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A NOVEL INTERACTING MULTIPLE MODEL ALGORITHM FOR MANEUVERING TARGET TRACKING

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Abstract. *To deal with computerized intractability and imprecise estimates of the standard interacting multiple model (IMM) algorithm, a novel IMM algorithm is presented in this paper. First we introduce the adaptive adjustment coefficient to simplify calculation of the Markov transition probability matrix. Afterwards the covariance matrix of process noise is modified to improve precision of estimates when target is maneuvering. Finally a typical scenario of maneuvering target tracking is performed to validate performance of the proposed IMM algorithm. Simulation results show that the proposed IMM algorithm can complete maneuvering target tracking with better efficiency and reliability.*
Keywords: Interacting multiple model, Maneuvering target, Kalman filter, Transition probability

1. **Introduction.** The interacting multiple model (IMM) algorithm has perfect tracking performance regarding to maneuvering target tracking, which can estimate the dynamic state of targets with many motion models [1,2]. Although the standard IMM algorithm is widely used in various applications, there are some inherent defects in calculation of the Markov transition probability matrix and the process noise.

Recently many scholars have studied the IMM algorithm with a great deal of success and many papers have been published in important international journals [3-6]. In [3] an adaptive IMM algorithm was designed to complete integration of inertial navigation system/global positioning system data using a limited number of subfilters formed based on the rough values obtained from fuzzy adaptive Kalman filter (KF). In [4] the fuzzy adaptive KF and the standard IMM algorithm were integrated to estimate various states. A modified IMM algorithm in [5] was constructed for maneuvering occurrence and switching initialization. In [6] a fuzzy adaptive IMM algorithm was put forward to adjust model transition probability by fuzzy reasoning in order to adaptively change motion model set. However these works all involve fuzzy theory, which bring about extra computational complexity. In addition, the precision of state estimates should be further improved when the target is maneuvering.

Aiming at the above problems, we present a novel IMM algorithm for maneuvering target tracking in this note. The main contributions of this work are summarized as: i) the calculation of the Markov transition probability matrix is simplified with the adaptive adjustment coefficient; ii) the process noise is regulated in order to enhance the estimation precision when the target is maneuvering. The remainder of this note is as follows. Section 2 presents problem definitions of the standard IMM algorithm. In Section 3, the principle of the novel IMM algorithm is proposed. The implementation of the proposed IMM algorithm is presented in Section 4. Section 5 shows the numerical study to validate tracking performance of the proposed IMM algorithm. In Section 6, we draw the conclusions with the future work.

2. Problem Definitions. Assume $\boldsymbol{X}_k$ and $\boldsymbol{Z}_k$ are the state vector and the measurement vector respectively, then the equations of the state and measurement are given by

$$\boldsymbol{X}_k = \boldsymbol{F}_{k|k-1}\boldsymbol{X}_{k-1} + \boldsymbol{\Gamma}_k\boldsymbol{W}_k \tag{1}$$

$$\boldsymbol{Z}_k = \boldsymbol{H}_k\boldsymbol{X}_k + \boldsymbol{V}_k \tag{2}$$

In (1) and (2), $\boldsymbol{F}_{k|k-1}$ and $\boldsymbol{H}_k$ are the state transition matrix and the measurement matrix, $\boldsymbol{\Gamma}_k$ is the state noise input matrix, and $\boldsymbol{W}_k$ and $\boldsymbol{V}_k$ are the state noise vector and the process noise vector with the following statistic characteristics [7]:

$$\begin{cases} \mathrm{E}\left[\boldsymbol{W}_k\right] = 0 \\ \mathrm{E}\left[\boldsymbol{W}_k\boldsymbol{W}_k^{\mathrm{T}}\right] = \boldsymbol{R}_k \end{cases}, \quad \begin{cases} \mathrm{E}\left[\boldsymbol{V}_k\right] = 0 \\ \mathrm{E}\left[\boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{T}}\right] = \boldsymbol{Q}_k \end{cases}$$

where $\mathrm{E}[\cdot]$ denotes the mathematical expectation, $(\cdot)^{\mathrm{T}}$ denotes the transpose of matrix, and $\boldsymbol{R}_k$ and $\boldsymbol{Q}_k$ are the covariance matrices of the state noise and the process noise respectively.

