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ADAPTIVE CONTROL FOR AN OMNIDIRECTIONAL REHABILITATIVE TRAINING WALKER WITH CENTER OF GRAVITY SHIFTS AND UNKNOWN VISUAL SERVO PARAMETERS

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Abstract. *This paper investigates an adaptive nonlinear tracking control method for an omnidirectional rehabilitative training walker with center of gravity shifts in the camera space. A stochastic model is constructed to describe the influence of the center of gravity shift and unknown visual servo parameters. An adaptive controller is designed to make the tracking error system exponentially practically stable in mean square. It is proved that the mean square of the tracking error can be made arbitrarily small by choosing appropriate design parameters. The simulation results demonstrate the feasibility and effectiveness of the proposed method.*
Keywords: Omnidirectional walker, Center of gravity shifts, Adaptive control, Visual servo

1. **Introduction.** An omnidirectional rehabilitative training walker (ODW) [1,2] is being developed to provide both walking rehabilitation and walking support to people with walking impairments. To enhance the effectiveness of walking rehabilitation, the accuracy of path tracking needs to be improved. However, in practical rehabilitative robot applications, inevitable influence factors such as the center of gravity shifts and uncertainty of sensor measurement parameters, can seriously affect trajectory tracking.

In the control of an ODW, the robot states are usually assumed to be exactly detected and constructed through sensor measurements. However, the ODW states from sensor measurements were affected by perturbations. In this paper, a vision system using a camera is introduced to obtain the Cartesian position information directly. It is well known that deterministic models have limited ability to predict the behavior of a robot as it operates in its environment, and so the research of systems with random disturbances is meaningful. Other study [3] has proposed tracking control approaches for robot manipulators with external disturbances. In [4], a stochastic Hamiltonian dynamic model and an adaptive backstepping state feedback controller were reported. Note that all of the above stochastic models considered only the random noises that came from control input channels. However, moving robots can have many unknown parameters, such as, a variable arm-length robot manipulator [3] and a center of gravity shift in an ODW [1]. Due to the random parameters, it is nontrivial to design a controller achieving accurate tracking with a fast response.

Motivated by the above observations, we list the main contributions of this paper as follows. (I) With the application of stochastic theory, the random variation of the center of gravity shift in ODW operation is investigated. (II) By introducing a monocular camera model to measure the posture information of the ODW, the stochastic model becomes more complex when including unknown visual parameters, but the model has more practical significance and will help us design a tracking controller in the camera space directly. (III) An adaptive controller is proposed to maintain the tracking error tends to an arbitrarily small neighborhood of zero, and so does its derivative. The simulation results demonstrate the efficiency of the proposed scheme.

The remainder of this paper is organized as follows. The stochastic ODW model is formulated in Section 2. The design of the adaptive tracking controller is proposed in Section 3, and a stability analysis is conducted in Section 4. Simulation results are presented in Section 5, and concluding remarks are given in Section 6.

2. Model of the ODW with a Monocular Camera and the Center of Gravity Shifts. An image of the ODW is illustrated in Figure 1. The coordinate settings and structure used to develop the tracking control for the ODW are shown in Figure 2.

