

## SENSORLESS VECTOR CONTROLLED INDUCTION MOTOR DRIVE WITH FULL-ORDER ROTOR FLUX OBSERVER SPEED IDENTIFICATION

YUNG-CHANG LUO\*, YU-HSIANG CHEN, YING-PIAO KUO AND CHENG-TAO TSAI

Department of Electrical Engineering  
National Chin-Yi University of Technology  
No. 57, Sec. 2, Zhongshan Rd., Taiping District, Taichung City 41170, Taiwan  
\*Corresponding author: luoyc@ncut.edu.tw

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**ABSTRACT.** *A speed identification method of the vector-controlled induction motor drive utilizing rotor flux observer is presented. The decoupling vector controlled approach based on the stator current and rotor flux is established to attain superior performance of the induction motor drive. The speed identification scheme applying the full-order rotor flux observer is developed for the sensorless vector controlled IM drive and the observer gain matrix is designed by Lyapunov stability theorem. Simulation and experimental results confirm the effectiveness of the proposed approach.*

**Keywords:** Sensorless vector control, Flux observer, Lyapunov stability observer gain design

1. **Introduction.** Induction motor (IM) is generally applied in industrial facility applications because of fewer maintenance requirements and cost, robust frame, and better reliability. However, the mathematical model of an IM is nonlinear, coupling, and time-varying, and the control of an IM drive is more complicated than that of a DC drive. The vector control (VC) approach utilizes suitable coordinate transformation [1], the 3-phase AC components (voltage, current, flux) of an IM are separated into the orthogonal 2-axis DC ones, and these enable IM drive to accomplish similar DC motor drive performance. Nevertheless, the implementation of VC IM drive requires rotor-shaft position sensor to detect the rotor speed. This sensor, however, reduces the drive reliability and is not suitable for antagonistic environment. Hence, the sensorless VC approaches adopt flux estimator to identify rotor speed, which have been used to replace the conventional VC IM drives. The stator current model rotor flux estimator is designed based on model reference adaptive system skill to identify the rotor speed [2]. The adaptive flux scheme is established utilizing the stator current and flux vector components to estimate the motor speed [3]. The reactive power mode artificial neural network adaptive estimator is proposed to recognize the shaft speed [4]. The band pass filter based modified stator model is used to solve the problem associated with the integration of the back electromotive force for sensorless direct torque control IM drive running low-speed range [5]. The instantaneous reactive power based model reference adaptive system is established to overcome the stator resistance sensitivity at low speed operation for sensorless IM drive [6]. The fuzzy self-tuning IP speed controller is developed to improve motor parameters variation of sensorless VC IM drive [7]. In this paper, the full-order rotor flux observer (FORFO) is developed that applies the stator current and rotor flux, and then the rotor-shaft speed is on-line identified based on the established FORFO. The proposed rotor speed identification skill, which compares with the above-mentioned methods, has advantages of simple scheme, low parameter sensitivity, and facile implementation.

**2. Rotor Field VC IM Drive.** The state equations of the stator current and rotor flux of an IM in the synchronous reference coordinate frame are derived as [8]

$$p\vec{i}_s^e = -\left(\frac{R_s}{L_\sigma} + \frac{1-\sigma}{\sigma\tau_r} + j\omega_e\right)\vec{i}_s^e + \frac{L_m}{L_\sigma L_r}\left(\frac{1}{\tau_r} - j\omega_r\right)\vec{\lambda}_r^e + \frac{1}{L_\sigma}\vec{v}_s^e \quad (1)$$

$$p\vec{\lambda}_r^e = \frac{L_m}{\tau_r}\vec{i}_s^e - \left(\frac{1}{\tau_r} + j\omega_{sl}\right)\vec{\lambda}_r^e \quad (2)$$