As we know, the standard IMM algorithm requires many models to match the dynamic states of maneuvering target. The tracking performance depends on the set of models. Therefore, the number of models can lead to additional computational cost in the measurement update step [8]. In addition, $\boldsymbol{V}_k$ varies with the dynamic state of targets. However, the standard IMM algorithm considers $\boldsymbol{Q}_k$ as a constant during the tracking process and cannot represent the actual dynamics of maneuvering target.

3. Principle of the Proposed IMM Algorithm. Assume the transition probability matrix from models $m_{k-1}^{(j)}$ to $m_k^{(i)}$ is determined by the Markov transition probability $\pi_{k-1}^{(ji)}$, we have

$$\boldsymbol{\Pi}_{k-1}^{(ji)} = \begin{bmatrix} \pi_{k-1}^{(11)} & \cdots & \pi_{k-1}^{(1i)} \\ \vdots & \ddots & \vdots \\ \pi_{k-1}^{(j1)} & \cdots & \pi_{k-1}^{(ji)} \end{bmatrix} \tag{3}$$

Define the change rate of the posterior probability between two adjacent scans as

$$\Delta_k^{(ji)} = \mu_k^{(i)} \Big/ \mu_{k-1}^{(j)} \tag{4}$$

where $\mu_{k-1}^{(j)}$ and $\mu_k^{(i)}$ are the posterior probability densities of $m_{k-1}^{(j)}$ and $m_k^{(i)}$. There is $\Delta_k^{(ji)} > 1$ when $\mu_k^{(i)} > \mu_{k-1}^{(j)}$, which means the posterior probability density from other models at the previous scan to the current model is increasing, and vice versa.

According to (3) and (4), the updated Markov transition probability $\pi_k^{(ji)}$ is given by

$$\pi_k^{(ji)} = \Delta_k^{(ji)}\pi_{k-1}^{(ji)} \Bigg/ \sum_{i=1}^{l} \Delta_k^{(ji)}\pi_{k-1}^{(ji)} \tag{5}$$

In (5), the adaptive adjustment coefficient $\rho_k^{(ji)}$ is defined as

$$\rho_k^{(ji)} = \Delta_k^{(ji)} \Bigg/ \sum_{i=1}^{l} \Delta_k^{(ji)}\pi_{k-1}^{(ji)} \tag{6}$$

Combining (6) with (5), we get the sum of $\pi_k^{(ji)}$.

$$\sum_{i=1}^{l} \pi_k^{(ji)} = \sum_{i=1}^{l} \rho_k^{(ji)}\pi_{k-1}^{(ji)} = 1 \tag{7}$$

Then $\pi_k^{(ji)}$ satisfies the condition of the Markov transition probability matrix because the sum of $\pi_k^{(ji)}$ in each row of $\boldsymbol{\Pi}_{k-1}^{(ji)}$ equals 1.

Subsequently we adjust $\boldsymbol{V}_k$ to represent the dynamic state of targets. It is necessary to reduce the tracking error with the increasing $\boldsymbol{V}_k$ when the target is maneuvering. On the other hand, we should enhance the tracking precision with the decreasing $\boldsymbol{V}_k$ when the target keeps non-maneuvering dynamics. Assume $\mu_{\max,k-1}^{(j)}$ is the maximum value of $\mu_{k-1}^{(j)}$, then the covariance matrix of $\boldsymbol{V}_k$ is given by

$$\boldsymbol{Q}_k^{(i)} = \begin{cases} \left(1 - \mu_{\max,k-1}^{(j)}\right)^2 \boldsymbol{Q}_{k-1}^{(j)}, & i = j \\ \left(1 + \mu_{\max,k-1}^{(j)}\right)^2 \boldsymbol{Q}_{k-1}^{(j)}, & i \neq j \end{cases} \tag{8}$$

Note that $m_{k-1}^{(j)}$ can match the current state and $\boldsymbol{Q}_k^{(i)}$ can be taken to the minimal value when $m_k^{(i)}$ and $m_{k-1}^{(j)}$ represent the same dynamic models. Otherwise $\boldsymbol{Q}_k^{(i)}$ is set to the maximal value under the condition of $i \neq j$. Therefore, the different values of $\boldsymbol{Q}_k^{(i)}$ can effectively meet various levels of maneuvers.