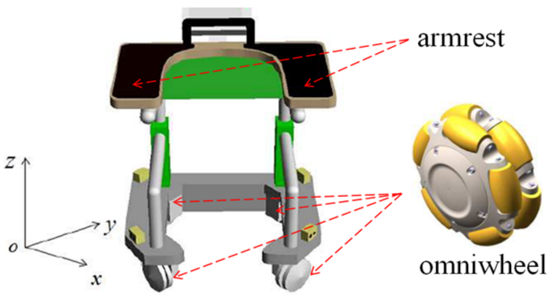


FIGURE 1. ODW and omniwheel

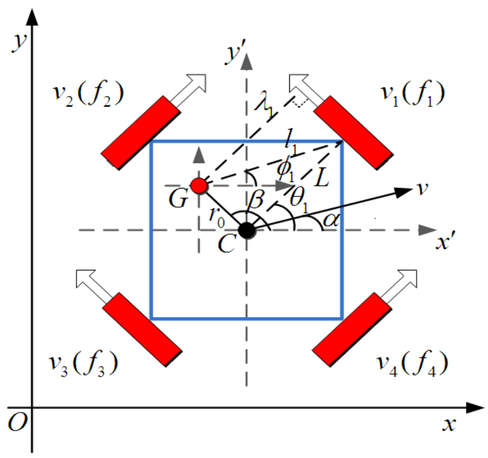


FIGURE 2. Structure of ODW

The stochastic model with the center of gravity shift, in [5], is expressed as

$$d\dot{X}(t) = M_1^{-1}B^*(\theta)u(t)dt + M_1^{-1}N\Sigma dw \tag{1}$$

where

$$X(t) = \begin{bmatrix} x(t) \\ y(t) \\ \theta(t) \end{bmatrix} \quad M_1 = \begin{bmatrix} M+m & 0 & 0 \\ 0 & M+m & 0 \\ 0 & 0 & I_0 \end{bmatrix}$$

$$B^*(\theta) = \begin{bmatrix} -\sin\theta_1 & \sin\theta_2 & \sin\theta_3 & -\sin\theta_4 \\ \cos\theta_1 & -\cos\theta_2 & \cos\theta_3 & \cos\theta_4 \\ L & L & L & L \end{bmatrix} \quad u(t) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}$$

$$N = \begin{bmatrix} -(M+m)\sin\theta & \dot{\theta}^2(M+m)\sin\theta & -\dot{\theta}^2(M+m)\cos\theta & -(M+m)\cos\theta & 0 \\ (M+m)\cos\theta & -\dot{\theta}^2(M+m)\cos\theta & -\dot{\theta}^2(M+m)\sin\theta & -(M+m)\sin\theta & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Here, M is the mass of the ODW, m is the user's equivalent mass, and I_0 is the inertia mass of the walker. The input forces are f_1, f_2, f_3, and f_4. w is a 5-dimensional independent standard Wiener process. $M_1^{-1}N\Sigma dw$ depicts the influence of the center of gravity shift, and $\Sigma/2\pi$ is the power spectral density of the white noise. The angle between the x' axis and the position of the first omniwheel is θ; as $\theta = \theta_1$, then we have $\theta_2 = \theta + \pi/2$, $\theta_3 = \theta + \pi$, and $\theta_4 = \theta + 3\pi/2$. The distance from the center of the ODW to each omniwheel is L.

