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