4. **Implementation of the Proposed IMM Algorithm.** The KF can yield perfect tracking performance for non-maneuvering target with the constant velocity (CV). However, the KF has worse precision when the target is maneuvering to some extent, such as the constant turn (CT) dynamics [9]. By comparison, the unscented KF (UKF) is competent because the unscented transform (UT) is easy to approximate nonlinear dynamics of targets. Considering the modified parameters $\pi_k^{(ji)}$ and $\boldsymbol{Q}_k^{(i)}$, we present the implementation of the proposed IMM algorithm that combined the KF with the UKF in one cycle.

Step 1. Model interaction. Let the combined state $\boldsymbol{X}_{k-1|k-1}^{(oi)}$ and its covariance $\boldsymbol{P}_{k-1|k-1}^{(oi)}$ be

$$\boldsymbol{X}_{k-1|k-1}^{(oi)} = \sum_{j=1}^{2} \mu_{k-1}^{(ji)} \boldsymbol{X}_{k-1|k-1}^{(j)} \tag{9}$$

$$\boldsymbol{P}_{k-1|k-1}^{(oi)} = \sum_{j=1}^{2} \mu_{k-1}^{(ji)} \left(\boldsymbol{P}_{k-1|k-1}^{(j)} + \left(\boldsymbol{X}_{k-1|k-1}^{(j)} - \boldsymbol{X}_{k-1|k-1}^{(oi)}\right)\left(\boldsymbol{X}_{k-1|k-1}^{(j)} - \boldsymbol{X}_{k-1|k-1}^{(oi)}\right)^{\mathrm{T}}\right) \tag{10}$$

Step 2. Time update. For the KF, the time-updated state $\boldsymbol{X}_{k|k-1}^{(1)}$ and its covariance $\boldsymbol{P}_{k|k-1}^{(1)}$ are

$$\boldsymbol{X}_{k|k-1}^{(1)} = \boldsymbol{F}_{k|k-1}^{(1)} \boldsymbol{X}_{k-1|k-1}^{(1)} + \boldsymbol{\Gamma}_{k-1}^{(1)} \boldsymbol{W}_{k-1}^{(1)} \tag{11}$$

$$\boldsymbol{P}_{k|k-1}^{(1)} = \boldsymbol{F}_{k|k-1}^{(1)} \boldsymbol{P}_{k-1|k-1}^{(1)} \left(\boldsymbol{F}_{k|k-1}^{(1)}\right)^{\mathrm{T}} + \boldsymbol{\Gamma}_{k-1}^{(1)} \boldsymbol{Q}_{k-1}^{(1)} \left(\boldsymbol{\Gamma}_{k-1}^{(1)}\right)^{\mathrm{T}} \tag{12}$$

We have the time-updated state $\boldsymbol{X}_{k|k-1}^{(2)}$ and its covariance $\boldsymbol{P}_{k|k-1}^{(2)}$ using the UKF

$$\boldsymbol{X}_{k|k-1}^{(2)} = \sum_{n=0}^{2n_{\boldsymbol{X}}} w_n \boldsymbol{\xi}_{n,k|k-1} \tag{13}$$

$$\boldsymbol{P}_{k|k-1}^{(2)} = \sum_{n=0}^{2n_{\boldsymbol{X}}} w_n \left(\boldsymbol{\xi}_{n,k|k-1} - \boldsymbol{X}_{k|k-1}^{(2)}\right)\left(\boldsymbol{\xi}_{n,k|k-1} - \boldsymbol{X}_{k|k-1}^{(2)}\right)^{\mathrm{T}} + \boldsymbol{\Gamma}_{k-1}^{(2)} \boldsymbol{Q}_{k-1}^{(2)} \left(\boldsymbol{\Gamma}_{k-1}^{(2)}\right)^{\mathrm{T}} \tag{14}$$

where $\boldsymbol{\xi}_{n,k|k-1}$ is the n^{th} sigma sampling point $(n = 0, \cdots, 2n_{\boldsymbol{X}})$ and w_n is the related weight.