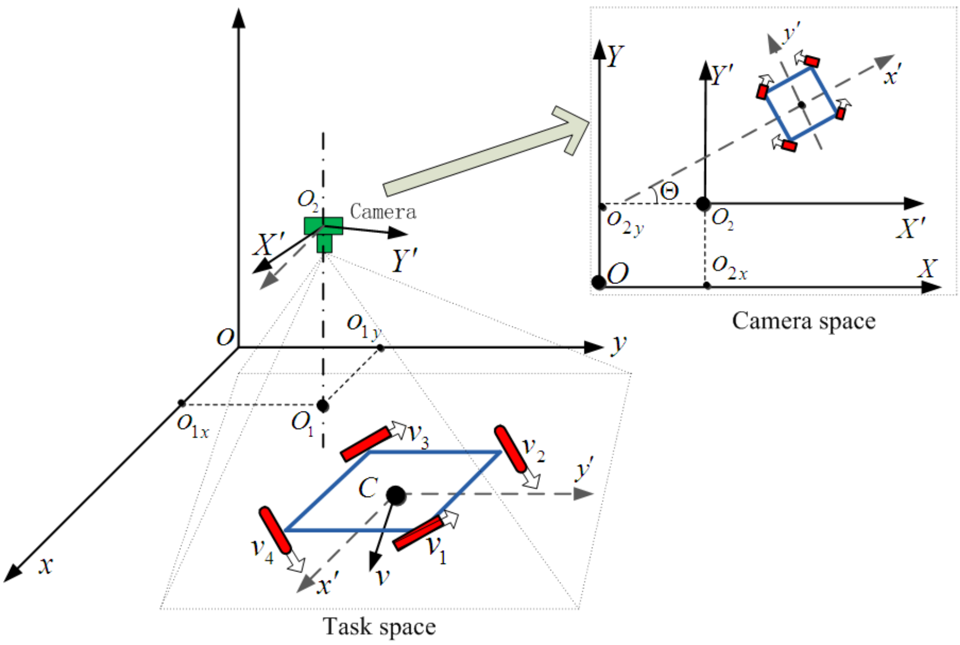


FIGURE 3. ODW with a monocular camera

Generally, the configuration of robots can be obtained from the encoders of motors, ultrasonic sensors, and other infrared sensors. However, for complex environments, it is difficult to implement this strategy. Instead, we take advantage of the vision information to deal with this challenge. As shown in Figure 3, a monocular camera is fixed to a ceiling and the ODW is under the camera. The world frame $x - y - o$ and the local frame $x' - y' - C$ are assumed to be parallel in the task space. Similarly, the image frame $X - Y - O$ and the camera frame $X' - Y' - O_2$ are assumed to be parallel in the camera space $O_1(o_{1x}, o_{1y})$ is the crossing point between the optical axis of the camera and the task space. The coordinate of the original point of the camera frame with respect to the image frame is defined by $O_2(o_{2x}, o_{2y})$, and Θ denotes the angle between the X'-axis and the x'-axis. The monocular camera model [6] yields.

$$X_m = \Lambda R(\Theta)\left[\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} - \begin{bmatrix} o_{1x} \\ o_{1y} \\ 0 \end{bmatrix}\right] + \begin{bmatrix} o_{2x} \\ o_{2y} \\ 0 \end{bmatrix} \tag{2}$$

where

$$X_m = \begin{bmatrix} x_m \\ y_m \\ \theta_m \end{bmatrix}, \quad \Lambda = \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R(\Theta) = \begin{bmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where α_1 and α_2 are unknown constants, which are dependent on the depth information, focus length and scalar factors; x_m, y_m, and θ_m are the X-axis position, the Y-axis position, and the orientation angle, respectively, in the image frame $X-Y-O$.

From (1) and (2), we can obtain the following stochastic model of the ODW with a monocular camera:

$$d\dot{X}_m = \Lambda R(\Theta) M_1^{-1} B^*(\theta) u(t) dt + \Lambda R(\Theta) M_1^{-1} N \Sigma dw \tag{3}$$

Assumption 2.1. *From the physical significance, camera parameters* Λ *and* $R(\Theta)$*, angular velocity* $\dot{\theta}$*, ODW mass* M *and user's equivalent mass* m*, are bounded, so a constant* h *exists such that*

$$\|\Lambda\|_F^2 \|R(\Theta)\|_F^2 \left\|M_1^{-1}\right\|_F^2 \left[2\dot{\theta}^4 (M+m)^2 + 2(M+m)^2 + 1\right] \|\Sigma\|_F^2 \le h \tag{4}$$

3. **Adaptive Visual Servo Controller Design.** The desired motion trajectory in the camera space is X_{dm}, and the actual motion trajectory is X_m; then, we define the tracking error as

$$e_1 = X_m - X_{dm} \tag{5}$$

$$e_2 = \dot{e}_1 + c_1 e_1 = \dot{X}_m - \dot{X}_{dm} + c_1 e_1 \tag{6}$$

where $c_1 > 0$ is a design parameter.