where “ $j$ ” stands for the imaginary part,  $\vec{v}_s^e = v_{ds}^e + jv_{qs}^e$  is the stator voltage,  $\vec{i}_s^e = i_{ds}^e + ji_{qs}^e$  is the stator current,  $\vec{\lambda}_r^e = \lambda_{dr}^e + j\lambda_{qr}^e$  is the rotor flux,  $L_s$  and  $L_r$  are the stator and rotor inductance, respectively,  $L_m$  is the mutual inductance between the stator and rotor,  $R_s$  and  $R_r$  are the stator and rotor resistance, respectively,  $\sigma = 1 - (L_m^2/L_s L_r)$  is the leakage inductance coefficient,  $L_\sigma = \sigma L_s$  is the stator leakage inductance,  $\tau_r = L_r/R_r$  is the rotor time constant,  $\omega_e$  is speed of the synchronous reference coordinate frame,  $\omega_r$  is the electric speed of the rotor,  $\omega_{sl} = \omega_e - \omega_r$  is the slip speed, and  $p = d/dt$  is the differential operator. The developed electromagnetic torque of an IM is acquired as

$$T_e = \frac{3P}{4} \frac{L_m}{L_r} (i_{qs}^e \lambda_{dr}^e - i_{ds}^e \lambda_{qr}^e) \quad (3)$$

where  $P$  is the pole number of the motor. The mechanical equation of the motor is

$$J_m p\omega_{rm} + B_m \omega_{rm} + T_L = T_e \quad (4)$$

where  $T_L$  is the load torque,  $J_m$  is the inertia of the motor,  $B_m$  is the viscous friction coefficient, and  $\omega_{rm} = (2/P)\omega_r$  is the mechanical speed of the motor rotor-shaft.

Under the rotor field vector control (RFVC) condition [9], set  $\lambda_{qr}^e = 0$  in (2), and the  $d$ -axis rotor flux is given by

$$\lambda_{dr}^e = \frac{L_m}{1 + \tau_r s} i_{ds}^e \quad (5)$$

where  $s$  is the Laplace operator.

The developed electromagnetic torque of an IM under RFVC condition is acquired as

$$T_e = \frac{3P}{4} \frac{L_m}{L_r} i_{qs}^e \lambda_{dr}^e \quad (6)$$

Moreover, also set  $\lambda_{qr}^e = 0$  in (1), and the  $d$ -axis and  $q$ -axis stator current state equations are given by, respectively

$$p i_{ds}^e = -\left(\frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma\tau_r}\right) i_{ds}^e + \omega_e i_{qs}^e + \frac{1-\sigma}{\sigma\tau_r L_m} \lambda_{dr}^e + \frac{1}{\sigma L_s} v_{ds}^e \quad (7)$$

$$p i_{qs}^e = -\frac{R_s}{\sigma L_s} i_{qs}^e - \omega_e i_{ds}^e - \frac{1-\sigma}{\sigma L_m} \omega_e \lambda_{dr}^e + \frac{1}{\sigma L_s} v_{qs}^e \quad (8)$$

In order to attain linear control of the stator current control loop, the feed-forward compensation skill is applied. According to (7) and (8), the  $d$ -axis and  $q$ -axis feed-forward voltage compensations are defined as, respectively [8]

$$\sigma L_s \left( -\omega_e i_{qs}^e - \frac{1-\sigma}{\sigma\tau_r L_m} \lambda_{dr}^e \right)$$

$$\sigma L_s \left( \omega_e i_{ds}^e + \frac{1-\sigma}{\sigma L_m} \omega_e \lambda_{dr}^e \right)$$

Hence, the voltage command of the  $d$ -axis and  $q$ -axis stator current control loops are obtained as, respectively

$$v_{ds}^{e*} = \sigma L_s \left( v_{ds}^{e'} - \omega_e i_{qs}^e - \frac{1-\sigma}{\sigma\tau_r L_m} \lambda_{dr}^e \right) \quad (9)$$

$$v_{qs}^{e*} = \sigma L_s \left( v_{qs}^{e'} + \omega_e i_{ds}^e + \frac{1-\sigma}{\sigma L_m} \omega_e \lambda_{dr}^e \right) \quad (10)$$

where  $v_{ds}^{e'}$  and  $v_{qs}^{e'}$  are the outputs of the  $d$ -axis and  $q$ -axis stator current controller, respectively.

