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AN APPROXIMATED LINEAR PROGRAMMING APPROACH FOR MEDIA RESOURCE ALLOCATION PROBLEM

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Abstract. *Currently, media planners have to face the media resource allocation problem (MRAP) such that the effectiveness of advertisements can be maximized. The MRAP includes independent types of media and different segments of advertisements for a given period of time. Subject to budgets and other constraints of advertisements, the media planners have to schedule the appropriate number of media in appropriate segments to achieve the maximal effectiveness of advertisements. In the past, based upon the weapon-target assignment problem, a nonlinear integer programming has been proposed for solving the MRAP. In this paper we show that this nonlinear MRAP can be approximately converted into a linear programming. In the new approach, several new constraints are added to replace the objective function in the MRAP. After replacing the objective function with the new constraints, we can then easily solve the problem. Numerical results show the effectiveness of the proposed new approach.*
Keywords: Media resource allocation, Linear programming, Optimization

1. **Introduction.** In the media industry, the types of media include television, radio, Internet, newspapers, printed materials and electronic mail (E-mail), etc. Advertisers often use the media to convey their commercial messages to audiences and potential customers for better sales and profits. Media planners usually schedule advertisement segments and advertisement quantity to maximize the effectiveness of all advertisements (ads) for the maximal profit. Typically, media planners aim to maximize the coverage of potential audience and firstly collect information related media ratings to determine the number of ads assigned to each segment [1]. In addition, media planners will cooperate with different media to achieve their best benefits. The so-called media resource allocation problem (MRAP) aims to maximize the effectiveness of ads by determining the ads types and the number of ads subject to the constraints of limited budgets and capacity of ads.

The first study of MRAP appeared in the early 1960s. Little and Lodish [2] proposed a dynamic programming method for the media distribution. In addition, some studies, for example, Charnes et al. [3] studied the MRAP using goal programming. Zufryden [4] investigated a probabilistic model for the MRAP. Locander et al. [1] used a nonlinear efficiency curve to structure an MRAP model. Fruchter and Kalish [5] and Berkowitz et al. [6] adopted various approaches to study the allocation of advertising budgets for media.

Additionally, Berkowitz et al. [6] also investigated the impact of differential lag effects for the allocation of advertising budgets. In 2006, based upon weapon-target assignment problem, Cetin and Esen [7] developed a nonlinear integer programming for solving the problem. Recently, Pérez-Gladish et al. [8] proposed a two-objective model to investigate the advertising on TV for products with fuzzy information. Jha et al. [9] studied the optimal advertising for multiple products with budget constraint. Jha and Aggarwal [10] further extended the model of [9] to one with fuzzy environment. Note that the models of [9,10] are multi-objective and linear programming.

In the past, most of approaches for MRAP are time consuming, e.g., dynamic programming method. Besides, several researchers were devoted to the optimal planning of media for products with linear programming models. In this study, we will propose a new solution procedure for the MRAP that is considered by Cetin and Esen [7]. This considered MARP is formulated as a nonlinear integer programming. Our new approach will convert the nonlinear objective function into new linear constraints, and then obtain the approximated real solution for the considered MRAP. Next, a procedure is used to improve the approximated real solution into integer solution for MRAP. In other words, this nonlinear integer programming can be approximately solved by a linear programming. Numerical experiments show that the proposed method in this paper can effectively solve the MRAP.

2. Media Resource Allocation Problem (MRAP).

2.1. Notations [7].

- W the number of kinds of advertisements,
- w the number of renewable (can be updated during the day) media type,
- t the number of TV appropriate to be advertised,
- T the number of segments,
- W_i the number of advertisements type i available,
- T_j the minimum number of ads required for target audience j,
- U_j the relative segment weights,
- p_{ij} the probability of reaching the target audience j by a single ad type i,
- c_{ij} the unit variable cost of an ad i to the target audience j,
- x_{ij} the number of advertisements of type i assigned to target audience j,
- B the total advertising campaign budget,
- ρ_1 the upper limit for percentage of total budget invested to TV,
- ρ_2 the upper limit for percentage of total budget invested to radio,
- ρ_3 the upper limit for percentage of total budget invested to newspaper.

