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A NEW LOW RANK MATRIX COMPLETION METHOD FOR VIDEO DENOISING IN WIRELESS MULTIMEDIA SENSOR NETWORKS

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Abstract. *As a central problem in image processing, denoising was studied widely in the literature. However, most existing denoising algorithms have assumed the additive white Gaussian noise. In this paper, we propose an efficient video denoising method that can handle non-homogeneous noise in Wireless Multimedia Sensor Networks (WMSN). Our methods are mainly based on low-rank matrix completion as follows. First, a noisy video is processed in blockwise manner and for each processed block we find candidate match pixels on other images using a fast block matching (BM) method. The blocks centered around these candidate pixels then will stack together to form a matrix and unreliable pixels will be removed using matrix completion method solved by penalty decomposition method. Experimental results show that the proposed method is capable of robustly denoising the mixed noise in WMSN video.*

Keywords: Denoising, Block matching, Matrix completion, Penalty decomposition

1. **Introduction.** As new wireless sensor devices with the integration of multimedia capabilities, WMSN [1] can perceive and transmit multimedia data from the physical world through a group of distributed multimedia sensor nodes, which has shown its potential in many areas. However, due to the complexity of WMSN monitoring scenes and inclement weather, the video images inevitably suffer from various noise types. Therefore, it is imperative to study denoising methods for removing mixed noise in WMSN video to ensure the reliability and robustness of video surveillance. In a nutshell, a general expression for the WMSN video image degradation can be written as:

$$g(x, y) = z(x, y) + n(x, y) \tag{1}$$

where $z(x, y)$ denotes the original video image, $g(x, y)$ is the noisy video image, and $n(x, y)$ represents the noise.

Many image/video denoising methods have been proposed in the last few decades, e.g., [2-4]. The non-local method [2] was presented by Mahmoudi and Sapiro to address the denoising problem while it does not perform well under strong noise intensity. Yang et al. [3] proposed adaptive dual-tree discrete wavelet packets which gives a multi-scale decomposition for video. Recently, patch-based methods [4,5] have been applied on video denoising as a powerful tool. Li et al. in [4] show that similar patches share the same dictionary elements in their sparse decomposition on denoising. As an extension to BM3D, the VBM3D method [5] further leverages the temporal redundancy of video data for better performance.

Unfortunately, the aforementioned methods have been limited to the one specific type of noise, especially when the noise is mostly Gaussian white noise. In WMSN video surveillance, the existence of outliers in the video data is not rare and can be caused by many factors.

In this paper, we aim at developing an effective algorithm to remove mixed noise from WMSN video. The proposed video denoising method is built upon many patch-based methods, whose basic idea is to convert the denoising problem from the stack of matched patches to a low rank matrix completion problem [6]. For each patch in the reference frame, we find the similar patches in the other frames using a fast block matching algorithm [7]. The found matches will be vectorized and then stacked into a matrix. The main stage will be done by applying our proposed approach to the incomplete matrix. A nearly noise free block will be output and a denoised patch will be constructed as the average value of each row in the completed matrix. It is shown in the experiments that our low rank matrix completion based approach can efficiently remove complex noise in WMSN video.

The reminder of this paper is structured as follows. Problem formulation and the proposed scheme will be introduced in Section 2 and Section 3, respectively. We show and discuss our simulation results in Section 4, followed by a brief conclusion in Section 5.

2. **Problem Formulation.** According to the Formula (1), the denoising model can be described as:

$$g_k = z_k + n_k \tag{2}$$

where g_k presents one frame of the noised WMSN video sequence and $G = \{g_K\}_{K=1}^{K}$ denotes K frames, z_k is the clean image and n_k is the noise.