**3. FORFO Based Rotor Speed Identification.** In the two-axis stationary reference coordinate frame ( $\omega_e = 0$ ), according to (1) and (2), FORFO is defined as

$$p \begin{bmatrix} \hat{i}_s^s \\ \hat{\lambda}_r^s \end{bmatrix} = \begin{bmatrix} -\left(\frac{R_s}{L_\sigma} + \frac{1-\sigma}{\sigma\tau_r}\right) & \frac{L_m}{L_\sigma L_r} \left(\frac{1}{\tau_r} - j\hat{\omega}_r\right) \\ \frac{L_m}{\tau_r} & -\left(\frac{1}{\tau_r} - j\hat{\omega}_r\right) \end{bmatrix} \begin{bmatrix} \hat{i}_s^s \\ \hat{\lambda}_r^s \end{bmatrix} + \begin{bmatrix} \frac{1}{L_\sigma} \\ 0 \end{bmatrix} \vec{v}_s^s + G \left( \vec{i}_s^s - \hat{\vec{i}}_s^s \right) \quad (11)$$

where “ $\wedge$ ” stands for the estimated value, and  $G$  is the observer gain matrix.

The proposed sensorless VC IM drive with FORFO rotor speed identification structure is shown in Figure 1, where the speed difference between the rotor speed command  $\omega_r^*$  and the estimated rotor speed  $\hat{\omega}_r$  is conducted by RFVC IM drive and the coordinate transformation two-axis synchronous to three-phase stationary ( $2^e \Rightarrow 3^s$ ), and then trigger the inverter to actuate IM. Furthermore, the current difference between the estimated stator current  $\hat{\vec{i}}_s^s$  (deriving from FORFO) and the measured stator current  $\vec{i}_s^s$  (deriving from IM by using the coordinate transformation three-phase stationary to two-axis synchronous ( $2^s \Leftarrow 3^s$  and  $2^e \Leftarrow 2^s$ )) is modulated by the observer gain matrix  $G$  to identify the estimated rotor speed.

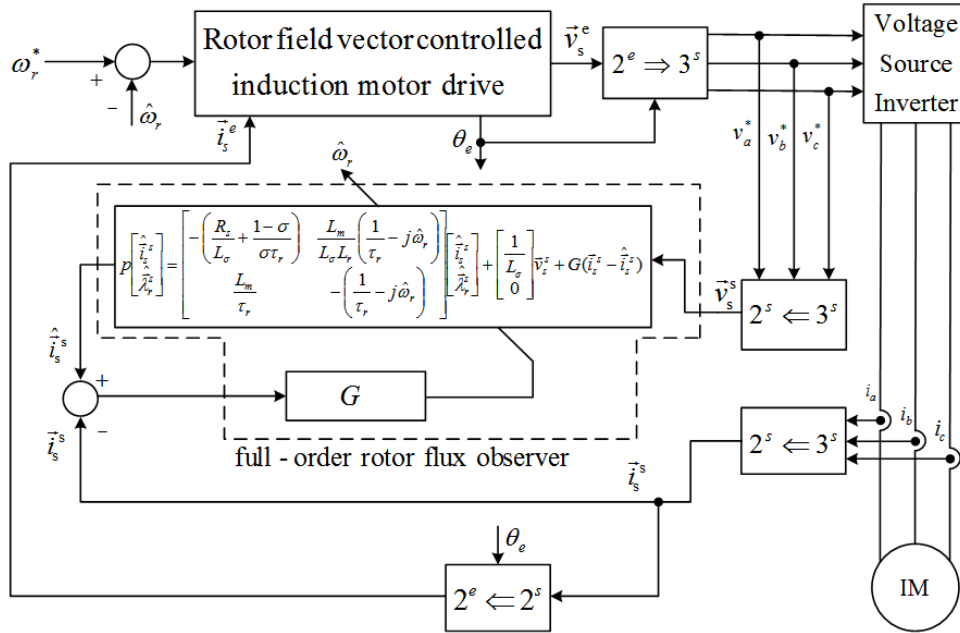


FIGURE 1. Sensorless VC IM drive with FORFO rotor speed identification structure

The synchronous position angle for execution coordinate transformation between synchronous and stationary reference coordinate frames is acquired by