Combining (5) and (6) with system (3), we arrive at the following error systems

$$\begin{aligned} de_1 &= (-c_1 e_1 + e_2) dt \\ de_2 &= d\dot{X}_m - \ddot{X}_{dm} dt + c_1 de_1 \\ &= \left[\Lambda R(\Theta) M_1^{-1} B^*(\theta) u(t) - \ddot{X}_{dm} - c_1^2 e_1 + c_1 e_2\right] dt + \Lambda R(\Theta) M_1^{-1} N \Sigma dw \end{aligned} \tag{7}$$

The Lyapunov function is defined as

$$V(x,t) = \left(e_1^T e_1\right)^2/4 + \left(e_2^T e_2\right)^2/4 + \tilde{a}^T \mathrm{H}^{-1} \tilde{a}/(2\gamma) \tag{8}$$

where the positive constant γ and the positive diagonal matrix H are the designed parameters. Vector a is composed by the diagonal elements of matrix Λ, which is defined as $a = \begin{bmatrix} \alpha_1 & \alpha_2 & 1 \end{bmatrix}^T$. The estimation of a is $\hat{a} = \begin{bmatrix} \hat{\alpha}_1 & \hat{\alpha}_2 & 1 \end{bmatrix}^T$ and $\tilde{a} = \hat{a} - a = \begin{bmatrix} \tilde{\alpha}_1 & \tilde{\alpha}_2 & 0 \end{bmatrix}^T$ is the estimate error. The infinitesimal generator of $V(x,t)$ along (7) satisfies

$$\begin{aligned} LV(x,t) = &-c_1 e_1^T e_1 e_1^T e_1 + e_1^T e_1 e_1^T e_2 + e_2^T e_2 e_2^T \Lambda R(\Theta) M_1^{-1} B^*(\theta) u(t) \\ &- c_1^2 e_2^T e_2 e_2^T e_1 + c_1 e_2^T e_2 e_2^T e_2 - e_2^T e_2 e_2^T \ddot{X}_{dm} \\ &+ Tr\left\{\Sigma^T N^T M_1^{-1} R^T(\Theta) \Lambda^T \left(2e_2 e_2^T + e_2^T e_2 I\right) \Lambda R(\Theta) M_1^{-1} N \Sigma\right\}/2 \\ &+ \tilde{a}^T \mathrm{H}^{-1} \dot{\hat{a}}/\gamma \end{aligned} \tag{9}$$

We analyze the terms on the right-hand side of (9). Using Young's inequality, we have

$$e_1^T e_1 e_1^T e_2 \le c_1 \left(e_1^T e_1\right)^2/4 + 27\left(e_2^T e_2\right)^2/\left(4c_1^3\right) \tag{10}$$

Furthermore, according to the definition of the Frobenius norm, the norm compatibility, (4), and Young's inequality, we have

$$\begin{aligned} &Tr\left\{\Sigma^T N^T(\theta) M_1^{-1} R^T(\Theta) \Lambda^T \left(2e_2 e_2^T + e_2^T e_2 I\right) \Lambda R(\Theta) M_1^{-1} N(\theta) \Sigma\right\}/2 \\ &\le 3\left(e_2^T e_2\right) \|\Lambda\|_F^2 \|R(\Theta)\|_F^2 \left\|M_1^{-1}\right\|_F^2 \|N(\theta)\|_F^2 \|\Sigma\|_F^2/2 \\ &\le 3\left(e_2^T e_2\right) \|\Lambda\|_F^2 \|R(\Theta)\|_F^2 \left\|M_1^{-1}\right\|_F^2 \left[2\dot{\theta}^4(M+m)^2 + 2(M+m)^2 + 1\right] \|\Sigma\|_F^2/2 \\ &\le 3h\left(e_2^T e_2\right)/2 \le 9\varepsilon^2 \left(e_2^T e_2\right)^2/4 + h^2/\left(2\varepsilon^2\right) \end{aligned} \tag{11}$$

where $\varepsilon > 0$ is a design parameter. Now, the control input $u(t)$ is designed as

$$u(t) = \widehat{B}^*(\theta) M_1 R^{-1}(\Theta) \hat{\Lambda}^{-1} \bar{u}(t) \tag{12}$$

where $\widehat{B}^*(\theta) = B^{*^T}(\theta) \left(B^*(\theta) B^{*^T}(\theta)\right)^{-1}$ is a pseudo inverse of $B^*(\theta)$. Substituting (10)-(12) into (9), we have