2.2. Assumptions.

(a) There are W kinds of advertisements.
(b) There are T segments for ads.
(c) The probability of reaching the target audience j by a single ad type i is p_{ij}.
(d) The unit variable cost of an ad i to the target audience j is c_{ij}.
(e) Subject to the constraints of budgets and capacity of ads, the planner aims to maximize the effectiveness of ads, i.e., $\sum_{j=1}^{T} U_j \left(1 - \prod_{i=1}^{W} (1-p_{ij})^{x_{ij}}\right)$.

2.3. Model.

$$\text{Max} \quad \sum_{j=1}^{T} U_j \left(1 - \prod_{i=1}^{W} (1-p_{ij})^{x_{ij}}\right) \tag{1}$$

$$\text{s.t.} \quad \sum_{j=1}^{T}\left(\sum_{i=1}^{w} c_{ij}x_{ij} + \sum_{i=w+1}^{W} \frac{1}{T}c_{ij}x_{ij}\right) \leq B \tag{2}$$

$$\sum_{j=1}^{T}\sum_{i=1}^{t} c_{ij}x_{ij} \leq B\rho_1 \tag{3}$$

$$\sum_{j=1}^{T}\sum_{i=t+1}^{r} c_{ij}x_{ij} \leq B\rho_2 \tag{4}$$

$$\sum_{j=1}^{T}\sum_{i=w+1}^{n} \frac{1}{T}c_{ij}x_{ij} \leq B\rho_3 \tag{5}$$

$$\sum_{j=1}^{T} x_{ij} \leq W_i, \quad i = w+1, \ldots, W \tag{6}$$

$$\sum_{i=1}^{W} x_{ij} \geq T_j, \quad j = 1, 2, \ldots, T-1 \tag{7}$$

$$x_{ij} = x_{i,j+1}, \quad i = w+1, \ldots, W, \quad j = 1, 2, \ldots, T-1 \tag{8}$$

$$x_{ij} \geq 0 \text{ and integers}, \quad i = 1, 2, \ldots, W, \quad j = 1, 2, \ldots, T \tag{9}$$

Objective Function (1) is to maximize the coverage of potential audience. Constraint (2) is the available budget. Constraints (3)-(5) are the restrictions of budget ratios for TV, radio and newspapers. Constraint (6) is the restriction of the maximum number of ads for each media. Constraint (7) is the minimal number of ads for each segment. Constraint (8) shows that printing media has equal number of ads for each segment. Constraint (9) is the range for variables. Note that Objective Function (1) is a nonlinear function.

Below we demonstrate our solution procedure for the considered MRAP. An example is solved to illustrate the proposed approach. The numerical result is compared with that by the well-known optimization software LINGO and the effectiveness of the proposed approach for the considered MRAP is discussed.

3. Method.

3.1. Basic theory.

Our approach is mainly based upon the following Lemma.

Lemma 3.1. $\frac{x_1+x_2+x_3+\cdots+x_n}{n} \geq \sqrt[n]{x_1x_2x_3\ldots x_n}$ *for* $x_i \geq 0$. *When* $x_1 = x_2 = x_3 = \cdots = x_n$ *or* $Ln(x_1) = Ln(x_2) = Ln(x_3) = \cdots = Ln(x_n)$, *the equality holds and* $x_1 + x_2 + \cdots + x_n$ *is minimized.*

Proof: It is trivial to prove by mathematical induction and we omit it.

3.2. Linearization.

By Objective Function (1), we have:

$$\text{Max} \sum_{j=1}^{T} U_j\left(1 - \prod_{i=1}^{W}(1-p_{ij})^{x_{ij}}\right)$$

$$\Leftrightarrow \text{Max} \sum_{j=1}^{T} U_j - \sum_{j=1}^{T} U_j \prod_{i=1}^{W}(1-p_{ij})^{x_{ij}}$$

$$\Leftrightarrow \text{Min} \sum_{j=1}^{T} U_j \prod_{i=1}^{W}(1-p_{ij})^{x_{ij}} \quad \left(\text{Since } \sum_{j=1}^{T} U_j \text{ is a constant}\right)$$