$$\theta_e = \int \omega_e dt \quad (12)$$

### Observer gain matrix design

The observer gain matrix of the proposed FORFO is designed utilizing Lyapunov stability theorem [10]. In accordance with (1) and (2), rewrite the state equations of an IM

in the stationary reference coordinate frame as

$$\dot{\vec{x}} = [A + \omega_r A_\omega] \vec{x} + B \vec{v}_s \quad (13)$$

$$\vec{y} = C \vec{x} \quad (14)$$

where  $\vec{x} = \begin{bmatrix} \vec{i}_s^s \\ \vec{\lambda}_r^s \end{bmatrix}$ ,  $A = \begin{bmatrix} -\left(\frac{1}{\sigma} + \frac{1-\sigma}{\sigma\tau_r}\right) & \frac{1-\sigma}{\sigma L_m \tau_r} \\ -\frac{L_m}{\tau_r} & -\frac{1}{\tau_r} \end{bmatrix}$ ,  $A_\omega = \begin{bmatrix} 0 & -j\frac{1-\sigma}{\sigma L_m} \\ 0 & j \end{bmatrix}$ ,  $B = \begin{bmatrix} \frac{1}{L_\sigma} \\ 0 \end{bmatrix}$ ,  $C = [1 \ 0]$ . According to (13), the estimation state matrix of the proposed FORFO is derived as

$$\dot{\hat{\vec{x}}} = [A + \hat{\omega}_r A_\omega] \hat{\vec{x}} + B \vec{v}_s + G \left( \vec{i}_s^s - \hat{\vec{i}}_s^s \right) \quad (15)$$

The estimation difference between (13) and (15) is

$$\dot{\vec{e}} = (A + \omega_r A_\omega + GC) \vec{e} + \Delta\omega_r A_\omega \hat{\vec{x}} \quad (16)$$

where  $\vec{e} = \vec{x} - \hat{\vec{x}}$ , and  $\Delta\omega_r = \hat{\omega}_r - \omega_r$ . Then, define the Lyapunov function as

$$V(\vec{e}, \Delta\omega_r) = \vec{e}^T H \vec{e} + \frac{(\Delta\omega_r)^2}{\gamma_\omega} \quad (17)$$

where  $H$  is a symmetric positive definite matrix.

The time derivative of the defined Lyapunov function is derived as

$$\begin{aligned} \dot{V} = & \vec{e}^T [(A + GC)^T H + H(A + GC) + \omega_r (A_\omega^T H + H A_\omega)] \vec{e} \\ & + \Delta\omega_r \left[ \hat{\vec{x}}^T A_\omega^T H \vec{e} + \vec{e}^T H A_\omega \hat{\vec{x}} \right] - \frac{2(\Delta\omega_r)}{\gamma_\omega} \frac{d(\Delta\omega_r)}{dt} \end{aligned} \quad (18)$$

where  $\gamma_\omega > 0$  is a constant.

Set  $\Delta\omega_r \left[ \hat{\vec{x}}^T A_\omega^T H \vec{e} + \vec{e}^T H A_\omega \hat{\vec{x}} \right] = 2(\Delta\omega_r)/\gamma_\omega \cdot d(\Delta\omega_r)/dt$  in (18), if  $G$  and  $H$  are selected to guarantee Inequality (19) validity, then (16) is asymptotically stable. Hence, the proposed FORFO is asymptotically stable.

$$(A + GC)^T H + H(A + GC) + \omega_r (A_\omega^T H + H A_\omega) < 0 \quad (19)$$

The adaptive law is chosen as

$$\frac{d(\Delta\omega_r)}{dt} = \vec{e}^T H A_\omega \hat{\vec{x}} \quad (20)$$

Consequently, the adaptive law for the estimation rotor speed is acquired as

$$\hat{\omega}_r = K_{po} \left[ \vec{e}^T H A_\omega \hat{\vec{x}} \right] + \frac{K_{io} \left[ \vec{e}^T H A_\omega \hat{\vec{x}} \right]}{s} \quad (21)$$

where  $K_{po}$  and  $K_{io}$  are the adaptation gain.