According to (8), we can get the time-updated $\boldsymbol{Q}_k^{(i)}$ for the next cycle.

Step 3. Measurement update. The measurement-updated state $\boldsymbol{X}_{k|k}^{(i)}$ and its covariance $\boldsymbol{P}_{k|k}^{(i)}$ are given by

$$\boldsymbol{X}_{k|k}^{(i)} = \boldsymbol{X}_{k|k-1}^{(i)} + \boldsymbol{K}_k^{(i)} \left(\boldsymbol{Z}_k - \boldsymbol{H}_k^{(i)} \boldsymbol{X}_{k|k-1}^{(i)}\right) \tag{15}$$

$$\boldsymbol{P}_{k|k}^{(i)} = \boldsymbol{P}_{k|k-1}^{(i)} - \boldsymbol{K}_k^{(i)} \left(\boldsymbol{H}_k^{(i)} \boldsymbol{P}_{k|k-1}^{(i)} \left(\boldsymbol{H}_k^{(i)} \right)^{\mathrm{T}} + \boldsymbol{R}_k^{(i)} \right) \left(\boldsymbol{K}_k^{(i)} \right)^{\mathrm{T}} \tag{16}$$

where the Kalman gains of the KF and the UKF are respectively defined as

$$\boldsymbol{K}_k^{(1)} = \boldsymbol{P}_{k|k-1}^{(1)} \left(\boldsymbol{H}_k^{(1)} \right)^{\mathrm{T}} \left(\boldsymbol{H}_k^{(1)} \boldsymbol{P}_{k|k-1}^{(1)} \left(\boldsymbol{H}_k^{(1)} \right)^{\mathrm{T}} + \boldsymbol{R}_k^{(1)} \right)^{-1} \tag{17}$$

$$\begin{aligned} \boldsymbol{K}_k^{(2)} = {} & \left(\sum_{n=0}^{2n_{\boldsymbol{X}}} w_n \left(\boldsymbol{\xi}_{n,k|k-1} - \boldsymbol{X}_{k|k-1}^{(2)} \right) \left(\boldsymbol{\xi}_{n,k|k-1} - \boldsymbol{X}_{k|k-1}^{(2)} \right)^{\mathrm{T}} \right) \\ & \times \left(\sum_{i=0}^{2n_{\boldsymbol{X}}} w_n \left(\boldsymbol{Z}_k - \boldsymbol{H}_k^{(2)} \boldsymbol{X}_{k|k-1}^{(2)} \right) \left(\boldsymbol{Z}_k - \boldsymbol{H}_k^{(2)} \boldsymbol{X}_{k|k-1}^{(2)} \right)^{\mathrm{T}} + \boldsymbol{R}_k^{(2)} \right)^{-1} \end{aligned} \tag{18}$$

The likelihood function $L_k^{(i)}$ is given by

$$\begin{aligned} L_k^{(i)} = {} & \left(2\pi \left| \boldsymbol{H}_k^{(i)} \boldsymbol{P}_{k|k-1}^{(i)} \left(\boldsymbol{H}_k^{(i)} \right)^{\mathrm{T}} + \boldsymbol{R}_k^{(i)} \right| \right)^{-\frac{1}{2}} \\ & \times \exp \left(-\frac{1}{2} \boldsymbol{V}_k^{\mathrm{T}} \left(\boldsymbol{H}_k^{(i)} \boldsymbol{P}_{k|k-1}^{(i)} \left(\boldsymbol{H}_k^{(i)} \right)^{\mathrm{T}} + \boldsymbol{R}_k^{(i)} \right)^{-1} \boldsymbol{V}_k \right) \end{aligned} \tag{19}$$

Considering the parameters $\Delta_k^{(j)}$ and $\pi_k^{(ji)}$ from (4) and (5), we get $\mu_k^{(i)}$

$$\mu_k^{(i)} = L_k^{(j)} \sum_{j=1}^{r} \pi_{k-1}^{(ji)} \mu_{k-1}^{(j)} \bigg/ \sum_{j=1}^{r} L_k^{(j)} \sum_{j=1}^{r} \pi_{k-1}^{(ji)} \mu_{k-1}^{(j)} \tag{20}$$