Let $U_j \prod_{i=1}^{W} (1-p_{ij})^{x_{ij}} = x_j$, $1 \le j \le T$. By Lemma 3.1, $\sum_{j=1}^{T} U_j \prod_{i=1}^{W} (1-p_{ij})^{x_{ij}}$ is minimized if

$$Ln\left(U_1 \prod_{i=1}^{W}(1-p_{i1})^{x_{i1}}\right) = Ln\left(U_2 \prod_{i=1}^{W}(1-p_{i2})^{x_{i2}}\right) = \cdots = Ln\left(U_T \prod_{i=1}^{W}(1-p_{iT})^{x_{iT}}\right)$$

$$\Leftrightarrow Ln(U_1) + \sum_{i=1}^{W} x_{i1} Ln(q_{i1}) = Ln(U_2) + \sum_{i=1}^{W} x_{i2} Ln(q_{i2}) = \cdots = Ln(U_T) + \sum_{i=1}^{W} x_{iT} Ln(q_{iT}) \tag{10}$$

Note that Objective Function (1) is nonlinear; however, Constraint (10) is linear. Solving (2)-(10) we may obtain an approximated real solution for the MRAP. Then we may round-off the approximated real solution and minus one to obtain a feasible approximated integer solution. Next, we re-allocate the remaining resources and obtain the final integer solution for MRAP.

3.3. **Example.** Consider the example in Table 1. Table 1 shows the probability p_{ij} and cost c_{ij} of two media ($W = 2$), ATV and BTV, in four segments ($T = 4$), namely, morning, afternoon, prime, and night time. The numbers of ads required, segment weights, and ad capacities are also shown in Table 1. The steps by steps of our approach are listed as follows.

Step 1: Convert the probability matrix (p_{ij}) into a new probability matrix (q_{ij}) as Table 2.

Step 2: Solving (2)-(10), we have the approximated real solution shown in Table 3. Note that the approximated objective value = 8.198028863156 is an infeasible solution.

Step 3: Round-off the approximated real solution and minus one to obtain a feasible approximated integer solution as those in Table 3. The approximated objective value = 7.407115900234 is a feasible solution.

Step 4: Re-allocate the remaining resources by linear programming and obtain the final integer solution as those in Table 3. The new integer solution objective value = 8.195222998860 is a feasible solution.

Table 1. The probability and cost matrices for the example

$p_{ij}(c_{ij})$	morning time	afternoon time	prime time	night time	Ad capacity
ATV(1)	0.21 (0.140)	0.12 (0.120)	0.12 (0.140)	0.23 (0.150)	16
BTV(2)	0.35 (0.110)	0.24 (0.130)	0.12 (0.150)	0.07 (0.100)	13
ads required	3	4	6	5	–
Segment weights	2	3	4	1	–

Table 2. The new probability matrix (q_{ij})

q_{ij}	morning time	afternoon time	prime time	night time
ATV(1)	0.79	0.88	0.88	0.77
BTV(2)	0.65	0.76	0.88	0.93

Table 3. The approximated real/approximated integer/new improved integer solution

solution	morning time	afternoon time	prime time	night time	Ad capacity
ATV(1)	0.0/0.0/0.0	0.0/0.0/0.0	11.0/10.0/11.0	5.0/4.0/5.0	16
BTV(2)	4.76/3.0/5.0	7.30/6.0/7.0	0.94/0.0/1.0	0.00/0.0/0.0	13

Table 4. The probability and cost matrices for the test problem (30 media and 8 segments)