**4. Simulation and Experimental Test.** The block diagram of the proposed FORFO based sensorless VC IM drive applying Lyapunov stability theorem observer gain matrix design is shown in Figure 2, which includes FORFO, speed controller, flux controller,  $q$ -axis and  $d$ -axis stator current controllers,  $d$ -axis and  $q$ -axis stator voltage decoupling, and coordinate transformation. In this system, the proportion-integral (PI) type controllers for the speed control loop, flux control loop,  $d$ -axis, and  $q$ -axis stator current control loops are designed by the root-locus methods. The proportion gain ( $K_P$ ), integral gain ( $K_I$ ), and bandwidth (B.W.) for the four PI type controllers are shown in Table 1. The Bode-diagram of the designed speed controller is shown in Figure 3. The observer gain matrix of the proposed FORFO is designed using Lyapunov stability theorem.

To confirm the effectiveness of the proposed FORFO based sensorless VC IM drive applying Lyapunov stability theorem observer gain matrix design, a 3-phase, 220 V, 0.75 kW,  $\Delta$ -connected, standard squirrel-cage IM is used, which serves as the controlled plant

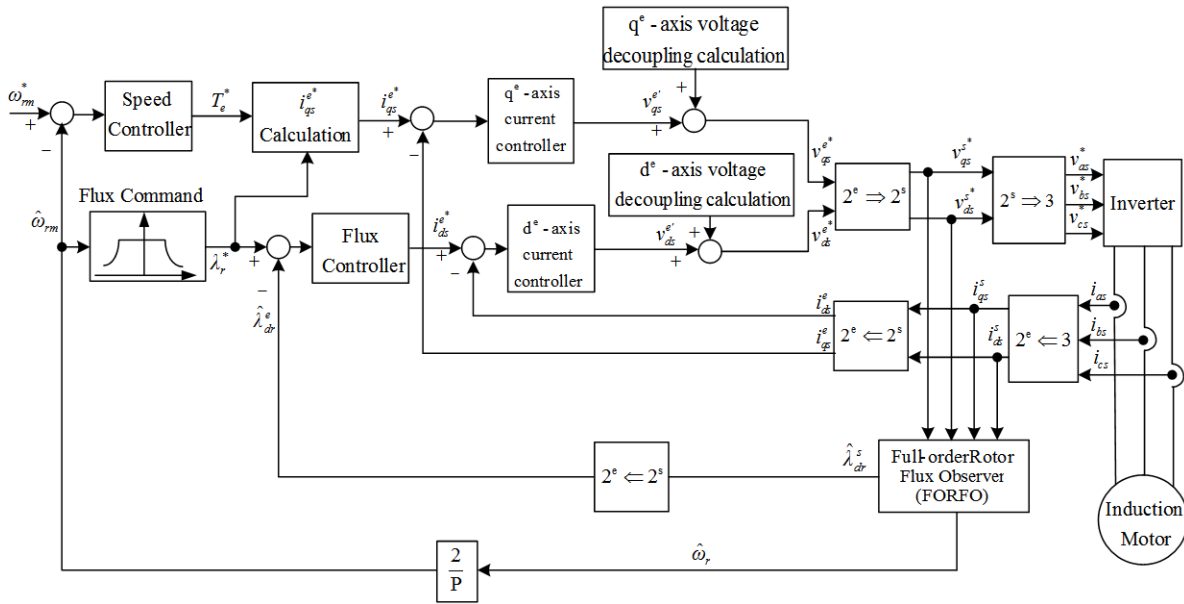


FIGURE 2. FORFO based rotor speed identification sensorless VC IM drive

TABLE 1. PI controller parameters and its bandwidth

	$K_P$	$K_I$	B.W. (rad./sec)
speed loop controller	1.35	63.4	193
flux loop controller	101.88	1063	225
$q$ -axis stator current loop controller	672.57	5945	427
$d$ -axis stator current loop controller	586.1	4427	542