In order to jointly remove the noise n_k, a patch-based method was utilized by fully exploiting the time redundancy in video. Image block $f_{i,j}$ is considered as reference patch with the size of $n \times n$ centered at pixel j in image g_k, and we will search for the patches similar to $f_{i,j}$ in other frames and within the neighborhood of the image g_k. Assuming that we found d patches $\{f_{i,j,k}\}_{i=1}^{d}$ similar to $f_{i,j}$ in both spatial and temporal domain, the vector $F_{i,j,k} \in R^{n^2}$ can be used to present $f_{i,j,k}$ by concatenating all columns of the patches. We define an $n^2 \times d$ matrix $F_{j,k} = (f_{1,j,k}, f_{2,j,k}, \cdots, f_{d,j,k})$. Then, the function (2) can be rewritten as:

$$F_{j,k} = P_{j,k} + N_{j,k} \tag{3}$$

where $P_{j,k}$ denotes the matrix corresponding to clean images and $N_{j,k}$ signifies the noise. According to the compressed sensing theory, the complete matrix can be recovered from a small amount of elements under the condition of sparseness. The main idea of our paper is based on low rank matrix completion by keeping the reliable elements and discarding other elements to recover the matrix $P_{j,k}$.

3. **WMSN Video Denoising Based on Low Rank Matrix Completion.**

3.1. **Preprocessing the WMSN video and BM.** In WMSN video surveillance applications, the monitoring scene is relatively fixed, and the adjacent frames are expected to be nearly identical and have a strong spatial-temporal correlation. Therefore, we can perform some pretreatments on the WMSN video due to its strong correlation. As a very simple but effective method for grouping similar patches, block matching plays an important role in video processing field, such as motion estimation, tracking and video compression. If the video is seriously damaged by large noise, the accuracy of BM cannot be guaranteed. Random-valued impulse noise obviously affects the accuracy of BM, so we apply center-weighted adaptive median filter to removing the outlier firstly [8]. The intermediate result is then used for BM further, which will greatly enhance the quality. Given a reference patch, the proposed block matching will find the d most similar patches to the reference patch in the neighbor frames and within the image frame. In this paper, we adopt a fast searching method to accelerate the progress of BM.

3.2. **Denoising the patched matrix by using penalty decomposition.** After performing the step above, most similar patches will form the matrix $F_{j,k}$ with the size of $n^2 \times d$. In order to estimate a low-rank matrix P from the given corrupted patch matrix F, the recovery of P is done by solving the following convex optimization problem:

$$\min_{P} \|P\|_*, \quad \text{s.t. } P|_\Omega = F \tag{4}$$

where Ω denotes the index set of the reliable pixels detected by preprocessing, $\cdot|_\Omega$ denotes the projection operator of a matrix, and $\|\cdot\|_*$ denotes the nuclear norm of a matrix.

Many authors have proposed efficient methods for solving such a nuclear norm-related minimization problem, and a fixed point iterative algorithm is used in [9]. By incorporating approximate singular value decomposition technique in [10], the solution to the matrix rank minimization problem is usually obtained by the fixed point continuation algorithm (FPCA). It is proved in [11] that the matrix can be exactly reconstructed on the condition that noise matrix is nonzero and within a certain range as well as the rank $r \ll \min(m, n)$ and the sample amount $m \geq Cn^{5/4} r \log n$, where n is the matrix dimension and C is a constant.

The function (4) can also be rewritten as:

$$\min_{P} rank(P) \quad \text{s.t. } P_{ij} = F_{ij},\ (i,j) \in \Omega \tag{5}$$

In the work of [12], the problem of (5) can be solved by using Lagrange described method while the performance is not so good. Candès and Recht [11] proved that most low rank matrices can be recovered by solving an NNM problem while costing a lot of time.

In order to better solve minimization problem, the Penalty Decomposition (PD) method [13] was used in our paper. Clearly, (5) can be equivalently reformulated as

$$\min_{X,P} \{f(X) + \mu rank(P) : X - P = 0,\ X \in \chi,\ P \in \Theta\} \tag{6}$$

given a penalty parameter $\rho > 0$, the associated penalty function for (6) is defined as

$$P_\rho(X, P) := f(X) + \mu rank(P) + \frac{\rho}{2}\|X - P\|_F^2 \tag{7}$$

The main steps are shown in Table 1.