Step 4. State estimation. The estimated state $\hat{\boldsymbol{X}}_{k|k}$ and its covariance $\hat{\boldsymbol{P}}_{k|k}$ are given by

$$\hat{\boldsymbol{X}}_{k|k} = \sum_{i=1}^{2} \mu_k^{(i)} \boldsymbol{X}_{k|k}^{(i)} \tag{21}$$

$$\hat{\boldsymbol{P}}_{k|k} = \sum_{i=1}^{2} \mu_k^{(i)} \left(\boldsymbol{P}_{k|k}^{(i)} + \left(\boldsymbol{X}_{k|k}^{(i)} - \boldsymbol{X}_{k|k} \right) \left(\boldsymbol{X}_{k|k}^{(i)} - \boldsymbol{X}_{k|k} \right)^{\mathrm{T}} \right) \tag{22}$$

5. **Numerical Study and Discussions.** This section presents the numerical study to validate the proposed IMM algorithm for maneuvering target tracking. Define the initial dynamic state of the target as $\boldsymbol{X}_0 = [5, 20, 5, 10]^{\mathrm{T}}$, the surveillance period is 50 s and the sampling period is 1 s. The target moves from initial position $(5, 5)$ m, the initial model probabilities of the CV and CT motion models are equivalent, and the dynamic states are as follows: i) the target keeps the CV motion with the velocity of $(20, 10)$ m/s during $1^{\mathrm{st}} \sim 10^{\mathrm{th}}$ s, $21^{\mathrm{st}} \sim 30^{\mathrm{th}}$ s and $41^{\mathrm{st}} \sim 50^{\mathrm{th}}$ s; ii) the target follows an anticlockwise 5 °/s CT with the velocity of $(20, 10)$ m/s during $11^{\mathrm{th}} \sim 20^{\mathrm{th}}$ s; iii) the target follows a clockwise 5 °/s CT with the velocity of $(20, 10)$ m/s during $31^{\mathrm{st}} \sim 40^{\mathrm{th}}$ s. The related parameters in (1) and (2) are as follows:

$$\boldsymbol{R}_k^{(i)} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, \quad \boldsymbol{Q}_0^{(i)} = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}, \quad \boldsymbol{\Pi}_0^{(ji)} = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$$

In addition, the root of mean square error (RMSE) is as a valid measure in this scenario.

Figure 1 shows the target track and estimates. In this figure, we note that the proposed IMM algorithm estimates the target in x and y positions accord with the true trajectory even if the target has many switching periods of dynamic states. For comparison, the standard IMM algorithm has position deviations in x and y positions to a certain extent. Figure 2 demonstrates the probability of models of the proposed IMM algorithm. It can be seen that the probability of the CV dynamic model or the CT motion model is greater