Ads/Segment	Day 1 Morning		Day 1 Afternoon		Day 1 Prime		Day 1 Night		Day 2 Morning		Day 2 Afternoon		Day 2 Prime		Day 2 Night		Ad capacity
	p_{i1}	c_{i1}	p_{i2}	c_{i2}	p_{i3}	c_{i3}	p_{i4}	c_{i4}	p_{i5}	c_{i5}	p_{i6}	c_{i6}	p_{i7}	c_{i7}	p_{i8}	c_{i8}	
ATV	0.21	0.140	0.12	0.120	0.12	0.140	0.23	0.150	0.21	0.140	0.12	0.120	0.12	0.140	0.23	0.150	16
BTV	0.35	0.110	0.24	0.130	0.12	0.150	0.07	0.100	0.35	0.110	0.24	0.130	0.12	0.150	0.07	0.100	14
CTV	0.19	0.100	0.04	0.043	0.00	0.100	0.19	0.085	0.19	0.100	0.04	0.043	0.00	0.100	0.19	0.085	18
DTV	0.00	0.130	0.26	0.130	0.19	0.130	0.13	0.125	0.00	0.130	0.26	0.130	0.19	0.130	0.13	0.125	10
ETV	0.13	0.130	0.19	0.135	0.25	0.140	0.00	0.140	0.13	0.130	0.19	0.135	0.25	0.140	0.00	0.140	12
FTV	0.24	0.150	0.14	0.150	0.22	0.150	0.09	0.150	0.24	0.150	0.14	0.150	0.22	0.150	0.09	0.150	16
GTV	0.09	0.160	0.00	0.160	0.18	0.160	0.28	0.180	0.09	0.160	0.00	0.160	0.18	0.160	0.28	0.180	6
K-RADIO	0.39	0.015	0.17	0.010	0.47	0.010	0.00	0.010	0.39	0.015	0.17	0.010	0.47	0.010	0.00	0.010	20
L-RADIO	0.24	0.025	0.31	0.020	0.14	0.020	0.43	0.027	0.24	0.025	0.31	0.020	0.14	0.020	0.43	0.027	30
Internet	0.10	0.050	0.23	0.050	0.03	0.050	0.35	0.065	0.10	0.050	0.23	0.050	0.03	0.050	0.35	0.065	24
P-Newspaper	0.12	0.100	0.11	0.100	0.03	0.100	0.09	0.100	0.12	0.100	0.11	0.100	0.03	0.100	0.09	0.100	16
R-Newspaper	0.32	0.160	0.23	0.160	0.09	0.160	0.21	0.160	0.32	0.160	0.23	0.160	0.09	0.160	0.21	0.160	8
Billboard	0.32	0.096	0.10	0.096	0.28	0.096	0.02	0.096	0.32	0.096	0.10	0.096	0.28	0.096	0.02	0.096	8
Printings	0.23	0.020	0.12	0.020	0.08	0.020	0.03	0.020	0.23	0.020	0.12	0.020	0.08	0.020	0.03	0.020	8
E-mail	0.29	0.008	0.07	0.008	0.04	0.008	0.32	0.008	0.29	0.008	0.07	0.008	0.04	0.008	0.32	0.008	8
ATV-2nd	0.21	0.140	0.12	0.120	0.12	0.140	0.23	0.150	0.21	0.140	0.12	0.120	0.12	0.140	0.23	0.150	16
BTV-2nd	0.35	0.110	0.24	0.130	0.12	0.150	0.07	0.100	0.35	0.110	0.24	0.130	0.12	0.150	0.07	0.100	14
CTV-2nd	0.19	0.100	0.04	0.043	0.00	0.100	0.19	0.085	0.19	0.100	0.04	0.043	0.00	0.100	0.19	0.085	18
DTV-2nd	0.00	0.130	0.26	0.130	0.19	0.130	0.13	0.125	0.00	0.130	0.26	0.130	0.19	0.130	0.13	0.125	10
ETV-2nd	0.13	0.130	0.19	0.135	0.25	0.140	0.00	0.140	0.13	0.130	0.19	0.135	0.25	0.140	0.00	0.140	12
FTV-2nd	0.24	0.150	0.14	0.150	0.22	0.150	0.09	0.150	0.24	0.150	0.14	0.150	0.22	0.150	0.09	0.150	16
GTV-2nd	0.09	0.160	0.00	0.160	0.18	0.160	0.28	0.180	0.09	0.160	0.00	0.160	0.18	0.160	0.28	0.180	6
K-RADIO-2nd	0.39	0.015	0.17	0.010	0.47	0.010	0.00	0.010	0.39	0.015	0.17	0.010	0.47	0.010	0.00	0.010	20
L-RADIO-2nd	0.24	0.025	0.31	0.020	0.14	0.020	0.43	0.027	0.24	0.025	0.31	0.020	0.14	0.020	0.43	0.027	30
Internet-2nd	0.10	0.050	0.23	0.050	0.03	0.050	0.35	0.065	0.10	0.050	0.23	0.050	0.03	0.050	0.35	0.065	24
P-Newspaper-2nd	0.12	0.100	0.11	0.100	0.03	0.100	0.09	0.100	0.12	0.100	0.11	0.100	0.03	0.100	0.09	0.100	16
R-Newspaper-2nd	0.32	0.160	0.23	0.160	0.09	0.160	0.21	0.160	0.32	0.160	0.23	0.160	0.09	0.160	0.21	0.160	8
Billboard-2nd	0.32	0.096	0.10	0.096	0.28	0.096	0.02	0.096	0.32	0.096	0.10	0.096	0.28	0.096	0.02	0.096	8
Printings-2	0.23	0.020	0.12	0.020	0.08	0.020	0.03	0.020	0.23	0.020	0.12	0.020	0.08	0.020	0.03	0.020	8
E-mail-2	0.29	0.008	0.07	0.008	0.04	0.008	0.32	0.008	0.29	0.008	0.07	0.008	0.04	0.008	0.32	0.008	8
ads required	32		36		50		20		32		36		50		20		
Segment weights	2		3		4		1		4		6		8		2		