Table 1. The main steps of PD method

Algorithm 1 PD method for solving (7)

1) Let $\rho_0 > 0$, $\sigma > 1$ be given and set $k = 0$. Choose an arbitrary $P_0^0 \in \Theta$ and a constant γ such that $\gamma \geq \max\left\{f\left(X^{feas}\right) + \mu rank\left(X^{feas}\right),\ \min_{X\in\chi} P_{\rho_0}\left(X, P_0^0\right)\right\}$.
2) Apply the block coordinate descend (BCD) method [14] to converting (7) to the penalty sub-problem:

$$\min\left\{P_{\rho_k}(X, P) : X \in \chi,\ P \in \Theta\right\} \tag{8}$$

3) Find an approximate solution $\left(X^k, P^k\right) \in \chi \times \Theta$ to (8) by performing steps 3a)-3d).
  3a) Solve $X_{l+1}^k \in \arg\min_{X\in\chi} P_{\rho_k}\left(X, P_l^k\right)$.
  3b) Solve $P_{l+1}^k \in \arg\min_{P\in\Theta} P_{\rho_k}\left(X_{l+1}^k, P\right)$.
  3c) Set $\left(X^k, P^k\right) := \left(X_{l+1}^k, P_{l+1}^k\right)$.

If $\left(X^k, P^k\right)$ satisfies $\|P_\chi\left(X^k - \nabla_X Q_{\rho_k}\left(X^k, P^k\right)\right) - X^k\|_F \le \varepsilon_k$, where $Q_\rho(X,P) := f(X) + \frac{\rho}{2}\|X-P\|_F^2$, then go to step 4).

3d) Set $l \to l+1$ and go to step 3a).

4) Set $\rho_{k+1} := \sigma_{\rho_k}$.

5) If $\min P_{\rho_{k+1}}\left(X, P^k\right) > \gamma$, set $P_0^{k+1} := X^{feas}$. Otherwise, set $P_0^{k+1} := P^k$.

6) Set $l \to l+1$ and go to step 1).

End

Therefore, the main parts of this method lie in solving the sub-problems as follows

$$\min\left\{\|X-A\|_F^2 : X \in \chi\right\} \tag{9}$$

$$\min\left\{rank(P) + \rho\,\|P-B\|_F^2 : P \in R^{m\times n}\right\} \tag{10}$$

where $\chi = \{X \in R^{m\times n} : X_{ij} = F_{ij},\ (i,j) \in \Omega\}$, for some $\rho > 0$, $A, B \in R^{m\times n}$, respectively. According to the definition of χ, we observe that sub-problem (9) and sub-problem (10) both have a closed-form solution.

The appropriate initialization and the termination criteria for our PD method should be selected to denoise the WMSN video. In particular, we choose X^{feas} to be the $m \times n$ matrix, which satisfies $X_{ij}^{feas} = F_{ij}$ for all $(i,j) \in \Omega$ and $X_{ij}^{feas} = 0$ for all $(i,j) \notin \Omega$, and then set $P_0^0 = X^{feas}$. Besides, we set the initial penalty parameter $\rho_0 = 0.1$ and the parameter $\sigma = 5$.

3.3. **Reconstructing the WMSN video from denoised patches.** Each frame in WMSN video should follow the steps above, and most of impulse noise in the patches can be removed. The denoised patches will form the denoised image and thus a denoised video sequence can be reconstructed. It is noteworthy that the overlapping area in block will be sampled so that every pixel will be contained in several patches. Finally, boundary effect can be greatly restrained due to the average of the denoised patches' results.