$$\begin{aligned} LV(x,t) \leq & -c_1 \left(e_1^T e_1\right)^2 + c_1 \left(e_1^T e_1\right)^2 /4 + 27 e_2^T e_2 e_2^T e_2 / \left(4c_1^3\right) + e_2^T e_2 e_2^T \bar{u}(t) \\ & - c_1^2 e_2^T e_2 e_2^T e_1 + c_1 e_2^T e_2 e_2^T e_2 - e_2^T e_2 e_2^T \ddot{X}_d + 9\varepsilon^2 \left(e_2^T e_2\right)^2 /4 \\ & + h^2 / \left(2\varepsilon^2\right) - \tilde{a}^T \bar{u}(t) e_2^T e_2 e_2^T \hat{a} + \tilde{a}^T \mathrm{H}^{-1} \dot{\hat{a}} / \gamma \end{aligned} \tag{13}$$

We design the adaptive laws as follows

$$\dot{\hat{a}} = \gamma \mathrm{H} \bar{u}(t) e_2^T e_2 e_2^T \hat{a} \tag{14}$$

where $\bar{u}(t) = c_1^2 e_1 - c_1 e_2 + \ddot{X}_d - 27 e_2 / \left(4c_1^3\right) - 9\varepsilon^2 e_2 /4 - 3c_2 e_2 /4$, and $c_2 > 0$ is a constant.

The following inequality is obtained:

$$LV(x,t) \leq -cV(x,t) + d \tag{15}$$

where $c = \min(3c_1, 3c_2, 2/\gamma)$, $d = h^2/(2\varepsilon^2)$.

4. **Stability Analysis.**

Theorem 4.1. *For the stochastic model of the ODW with a monocular camera (3), the adaptive controller (12) is designed such that the error system (7) has a unique strong solution on* $[t_0, \infty)$*, and* e_1 *and* $\dot{e}_1$ *are exponentially practically stable in the mean square for initial values* $e_1(t_0) \in R^n$ *and* $e_2(t_0) \in R^n$*. The tracking errors* e_1 *and* $\dot{e}_1$ *satisfy*

$$\lim_{t\to\infty} E\left|e_1\right| \leq (4d/c)^{1/4} \tag{16}$$

$$\lim_{t\to\infty} E\left|\dot{e}_1\right| \leq 2\left(1+c_1^2\right)(4d/c)^{1/4} \tag{17}$$

Moreover, the right-hand sides of (16) and (17) can be made arbitrarily small by choosing the appropriate design parameters.

Proof: From the physical significance of the inertia matrix M_1, which is symmetric and positive definite, the inverse matrix M_1^{-1} exists and is smooth, as it does for the functions $u(t)$, $B^*(\theta)$, and $R(\Theta)$. Therefore, the error system (7) satisfies the local Lipschitz condition. From (8) and (15), according to Lemma 1 in [7], a unique strong solution to the error system (7) on $[t_0, \infty)$ exists for initial values $e_1(t_0) \in R^n$ and $e_2(t_0) \in R^n$, and the error system (7) is exponentially practically stable in mean square.

Moreover, by multiplying the inequality (15) by $e^{ct} > 0$ and integrating it from t_0 to t, we have

$$E\left(e^{ct} V(x,t)\right) \leq e^{ct_0} V(x_0, t_0) + E\int_{t_0}^{t} e^{cs} d \cdot ds \quad \forall t \geq t_0 \tag{18}$$