4. **Numerical Results.** To test the effectiveness of the proposed method in this paper, we solve two main test problems.

(a) Problem 1: This test problem ([7]) has 15 media and 4 segments;

(b) Problem 2: This test problem, with 30 media and 8 segments, is extended from Problem 1. The corresponding data are shown in Table 4.

The solution of Problem 1 obtained by the new approach is the same as that of [7]. The solution of Problem 2 is reported in Table 5. From Table 5, one observes the final integer solutions by the proposed method with objective value = 29.999993147261 which is better than the objective value = 29.999990387650 obtained by the well-known solver LINGO. From these two test problems, it implies that the proposed approach can be effective and efficient to solve the considered MRAP.

5. **Conclusions and Future Research.** In this paper, we have proposed an effective approach to approximately solve the MRAP. This approach converts the nonlinear MRAP into a linear programming which can be solved directly. In the future, one may attempt to use artificial intelligence methods to investigate the nonlinear MRAP.

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Table 5. The best solution by the proposed method

Ads/Segment	Day 1 Morning	Day 1 Afternoon	Day 1 Prime	Day 1 Night	Day 2 Morning	Day 2 Afternoon	Day 2 Prime	Day 2 Night
ATV	0	0	0	0	0	0	0	0
BTV	9	0	0	0	5	0	0	0
CTV	0	0	0	0	2	0	0	16
DTV	0	5	0	0	0	5	0	0
ETV	0	0	1	0	0	0	11	0
FTV	1	0	12	0	0	0	1	0
GTV	0	0	0	0	0	0	0	0
K-RADIO	0	0	6	0	0	0	14	0
L-RADIO	0	25	0	0	0	0	0	5
Internet	0	2	0	14	0	2	0	6
P-Newspaper	2	2	2	2	2	2	2	2
R-Newspaper	1	1	1	1	1	1	1	1
Billboard	1	1	1	1	1	1	1	1
Printings	1	1	1	1	1	1	1	1
E-mail	1	1	1	1	1	1	1	1
ATV-2nd	0	0	0	0	0	0	0	0
BTV-2nd	10	0	0	0	4	0	0	0
CTV-2nd	13	0	0	0	3	0	0	2
DTV-2nd	0	4	0	0	0	5	1	0
ETV-2nd	0	0	1	0	0	0	11	0
FTV-2nd	0	0	13	0	0	0	0	0
GTV-2nd	0	0	0	0	0	0	0	0
K-RADIO-2nd	0	0	6	0	14	0	0	0
L-RADIO-2nd	0	1	0	0	0	26	0	3
Internet-2nd	0	1	0	15	0	3	0	5
P-Newspaper-2nd	2	2	2	2	2	2	2	2
R-Newspaper-2nd	1	1	1	1	1	1	1	1
Billboard-2nd	1	1	1	1	1	1	1	1
Printings-2	1	1	1	1	1	1	1	1
E-mail-2	1	1	1	1	1	1	1	1

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