4. **Experimental Results and Discussions.** To evaluate our performance, we present some video denoising examples using existing sequences. All tests in this section were processed in the following manner. All 30 frames were involved in this processing of each image, these images are 256×256 pixels in size and contain different content and textures and the similar block size used for block matching is 16×16. The peak-signal-to-noise ratio (PSNR), subjective vision and rank level were employed as performance metrics.

In Figure 1, we show the PSNR result and clear visual comparison on the Car sequence and Rail sequence. In our experiment, the original video is seriously corrupted by mixed noise with Gaussian white noise of mean zero and variance 0.1, and impulsive noise of the noise density 0.01. Compared with VBM3D method, the proposed method yields most visually pleasant result, and most of detail structures are retained, while VBM3D method is not robust to impulsive noise or outliers. Moreover, the proposed method performs better which is also validated by its PSNR values, which surpasses the VBM3D method in all frames for all sequences with more than 2dB. Therefore, the results fully demonstrate the superiority of the proposed method.

As our aim is to find a successful recovery with the lowest possible rank, for each sample ratio, we compared PD method with the aforementioned method FPCA [10]. The computational results are shown in Table 2. It can be seen that running time becomes shorter with the increase of sample rate, and it follows that our PD method is capable of producing a matrix with smallest possible rank to successfully recover the denoised matrix, but the other cannot. In addition, the proposed method achieves higher reconstruction accuracy.

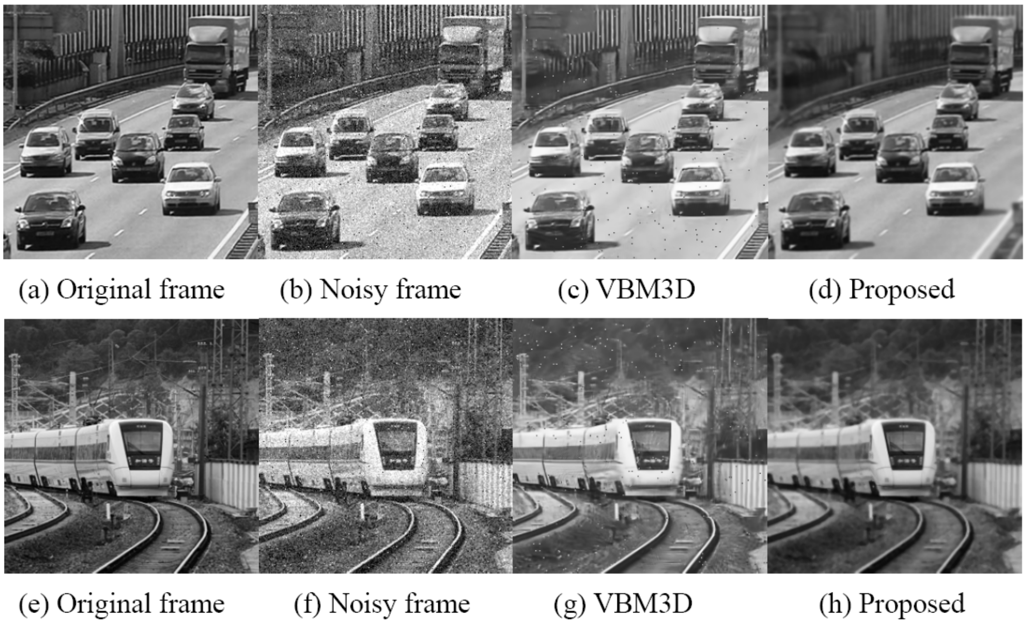

Figure 1. Visual quality comparison of denoising results for Car and Rail. PSNR for (b)-(d) are 16.61dB, 19.02dB and 22.73dB, PSNR for (f)-(h) are 16.44dB, 18.71dB, 22.27dB.