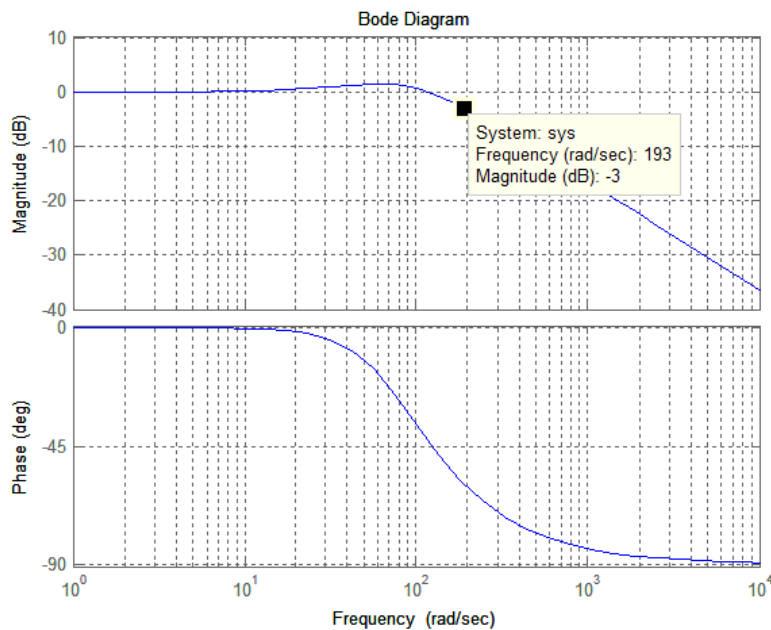


FIGURE 3. The Bode diagram of the designed speed loop controller

for experimentation. The simulated and measured response with loading 2 N-m at reversible steady-state speed command 1800 rpm and 300 rpm are shown in Figures 4-5 and 6-7, respectively, each figure including four responses: estimation speed, stator current, electromagnetic torque, and rotor flux locus.