From (8), we can deduce that

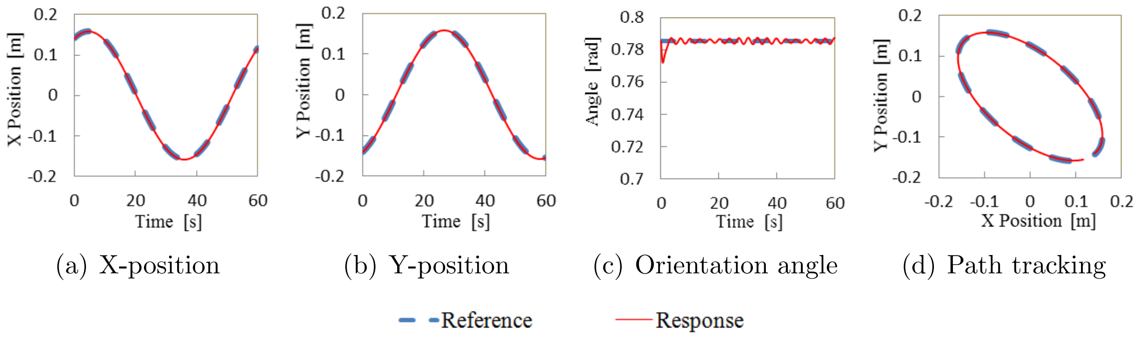
$$\begin{aligned} E\left|e_1\right|^2 \leq 2e^{c(t_0-t)/2} \Big(& \left(e_1^T(t_0) e_1(t_0)\right)^2/4 + \left(e_2^T(t_0) e_2(t_0)\right)^2/4 \\ & + \tilde{a}^T(t_0) \mathrm{H}^{-1} \tilde{a}(t_0)/(2\gamma)\Big)^{1/2} + (4d/c)^{1/2} \end{aligned} \tag{19}$$

$$\begin{aligned} E\left|e_2\right|^2 \leq 2e^{c(t_0-t)/2} \Big(& \left(e_1^T(t_0) e_1(t_0)\right)^2/4 + \left(e_2^T(t_0) e_2(t_0)\right)^2/4 \\ & + \tilde{a}^T(t_0) \mathrm{H}^{-1} \tilde{a}(t_0)/(2\gamma)\Big)^{1/2} + (4d/c)^{1/2} \end{aligned} \tag{20}$$

Using (19) and (20), and in view of $\left|\dot{e}_1\right|^2 = \left(\left|e_2\right| + c_1\left|e_1\right|\right)^2 \leq 2\left(1+c_1^2\right)\left(\left|e_2\right|^2 + \left|e_1\right|^2\right)$, it follows that (16) and (17) hold. Therefore, the right-hand side of (16) and (17) can be made small enough by appropriate choice of c_1, c_2, γ, ε, and H, because they are independent.

5. **Simulation Results.** In this section, the proposed adaptive tracking control algorithm is verified by a simulation of the ODW with center of gravity shifts and unknown visual servo parameters. To rigorously verify the tracking performance of the proposed method, we assume that the walker follows an elliptical path and that the random parameters caused by the center of gravity shift are $r_0 = 0.16(1 + \sin t)$ m, $\lambda_1 = L - r_0 \sin t$ m, $\lambda_2 = L + r_0 \cos t$ m, $\lambda_3 = L + r_0 \sin t$ m, and $\lambda_4 = L - r_0 \cos t$ m. The physical parameters of the ODW used in the simulation are $M = 58$ kg, $m = 80$ kg, $L = 0.4$ m, $I_0 = 27.7$ kg·m^2, and $\Theta = \pi/4$ rad. The design parameters are $c_1 = 2.08$, $c_2 = 0.0001$, $\varepsilon = 1.4$, $\gamma = 470000$, and $\mathrm{H} = \mathrm{diag}\{230, 4000, 75\}$. Initial values are chosen as $\hat{\alpha}_1(0) = 0.012$, $\hat{\alpha}_2(0) = 0.015$, $x_m(0) = 0.1414$, $y_m(0) = 0.1414$, $\theta_m(0) = \pi/4$, $x(0) = 20$, $y(0) = 0$, and $\theta(0) = \pi/4$. The trajectory X_{dm} is described by $x_{dm} = 20\cos(0.1t)$, $y_{dm} = 10\sin(0.1t)$, $\theta_{dm} = \pi/4$.