Table 2. The rank level and related-error of different algorithms

sample rate	FPCA			Proposed		
	Rank	Error	Time	Rank	Error	Time
0.1	11.9	3.04E-02	32.6	4.0	9.67E-04	54.2
0.2	5.0	5.16E-04	17.2	4.0	9.04E-04	20.9
0.3	5.0	3.97E-04	14.6	4.0	8.01E-04	16.1
0.4	5.0	3.66E-04	11.2	4.0	7.09E-04	13.6
0.5	5.5	3.26E-04	7.8	4.0	6.23E-04	8.4
0.6	5.6	3.22E-04	6.9	4.0	5.42E-04	6.4

5. **Conclusion.** In this paper, a new low rank completion method is proposed to be applied to video denoising in WMSN, which can successfully recover the denoised matrix via penalty decomposition method to solve the minimization problem. At the same time, in order to improve the computational efficiency, a fast block matching algorithm is proposed. Experimental results demonstrate that the proposed method significantly improves the peak-signal-to-noise ratio and reconstruction accuracy compared to other state-of-the-art methods. Our future work will focus on seeking more efficient ways to design a robust algorithm for low rank matrix completion.

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REFERENCES

[1] I. F. Akyildiz, T. Melodia and K. R. Chowdhury, Wireless multimedia sensor networks: Applications and testbeds, *Proc. of the IEEE*, vol.96, no.10, pp.1588-1605, 2008.

[2] M. Mahmoudi and G. Sapiro, Fast image and video denoising via nonlocal means of similar neighborhoods, *Signal Processing Letters*, vol.12, no.12, pp.839-842, 2005.

[3] J. Yang, Y. Wang, W. Xu et al., Image and video denoising using adaptive dual-tree discrete wavelet packets, *IEEE Trans. Circuits and Systems for Video Technology*, vol.19, no.5, pp.642-655, 2009.

[4] W. Li, J. Zhang and Q. Dai, Video denoising using shape-adaptive sparse representation over similar spatio-temporal patches, *Signal Processing: Image Communication*, vol.26, no.4, pp.250-265, 2011.

[5] D. Kostadin, A. Foi and K. Egiazarian, Video denoising by sparse 3D transform-domain collaborative filtering, *Proc. of the 15th European Signal Processing Conference*, Poznan, Poland, vol.1, no.2, 2007.

[6] R. Ma, N. Barzigar, A. Roozgard and S. Cheng, Decomposition approach for low-rank matrix completion and its applications, *IEEE Trans. Signal Processing*, vol.62, no.7, pp.1671-1683, 2014.

[7] Z. Shan and K. Ma, A new diamond search algorithm for fast block-matching motion estimation, *IEEE Trans. Image Processing*, vol.9, no.2, pp.287-290, 2000.

[8] H. Ibrahim, N. S. P. Kong and T. F. Ng, Simple adaptive median filter for the removal of impulse noise from highly corrupted images, *IEEE Trans. Consumer Electronics*, vol.54, no.4, pp.1920-1927, 2008.

[9] S. Ma, D. Goldfarb and L. Chen, Fixed point and Bregman iterative methods for matrix rank minimization, *Mathematical Programming*, vol.128, no.1, pp.321-353, 2011.

[10] D. Goldfarb and S. Ma, Convergence of fixed-point continuation algorithms for matrix rank minimization, *Foundations of Computational Mathematics*, vol.11, no.2, pp.183-210, 2011.

[11] E. J. Candès and B. Recht, Exact matrix completion via convex optimization, *Foundations of Computational Mathematics*, vol.9, no.6, pp.717-772, 2009.

[12] E. J. Candès and Y. Plan, Matrix completion with noise, *Proc. of the IEEE*, vol.98, no.6, pp.925-936, 2010.

[13] Z. Lu, Y. Zhang and X. Li, Penalty decomposition methods for rank minimization, *Optimization Methods and Software Ahead-of-Print*, pp.1-28, 2014.

[14] P. Tseng, Convergence of a block coordinate descent method for non-differentiable minimization, *Journal of Optimization Theory and Applications*, vol.109, no.3, pp.475-494, 2001.