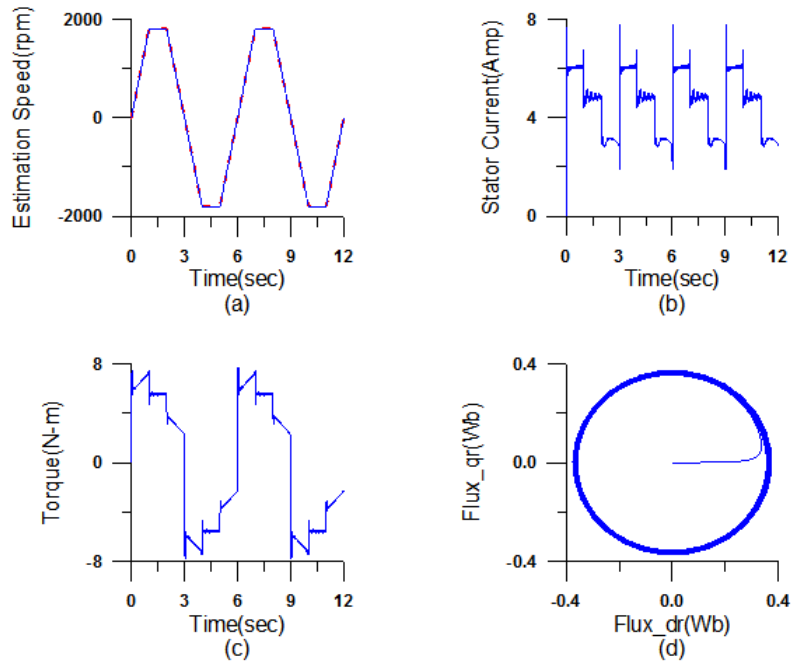


FIGURE 4. Simulated responses of the proposed FORFO sensorless VC IM drive with loading 2 N-m at reversible steady-state speed command 1800 rpm: (a) command (dotted line) and estimated (solid line) rotor-shaft speed, (b) stator current, (c) estimated electromagnetic torque, (d) rotor flux locus ( $q$ -axis vs.  $d$ -axis)

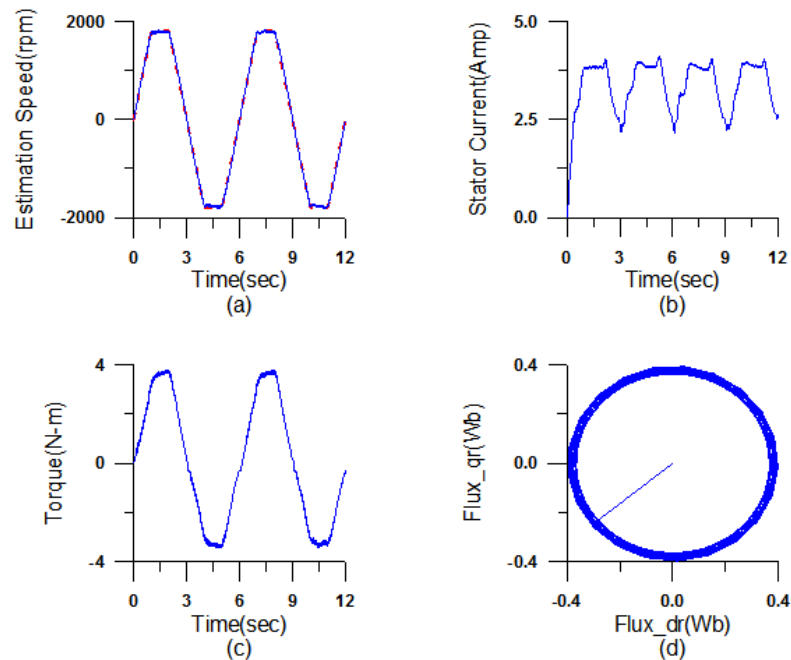


FIGURE 5. Measured responses of the proposed FORFO sensorless VC IM drive with loading 2 N-m at reversible steady-state speed command 1800 rpm: (a) command (dotted line) and estimated (solid line) rotor-shaft speed, (b) stator current, (c) estimated electromagnetic torque, (d) rotor flux locus ( $q$ -axis vs.  $d$ -axis)

Based on the simulated and experimental results for the reversible steady-state speed command 1800 rpm and 300 rpm which are shown in Figures 4-5 and 6-7, the proposed

FORFO rotor-shaft speed on line identification sensorless VC IM drive applying Lyapunov stability theorem observer gain matrix design strategy has shown that desired performance can be acquired.