The simulation results are given in the following figures.



(a) X-position (b) Y-position (c) Orientation angle (d) Path tracking

FIGURE 4. Tracking performance of the camera space

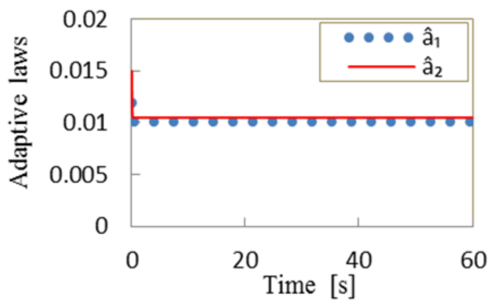


FIGURE 5. Adaptive laws

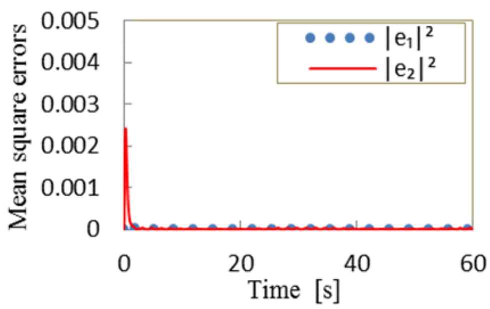
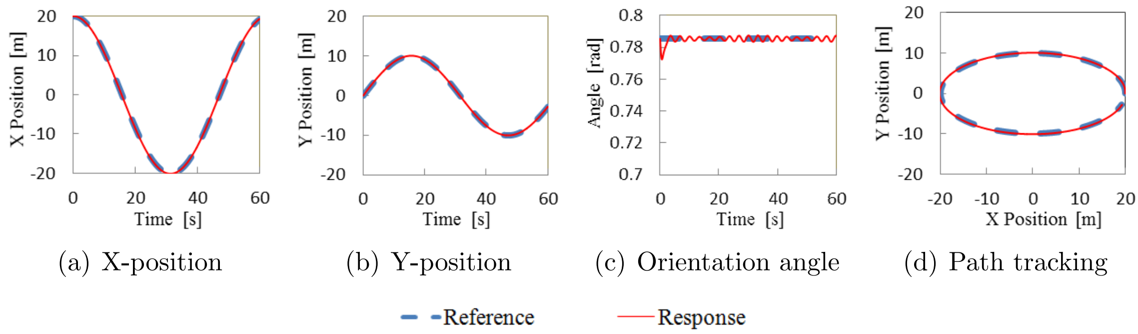


FIGURE 6. Mean square errors



(a) X-position (b) Y-position (c) Orientation angle (d) Path tracking

FIGURE 7. Tracking performance of the task space

Figures 4(a)-4(d) plot the tracking performance of the ODW for the X-position, the Y-position, the orientation angle, and the path tracking of the ellipse in the camera space, respectively. We can see that the closed-loop system can realize exponentially mean square stability and that the ODW can realize trajectory tracking with the adaptive controller (12). The adaptive laws are shown in Figure 5. Figure 6 shows that the mean square of the errors can be made arbitrarily small by choosing the appropriate design parameters. Figures 7(a)-7(d) give the tracking performance in the task space, respectively. The simulation results above show that the controller designed in the camera space can simultaneously complete the trajectory tracking in the camera space and the task space.

6. **Conclusions.** In this paper, we propose an effective scheme to address the center of gravity shift and unknown visual parameters of an ODW. To accomplish this, we first construct a reasonable stochastic model to describe the motion of an ODW subject to the uncertainty of random parameters in a monocular camera system. Then, we design an adaptive tracking controller to achieve the trajectory tracking of ODW with the center of gravity shift and unknown visual parameters. The stability of the system is analyzed and established by using a Lyapunov approach of the probability space. Finally, we demonstrate the effectiveness of our proposed controllers with simulation studies.

There are other problems under current investigation such as tracking control with incomplete measurements, bounded control force aiming to control the ODW's trajectory, and their application to other mechanical systems.

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