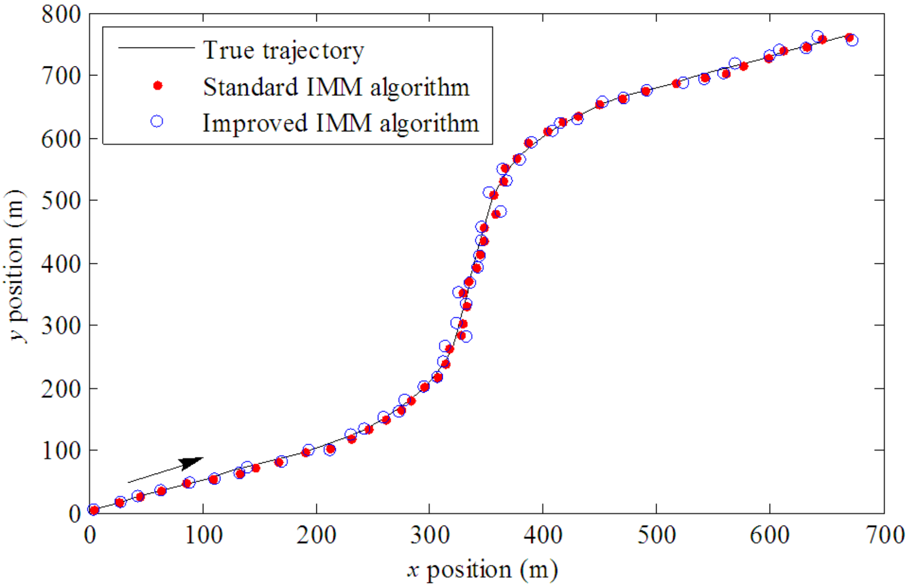

Figure 1. Target track and estimates

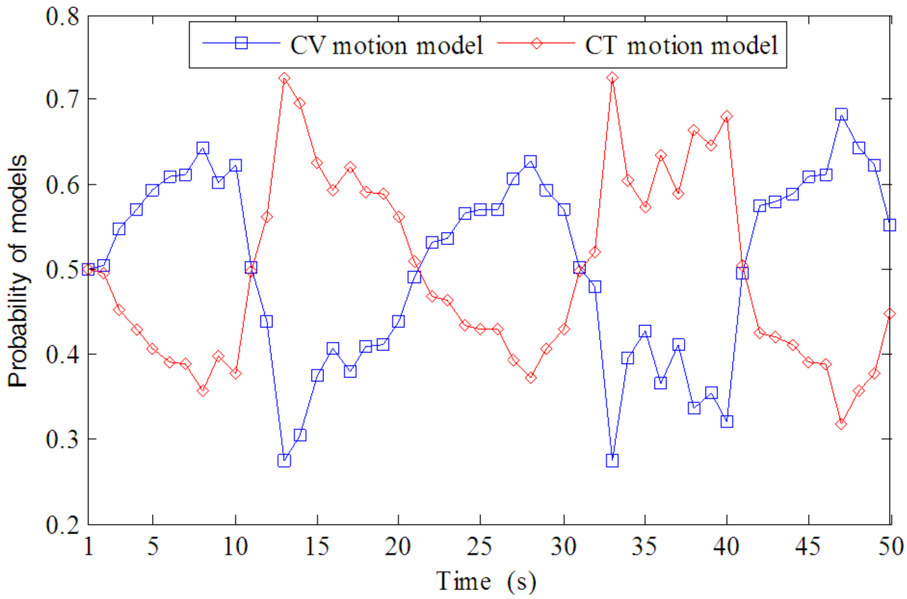

Figure 2. Probability of models

Table 1. Comparison of tracking performance

	RMSE of x position (m)	RMSE of y position (m)	running time (s)
Standard IMM algorithm	6.112	4.736	2.719
Proposed IMM algorithm	1.397	1.168	2.035

in the corresponding period. Two model probabilities are equivalent when the CV and CT dynamic states are switching. It is obvious that the sum of two model probabilities equals 100%. Figure 3 shows the transition probability of models using the proposed IMM algorithm. As seen, during the CV motion stage, the transition probability of the CV motion model is the greatest. In addition, the transition probability from the CT motion model to the CV model is increasing, and the peaks can be achieved on 10$^{\text{th}}$ s, 30$^{\text{th}}$ s and 50$^{\text{th}}$ s. On the other hand, we can arrive at the similar conclusion in the CT