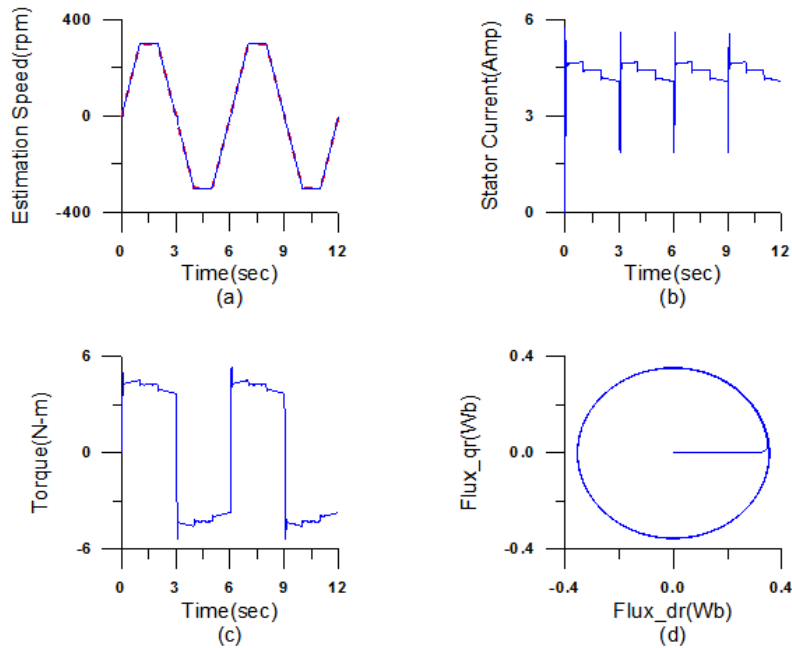


FIGURE 6. Simulated responses of the proposed FORFO sensorless VC IM drive with loading 2 N-m at reversible steady-state speed command 300 rpm: (a) command (dotted line) and estimated (solid line) rotor-shaft speed, (b) stator current, (c) estimated electromagnetic torque, (d) rotor flux locus ( $q$ -axis vs.  $d$ -axis)

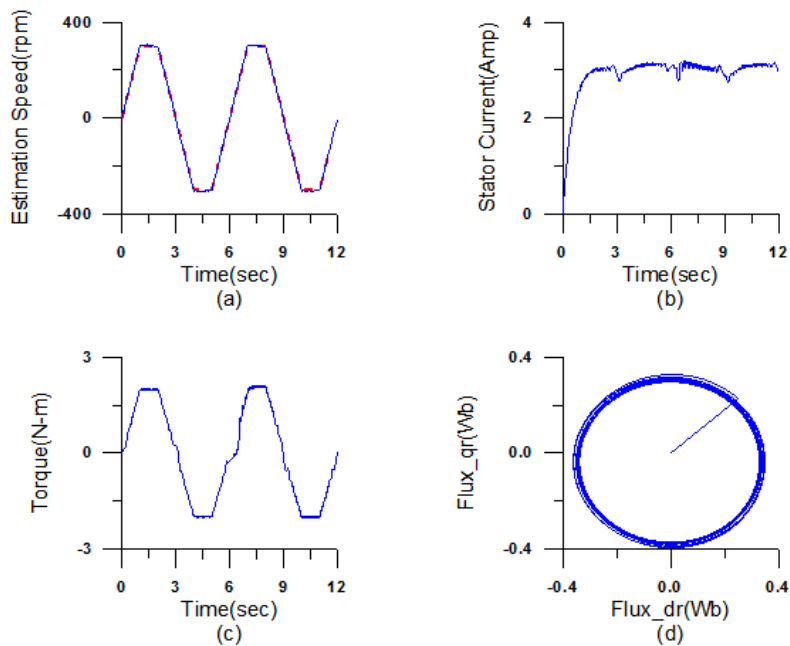


FIGURE 7. Measured responses of the proposed FORFO sensorless VC IM drive with loading 2 N-m at reversible steady-state speed command 300 rpm: (a) command (dotted line) and estimated (solid line) rotor-shaft speed, (b) stator current, (c) estimated electromagnetic torque, (d) rotor flux locus ( $q$ -axis vs.  $d$ -axis)

5. **Conclusion.** A FORFO based rotor-shaft speed on-line identification applying Lyapunov stability theorem observer gain matrix design strategy has been proposed to control a sensorless VC IM drive. The proposed estimation speed structure using FORFO has the advantages of simple structure, low parameter sensitivity, and facile implementation. The observer gain matrix of the proposed FORFO is designed applying Lyapunov stability theorem which guaranteed to acquire stable adaptation gain. The simulation and experimental responses at reversible steady-state speed command 1800 rpm and 300 rpm confirmed the effectiveness of the proposed approach. Applying the fuzzy logical control skill to design the observer gain matrix of FORFO is the future extension research of this system.

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