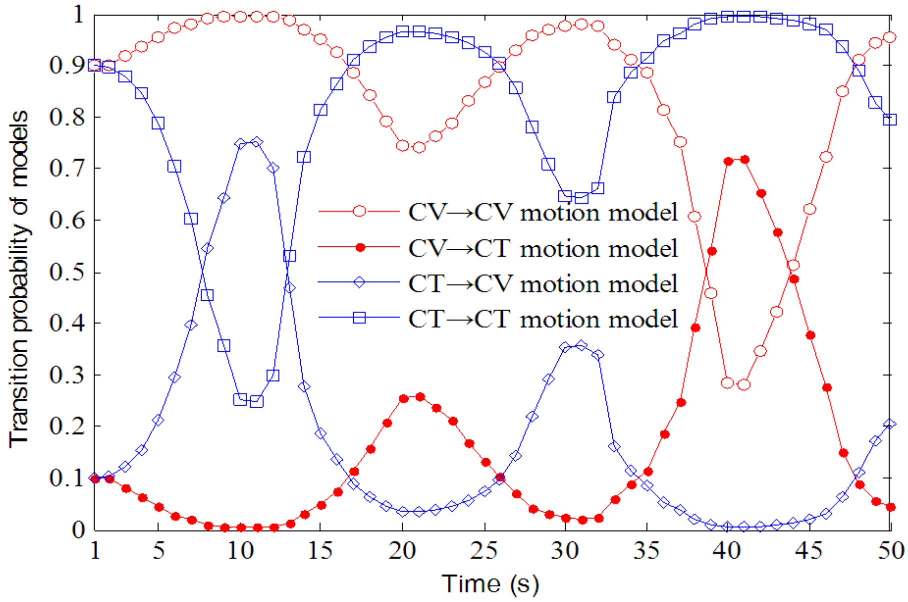

FIGURE 3. Transition probability of models

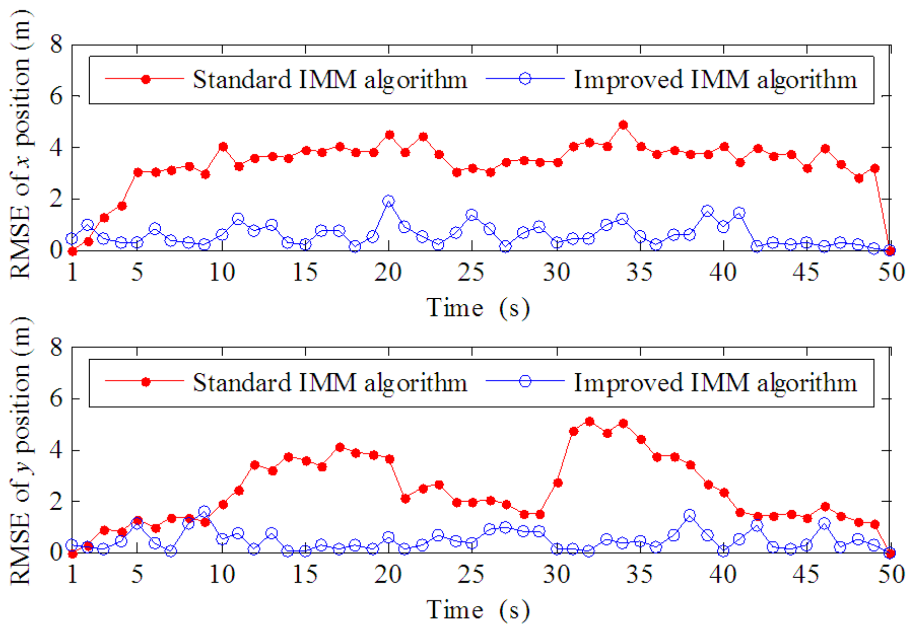

FIGURE 4. RMSE of x and y positions

motion stage. The transition probability of the CT motion model is the greatest and the peaks can be obtained on 20$^{\text{th}}$ s and 40$^{\text{th}}$ s. The reason can be explained that $\pi_k^{(ji)}$ is determined by $\rho_k^{(ji)}$, which approximates the dynamic state and presents the state change when the model is switching. Figure 4 demonstrates the RMSE of x and y positions of two IMM algorithms. It can be seen that the performance of the standard IMM algorithm is worse because it exaggerates the biased position especially when the dynamic state is changing. As for the proposed IMM algorithm, it adaptively adjusts $\boldsymbol{Q}_k^{(i)}$ to meet the change of target maneuvers and gets rid of the assumption of the white Gaussian noise with zero mean in the standard algorithm. Then the RMSE of x and y positions is small whether the motion state is maneuvering or non-maneuvering. Finally Table 1 shows the comparison results of tracking performance with the average values of the RMSE of x and y positions and the running time. It is reported that the parameters with the proposed IMM algorithm reduce by 77.14%, 75.33% and 25.16% respectively compared with the standard IMM algorithm. As a result, the substantial improvement of tracking performance is achieved using the proposed IMM algorithm.

6. **Conclusions.** The challenges are to handle computerized intractability and imprecise estimates of the standard IMM algorithm. We present a novel IMM algorithm to track maneuvering target which employed the improved Markov element and the modified covariance matrix of process noise. The numerical study shows that the proposed IMM algorithm has a significant improvement in both computing time and state estimates with promising tracking results. In the future research of this work, we plan to reduce computational complexity as much as possible under the current tracking precision.

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